

“Unsmooth” 1D inversion of frequency domain marine controlled source EM data

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SUMMARY

The goal of geophysical inversion of electromagnetic (EM) data is to recover a model of the geoelectrical properties of the sub-surface. The standard practice for 1D inversion of marine controlled source EM (CSEM) data is to generate smooth conductivity models using least squares (L_2 -norm) methods. However, sedimentary geology is stratified and piece-wise continuous. As such, smooth resistivity models cannot represent this character. In response to this inconsistency, a means was sought to generate more geologically plausible, piece-wise continuous models.

The common approach in the literature when generating piece-wise continuous inversion models is to regularize L_2 -norm methods in such a manner as to induce blocky behaviour. Although effective, these techniques are self-conflicting; forcing non-smooth behaviour from an implicitly smooth algorithm. In contrast, L_1 -norm inversion inherently produces piece-wise continuous models. To investigate the possible utility of this approach, a L_1 -norm inversion algorithm has been developed and tested on synthetic and real datasets. The L_1 -norm results were compared with those generated using an industry standard L_2 -norm algorithm.

The synthetic inversions focused on previously published examples. The real data inversions focused on electric and magnetic field measurements recorded over the main reservoir sand of the Pluto gas field in block WA350-P, North West Shelf, WA.

The L_1 -norm inversions recovered, to within the resolution limits of the CSEM method, the depth, thickness and resistivity of the synthetic geological models and the Pluto-1 resistivity well log, whilst fitting the input data to within noise. When compared against the L_2 -norm profiles, the L_1 -norm inversion more closely represented the stratified character of the sedimentary sequence. It was therefore concluded that L_1 -norm inversion is an attractive alternative to smooth L_2 -norm methods when blocky inversion models are desired.

Keywords: CSEM, inversion, L_1 , 1D

INTRODUCTION

The controlled source EM (CSEM) method has gained traction within the petroleum industry over the last eleven years for its capability to detect resistive anomalies which may be associated with hydrocarbons. A key component of the interpretation of CSEM data is its inversion into a geoelectrical model of the subsurface. Although higher dimensional

methods are now common, 1D inversion remains relevant due to its algorithmic simplicity and computational efficiency. Much of the CSEM literature has focused on the generation of smooth resistivity models using least squares (L_2 -norm) methods, e.g Constable et al. (2008), Key(2009), Key and Lockwood (2010). However, sedimentary geology is stratified and its resistivity is piece-wise continuous. If the goal of the inversion is to generate geologically representative profiles, smooth resistivity models are not be desirable. A common approach is to regularize L_2 -norm methods in such a manner as to induce blocky behaviour. Although effective, these techniques are self-conflicting: demanding non-smooth behaviour from an implicitly smooth algorithm. In contrast linear programming (L_1 -norm) inversion inherently produces piece-wise continuous models. However, with the exception of Christensen and Dodds (2007), little has been published regarding L_1 -norm inversion in CSEM. Therefore to investigate the possible utility of this approach, a L_1 -norm inversion algorithm has been developed. This algorithm has been tested on synthetic and real CSEM datasets and the results compared with the smooth inverted models produced by an industry standard L_2 -norm algorithm.

METHOD

Overview

In order to examine the capability and limitations of L_1 -norm methods, two studies were performed. For the synthetic study, L_1 and L_2 -norm inversions were run on two synthetic datasets using the same starting model and a target misfit of 1%. For the real data study, L_1 and L_2 -norm inversions were run using similar starting models and a target misfit of 5%. For the synthetic study the inverted models were evaluated against the known geoelectrical profile. For the real data study the inverted models were assessed against the resistivity logs from a well through the main Pluto reservoir.

The L_1 -norm inversions were performed using *VPcsem1D* (Fullagar Geophysics Pty Ltd), a 1D forward modelling and inversion program developed for purpose. The L_2 -norm inversions were performed using *Occam1DCSEM* (Key 2009). Both programs make use of the isotropic conductivity point dipole frequency domain full tensor E and B field formulation of Key (2009). *VPcsem1D* utilises a minimum L_1 -norm inversion scheme formulated below (Fullagar & Oldenburg, 1984). Variations of this scheme have previously been applied in Fullagar Geophysics Pty Ltd 1D TEM inversion program *AMITY*. For details of the *Occam1DCSEM* L_2 -norm inversion algorithm the reader is referred to Constable et al. (1987).

