



Issues related to determination of the horizontal centre of magnetization

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SUMMARY

The direction of magnetization of a compact source causing a magnetic field anomaly can be found without concern for the details of its shape using magnetic moment analysis (MMA). This provides assistance in the successful inversion of magnetic anomalies due in part to remanent magnetization of unknown direction. However, the success of MMA is dependant on the analysis being positioned appropriately over the horizontal centre of the source body. MMA itself can derive an estimate of the horizontal centre of magnetization and in this presentation we investigate the conditions for stable convergence of an iterative solution that progressively steps the analysis to a revised horizontal centre of magnetization. We find that reliable magnetization directions can be recovered for grid widths down to twice the source depth and less. One, or at most two iterations of the analysis should locate the horizontal centre of magnetization over a compact source to within 5% of its depth and with a resulting uncertainty in magnetization direction of the order of 5°.

Key words: magnetic moment, centre of magnetization, remanent magnetization, magnetization direction

INTRODUCTION

For compact sources magnetic moment analysis (MMA) provides estimates of resultant magnetization direction that can be input as a constraint to magnetic field inversion, reducing the complication of estimating both magnetization direction and geological structure from the inversion itself. MMA can be performed using vector components of the field (for example, Helbig, 1963; Schmidt and Clark, 1998; Phillips, 2005; Foss and McKenzie, 2011a) or from gradient tensor elements (for example, Phillips *et al*, 2007; Foss and McKenzie, 2011b). Gradient tensor MMA has the advantage that removal of the regional field is less problematic and that gradient data is better sampled in the case of adjacent and overlapping anomalies. However, as noted by Foss and McKenzie (2011b) the sharper fall-off of gradient anomalies can be expected to increase the penalty of errors that arise from any mislocation of the horizontal centre of magnetization. We have implemented an extension of MMA by Caratori Tontini and Pedersen (2008) to estimate the

horizontal location of the centre of magnetization in the hope that this may provide an iterative method to refine estimated magnetization directions. In this paper we report on systematic studies to evaluate convergence of successive estimates of the horizontal centre of magnetization and the reliability of magnetization directions obtained from MMA as a function of residual error in positioning. These results are required to establish resolution limits for MMA studies that we are now performing routinely on Australian magnetic field data as part of a collaborative study between CSIRO and Geoscience Australia (Foss *et al*, 2012). The objective of these studies is to catalogue resultant magnetization directions across Australia as an aid to magnetic field interpretation.

Caratori Tontini and Pedersen's method to locate the horizontal centre of magnetization

Consider a compact magnetic source with finite volume v , uniform magnetization \mathbf{J} centred at (x_c, y_c, z_c) whose anomalous magnetic field is $\mathbf{B}(\mathbf{r}) = [B_x(\mathbf{r}), B_y(\mathbf{r}), B_z(\mathbf{r})]^T$. Here the superscript T denotes transposition and the x, y, z coordinates are defined within a right-hand clockwise coordinate (NED) system, i.e. x -East, y -North and z -vertically downward, which is identical to the IGRF coordinate system. Then the zero order vector magnetic dipole moment $\mathbf{M} = (M_x, M_y, M_z)^T$ may be expressed (see for example, Medeiros and Silva, 1995; Caratori Tontini and Pedersen, 2008):

$$M_x = \iiint_v J_x(\mathbf{r}') dv; \quad M_y = \iiint_v J_y(\mathbf{r}') dv; \quad M_z = \iiint_v J_z(\mathbf{r}') dv \quad (1)$$

where $\mathbf{r}' = (x', y', z')^T$ is the position vector of a volume element dv within the magnetized source body and $\mathbf{r} = (x, y, z)^T$ the position vector of an arbitrary measurement point. Helbig (1963) has shown that the magnetic dipole moment \mathbf{M} of the disturbing magnetic source can be estimated using the first moments of its anomalous magnetic field $\mathbf{B}(\mathbf{r})$ at all points $\mathbf{r} = (x, y, z_0)$ on the horizontal plane $z = z_0$. For the M_x component we use the x moment of $B_z(\mathbf{r})$,

$$M_x = (2/\mu_0) I_{xBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x B_z(x, y, z_0) dx dy \quad (2a)$$

and for the M_y component moment we use the y moment of $B_z(\mathbf{r})$,

$$M_y = (2/\mu_0) I_{yBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y B_z(x, y, z_0) dx dy \quad (2b)$$

and for the M_z component we may use the x moment of $B_x(\mathbf{r})$ or the y moment of $B_y(\mathbf{r})$

$$M_z = (2/\mu_0) I_{xBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x B_x(x,y,z_0) dx dy \quad (2c)$$

$$M_z = (2/\mu_0) I_{yBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y B_y(x,y,z_0) dx dy \quad (2d)$$

and where C_m is a constant which is 10^{-2} for the SI system in which the magnetic fields are expressed in units of nanotesla and the magnetic moment is expressed in Am^2 . Caratori Tontini and Pedersen (2008) have shown that expressions for the centre of mass of magnetization distribution at (x_c, y_c, z_c) may be derived by taking combinations of the second order moments of the anomalous magnetic field components of $\mathbf{B}(\mathbf{r})$. For the horizontal centre of magnetization (x_c, y_c) the expressions are as follows :

$$x_c = [I_{xyBz} I_{yBz} + \frac{1}{2} (I_{xxBz} - I_{yyBz}) I_{xBz}] / (I_{xBz}^2 + I_{yBz}^2) \quad (3a)$$

$$y_c = [I_{xyBz} I_{xBz} - \frac{1}{2} (I_{xxBz} - I_{yyBz}) I_{yBz}] / (I_{xBz}^2 + I_{yBz}^2) \quad (3b)$$

where

$$I_{xyBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy B_z(x,y,z_0) dx dy \quad (4a)$$

$$I_{xxBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 B_z(x,y,z_0) dx dy \quad (4b)$$

$$I_{yyBz} = -(C_m/2\pi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^2 B_z(x,y,z_0) dx dy \quad (4c)$$