Inversion Methodology

The inversion module solves the inverse problem in the conventional manner by iteratively adjusting a layered starting model. Corrections ($\Delta\sigma$) to the layer conductivities (σ) are computed in order to reduce the misfit between the calculated

model response and the N data. The program utilises a linear programming routine to solve

$$[\vec{d}_N - \vec{c}_N] \circ \frac{1}{\vec{\epsilon}_N} = [J_{N,K} \quad I_{N,N}] \begin{bmatrix} \Delta \vec{\sigma}_K \\ \vec{\eta}_N \end{bmatrix} \circ \frac{1}{\vec{\epsilon}_N} \quad (1)$$

where \vec{d}_N is a vector containing the N observations, \vec{c}_N a vector containing the N calculated data, $\vec{\epsilon}_N$ a vector containing the N associated standard deviations, $J_{N,K}$ a Jacobian which relates the change for the k^{th} conductivity in model space to the n^{th} datum in data space, $I_{N,N}$ a order N identity matrix, $\Delta \vec{\sigma}_K$ a vector containing the conductivity corrections and $\vec{\eta}_N$ is a vector of data misfits. The fit of measured to calculated data is gauged using an L_1 -norm, i.e.

$$\sum_{n=1}^N \eta_n = \frac{1}{N} \sqrt{\frac{\pi}{2}} \sum_{n=1}^N \left| \frac{d_n - c_n}{\epsilon_n} \right| \quad (2)$$

If the data errors are realisation of independent Gaussian random variables with zero mean and standard deviations (ϵ), the expected value of Equation 2 is unity. The explicit inclusion of misfits allows for the imposition of the L_1 -norm condition on the solution and ensures that the problem is under-determined. Equation 1 is solved for the unknown conductivity corrections and misfits. Inversion proceeds iteratively, and the algorithm tries to reduce the L_1 -norm to some fraction (usually half) of its current value. For each iteration the optimal solution was defined as the one that minimises,

$$\Psi = \sum_{k=1}^K |\Delta \sigma_k| \quad (3)$$

The inversion continues until Equation 2 is less than or equal to unity or stalling has occurred. Derivatives for the inversion are computed via a two point centred difference approximation, i.e.

$$\frac{\partial F(\sigma_k)}{\partial \sigma_k} \approx \frac{f(\sigma_k + \delta \sigma_k) - f(\sigma_k - \delta \sigma_k)}{2\delta \sigma_k} \quad (4)$$

where $\delta \sigma_k$ is a small perturbation of the model parameter k .

Geological Models

Two geological models, previously examined by Key(2009) and others, were used for the synthetic study (Figure 1). The first, a canonical 1D reservoir model, consisted of a 100m thick, 100 Ω m resistive reservoir at 1 km depth below the sea floor. The second, a multiple reservoir model, consisted of the same reservoir with the addition of a 25 m thick, 5 Ω m gas hydrate on the ocean floor and a 50 m thick, 10 Ω m secondary reservoir at 500 m depth below ocean bottom. For both models the host is 1 Ω m saline water saturated sediment beneath a 0.3 Ω m, 1 km thick ocean column. An infinite basement extends below a depth of 3 km from the sea floor, its resistivity 1 Ω m for the first model and 10 Ω m for the second model.

Synthetic Data

After Key (2009), synthetic data were generated for a single receiver positioned on the sea floor. The data, normalised to the transmitter-moment, were generated at 50m stations for the fundamental (0.1 Hz) harmonic of a y-directed horizontal electric point dipole (HED) with a 100% duty cycle square wave source current ‘flown’ at 25m above the sea floor. The transmitter was towed away from the receiver over each of the geological models. For perfect in-line geometry and isotropic

1D conductivities, a y-directed HED excites in-line and vertical E-field and cross-line B-field components. Gaussian distributed random noise was generated with an amplitude of 1% of the signal. Independent realizations of the noise were added to the real and imaginary components subject to a minimum noise floor of $10^{-18} \text{TA}^{-1} \text{m}^{-1}$ for the normalised magnetic and $10^{-15} \text{VA}^{-1} \text{m}^{-2}$ for the normalised electric field data. Although near offset in-line data are well above this noise floor, the 1% error represents fluctuations due to imprecision in the source-receiver geometry as a result of navigational error. Data was observed to have dropped below the noise floor (i.e. S/N approached 0) at offsets greater than 7 km in-line and were therefore eliminated.

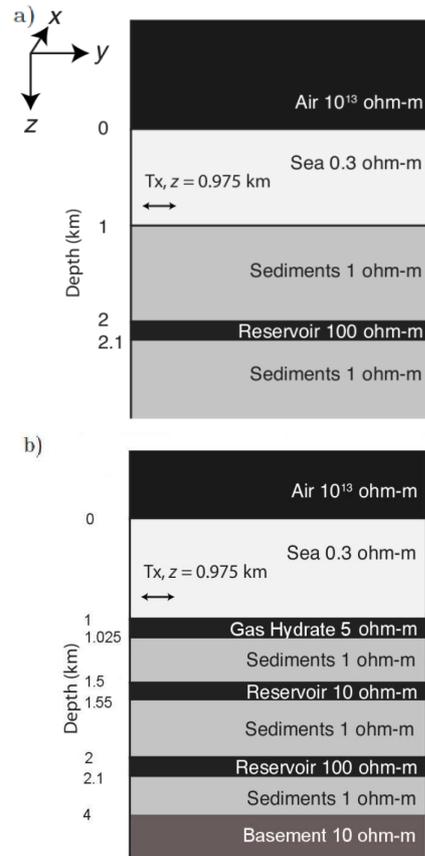


Figure 1. The (a) canonical and (b) multiple 1D reservoir models used for the synthetic inversion study. Modified from Key (2009).

Real Data

After Key and Lockwood (2010) the real data consisted of horizontal in-line E field, and cross-line magnetic field recorded by EMGS for a single receiver, number 11 on transmitter line 1 (01Rx11), positioned on the sea floor at 1003.9m water-depth over the main Pluto reservoir sand (209 m thick at 3.1 km depth below sea level) in block WA350-P, North West Shelf, Western Australia. Only the out-tow data (recorded whilst the transmitter is towed away from the receiver; positions north of 01Rx011) were used as the in-tow data suffered from significant noise due to irregular sea-floor bathymetry. The third (0.24 Hz) and fifth (0.4 Hz) harmonic were used in the subsequent inversion analysis for consistency with Key and Lockwood (2010). The data had been processed in the standard manner; which included normalisation by the

transmitter moment and correction for rotation of the receivers from in-line geometry during deployment (Wicklund, 2007). A relative error of 5% was assigned to the data subject to a minimum absolute noise floor of $10^{-18} \text{VA}^{-1} \text{m}^{-1}$ for the normalised magnetic and $10^{-15} \text{VA}^{-1} \text{m}^{-2}$ for the normalised electric field data. A polynomial fit of the data was performed along profiles. Points that showed large deviations from the trend were assigned standard deviations such that the error bars encompassed the trend line. Data from locations with an in-line offset less than 1.5 km were removed as they contained little information at reservoir depths and suffered from increased navigation error (Lockwood, 2010 *pers. comms.*). The electric field data was smoothly varying with in-line offset (i.e. exhibited good S/N) out to 8km at 0.24 Hz and 6 km at 0.4 Hz. The magnetic field data was smoothly varying with in-line offsets out to 5km for both frequencies. Large errors, in excess of the noise floor, were assigned to the magnetic field data with greater than 5 km in-line offsets to ensure they had minimal influence on the inversion result. This was done because *VPcsem1D* did not allow for differing numbers of data between components. Data from stations with in-line offsets greater than 8km were eliminated.

Starting Models

For the synthetic inversion study the *VPcsem1D* starting model consisted of forty $1 \Omega\text{m}$ layers, ranging from 1 to 4 km depth below sea level, with logarithmically increasing thicknesses ranging from 25 to 240m. The thickness of the layers at the depths of the target reservoirs were altered so that layer boundaries precisely matched those of the true model. Although an optimal case, the inversion algorithm was found to robustly recover the depth, resistivity and thickness of the reservoir without this modification (Godber, 2010). The *VPcsem1D* starting model for the Pluto inversion consisted of fifty $0.5 \Omega\text{m}$ layers between 870 and 6000 m with thicknesses increasing logarithmically from 40 to 220m. Three constant thickness layers at the base of the water column were allowed to vary. The remainder were fixed at $0.22 \Omega\text{m}$. For the synthetic study, the *Occam1DCSEM* starting model consisted of seventy five $1 \Omega\text{m}$ layers ranging from 1 to 4.5 km depth below sea level with logarithmically increasing thickness layers. This was chosen for consistency with Key (2009). The *Occam1DCSEM* starting model for the Pluto inversion consisted of one hundred and eight, $0.5 \Omega\text{m}$ layers ranging from 870 to 8000 m depth below sea with a constant thickness of 72m. This was chosen for consistency with Key and Lockwood (2010).

RESULTS

Synthetic study: Canonical reservoir inversion

The results of the canonical reservoir synthetic data inversion are presented in Figure 2 and Figure 3. The *VPcsem1D* inversion converged from an initial L_1 -norm misfit of 32 to an acceptable misfit of 0.8 in 17 iterations. The *Occam1DCSEM* inversion converged from an initial L_2 -norm misfit of 25 to 1 in 27 iterations.

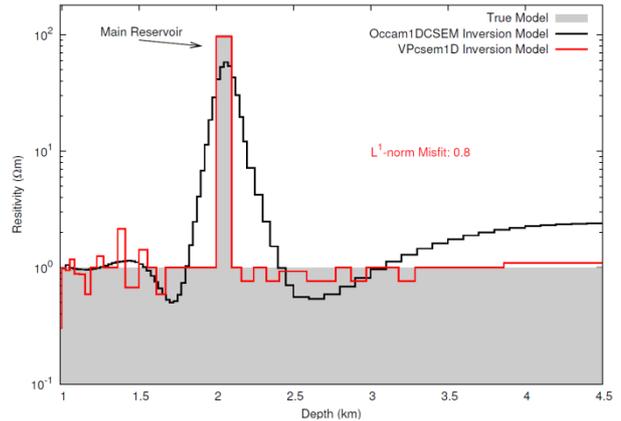


Figure 2. Synthetic study inverted model for 0.1 Hz canonical reservoir data with 1% Gaussian noise.

Both inversions fit the data to within 1% and recovered the approximate depth and resistivity of the target reservoir. The thickness of the target reservoir was closely approximated by the *VPcsem1D* inversion whilst the *Occam1DCSEM* suggested a thicker, broadly varying reservoir. Key (2009) found that the inclusion of a second frequency markedly improved the *Occam1DCSEM* thickness estimate. This was not observed to occur with *VPcsem1D* (Godber,2010). The L_1 -norm exhibited the piece-wise continuous character of the true profile whilst the smoothly varying L_2 -norm did not.

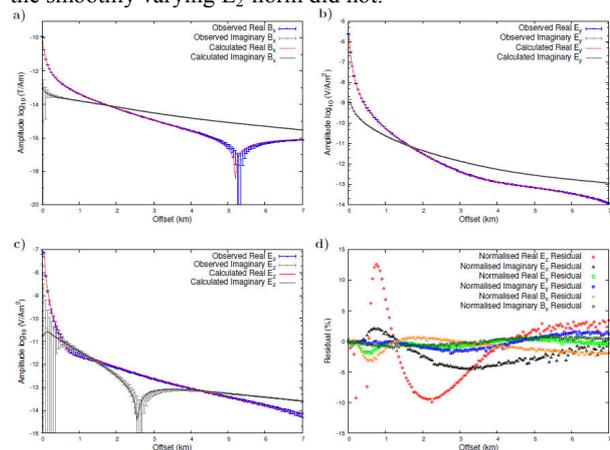


Figure 3. Observed and final *VPcsem1D* inversion model response for canonical reservoir synthetic inversion study. (a) B_x ; (b) E_y ; (c) E_z . (d) Normalized residuals. Error bars indicate attached standard deviation

Synthetic study: Multiple reservoir inversion

The results of the multiple reservoir synthetic data inversion are presented in Figure 4 and Figure 5. The *VPcsem1D* inversion converged monotonically from an initial L_1 -norm misfit of 53 to an acceptable misfit of 0.8 in 21 iterations. The *Occam1DCSEM* inversion converged from an initial L_2 -norm misfit of 28 to 1 in 27 iterations. Both inversions fit the data to within 1% and effectively recovered the depth and resistivity of the main reservoir and gas hydrate. Interestingly, both also underestimated the thin reservoirs resistivity and depth whilst overestimating its thickness. This result suggested that there may be insufficient resolution in the dataset. *Occam1DCSEM* more closely recovered the basement resistivity than *VPcsem1D*. Notably, *VPcsem1D* introduced a spurious resistive anomaly between the thin and main reservoir.

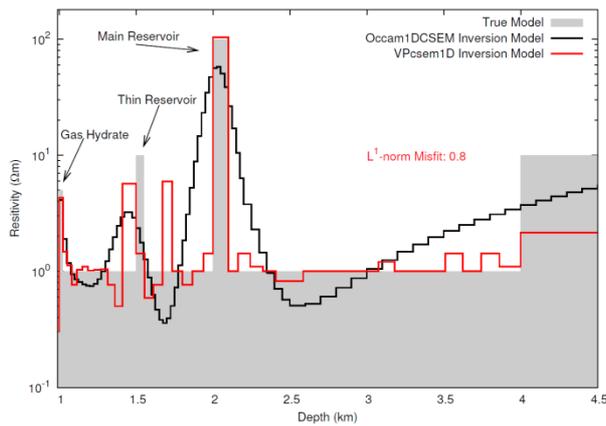


Figure 4. Synthetic study inverted model for 0.1 Hz multiple reservoir data with 1% Gaussian noise.

As was observed in the canonical inversion, the L_1 -norm inverted model exhibited the piece-wise continuous character of the true profile whilst the smoothly varying L_2 -norm did not. This behaviour was not observed in any other results and was not a consistent feature of the approach (Godber, 2010).

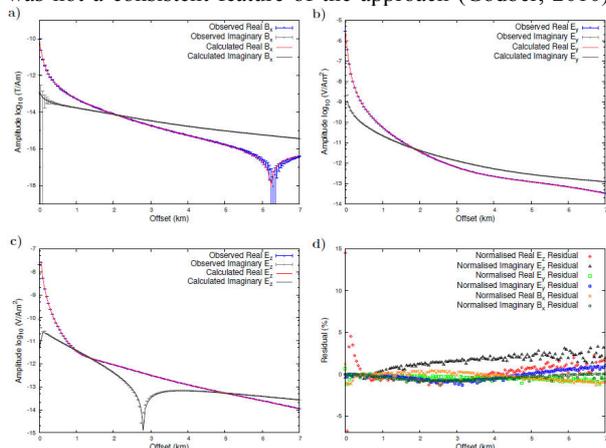


Figure 5. Observed and final *VPcsem1D* inversion model response for multiple reservoir synthetic inversion study. (a) B_x ; (b) E_y ; (c) E_z . (d) Normalized residuals. Error bars indicate attached standard deviation.

Real data study: Pluto inversion

The results of the real data inversion are presented in Figure 6. The *VPcsem1D* inversion converged from an initial L_1 -norm misfit of 4.0 to a final misfit of 1.4 in 64 iterations. The *Occam1DCSEM* inversion converged from an initial L_2 -norm misfit of 2.1 to 1.0 in 29 iterations. The *VPcsem1D* inversion managed to recover the bulk resistivity and thickness of the resistive reservoir package, as measured by the Pluto-1 resistivity well log, but underestimated the depth of burial by ~200m (representing 6% of the actual depth). The *Occam1DCSEM* inversion picked the same depth of the reservoir (measured by the peak resistivity in the target depth window) as *VPcsem1D*, but yielded a lower estimate of the bulk resistivity and an unclear assessment of the thickness.

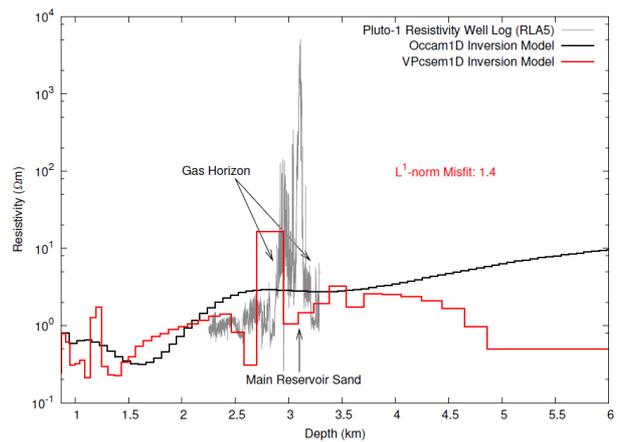


Figure 6. *VPcsem1D* (red) and *Occam1DCSEM* (black) inversion result for Pluto 1.5 to 7 km 0.24 and 0.4 Hz data with 5% assumed noise.

Notably the *Occam1DCSEM* results published in Key and Lockwood (2010) accurately recovered the depth, thickness and resistivity of the Pluto. Communications with the authors (Key & Lockwood, 2010 *pers. comms.*) indicated that additional processing of the data had yielded further usable far offsets which had improved the inversions depth and thickness estimate. Although the time limitation of honours prevented further investigation, it was considered probable that the lack of offset in the input data had yielded the shallow depth result. *VPcsem1D* raised the resistivity of two adjoining layers to indicate the presence of the target reservoir demonstrating thickness sensitivity in the methodology. The *VPcsem1D* inversion failed to achieve an L_1 -norm misfit of 1.0, but it was later realised that some cross-line 0.24 Hz magnetic field data had been assigned erroneously small standard deviations. With the data assigned appropriate uncertainties, an L_1 -norm misfit of 1.0 was achieved. As in the synthetic geological models, the reservoir consists of a hydrocarbon charged sedimentary formation with distinct boundaries. These boundaries, marked by the gas horizon pointers in Figure 6, exhibited as abrupt changes in the resistivity logs. The *VPcsem1D* inversion model recovered these abrupt boundaries, whilst the smooth *Occam1DCSEM* model did not.

CONCLUSIONS

A minimum L_1 -norm 1D CSEM inversion algorithm has been developed and implemented in the program *VPcsem1D*. When applied to synthetic and real marine CSEM data sets the algorithm recovered the depth, resistivity and thickness of resistive targets and fitted the data to within noise. When compared against smooth L_2 -norm inverted profiles, the L_1 -norm models exhibited the sharp contrasts in resistivity which typically occurs across reservoir boundaries and between some of the other sedimentary formations. It was therefore concluded that L_1 -norm inversion is an attractive alternative to smooth L_2 -norm methods when blocky (“unsmooth”) inversion models are desired.

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REFERENCES

Christensen, N. B., and K. Dodds, 2007, 1D inversion and resolution analysis of marine CSEM data: *Geophysics*, 72, 27-38.

Constable, S.C., R. L. Parker, and C. G. Constable, 1987, Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data: *Geophysics*, 52, 289-300.

Fullagar, P. K., and D. W. Oldenburg, 1984, Inversion of horizontal loop electromagnetic frequency sounds: *Geophysics*, 49, 150-164.

Godber, N., 2010, Minimum L^1 -Norm inversion of Marine Controlled Source EM data: Unpublished B.Sc.Hons. thesis, University of Queensland.

Key, K., 2009, 1D inversion of multicomponent, multifrequency marine CSEM data: Methodology and synthetic studies for resolving thin resistive layers: *Geophysics*, 74, 9-20.

Key, K., and A. Lockwood, 2010, Determining the orientations of marine CSEM receivers using orthogonal procrustes rotation analysis: *Geophysics*, 75, 63-70.

Wicklund, T.A., 2007, Seabed Logging Processing Report: Flanders, NWS Australia. EMGS