Influence of grid size on MMA

MMA involves the taking of integrals. These integrals should ideally have infinite extent but clearly this is never the case. Schmidt and Clarke (1998) reported that the estimated direction of magnetization is quite robust for grids of limited size (but which are still correctly centred) and provided a table of the scaling factor required to obtain the true moment amplitude of a dipole according to the ratio between grid size and source depth. They give a value of 73% for the moment recovered from a grid width to source depth ratio of eight. In previous studies (Foss and McKenzie 2011a, 2011b) we also found consistently correct magnetization direction estimates from small grids. At variance with these findings Caratori Tontini and Pedersen (2008) report that for a grid width twenty times the source depth only 60% of the moment is recovered and they report a 20% error in the estimated inclination of magnetization for a grid width to source depth ratio of 4. To resolve these issues we performed a more comprehensive set of MMA tests across a range of grid size to source depth ratios. Using exact component and tensor grids forward computed from a source model of specified magnetization direction we found that the amplitude of the dipole moment derived from the components method as shown in Figure 1 agrees reasonably well with the theoretical value of 73% for a grid width to dipole depth ratio of 8 as tabulated by Schmidt and Clark (1998). A little surprisingly we found systematically lower amplitudes estimated from the tensor method (we had expected higher values than from the

components analysis because of the greater compactness of tensor anomalies) but this may possibly be explained by the approximate nature of the tensor analysis (Phillips *et al*, 2007). More importantly we confirmed our earlier finding that MMA has the potential to reliably recover magnetization direction from small grids, and validated this across a wide range of magnetization directions. Figure 2 shows that the error in recovered magnetization direction increases from negligible values for large grids to values for a grid with width equal to source depth of only 0.2° and 1.2° from component and tensor MMA respectively. These results are consistent for magnetizations of low and steep, and positive and negative inclinations. Figure 3 shows the error in the MMA estimate of the horizontal centre of magnetization. This error is also small, but does vary with magnetization direction. The errors are

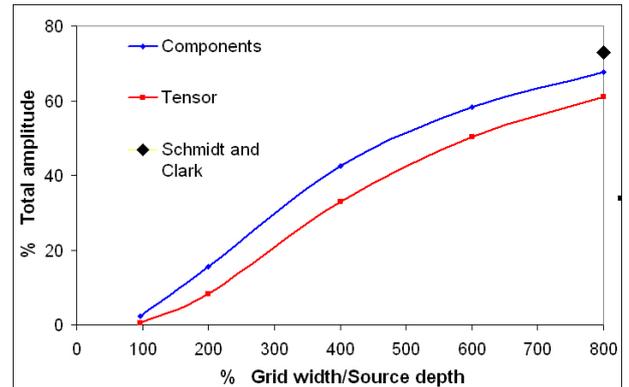


Figure 1. Recovered MMA as a function of grid size

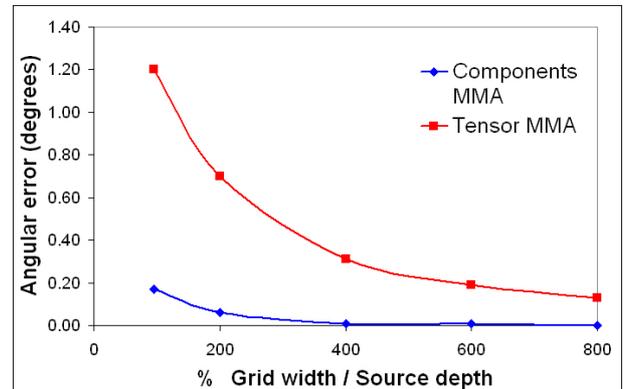


Figure 2. Error in magnetization direction from MMA of computed grids

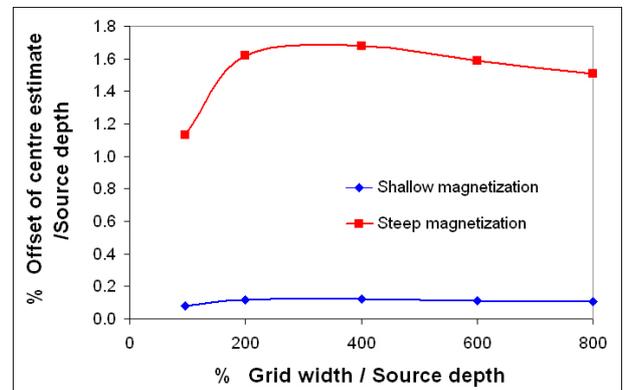


Figure 3. Error in magnetization direction from MMA of computed grids

negligible for the -15° low inclinations and larger (but still less than 2% of source depth) identically for both polarity steep inclinations of $\pm 75^\circ$ that were tested. MMA of field data requires estimation of the components by FFT methods (at least until tensor gradiometry surveys become more prevalent). Any limitations arising from the FFT methods must therefore be incorporated into any evaluation of practical resolution capabilities of MMA applied to field data. These FFT limitations clearly increase with decreasing grid size as illustrated by Foss and McKenzie (2011b). The error in estimation of the horizontal centre of magnetization from component grids derived by FFT filtering of TMI is plotted in Figure 4. For the low inclination magnetizations the FFT processes add an almost constant increase to the error in horizontal location of the centre of magnetization (to values still only up to 0.5% of source depth). For the steep inclination magnetizations the error in location increases to 2.5% and appears to diverge from the location estimated from analytic inputs at larger grid widths. This may illustrate limitations either in the analysis or in our implementation of it.

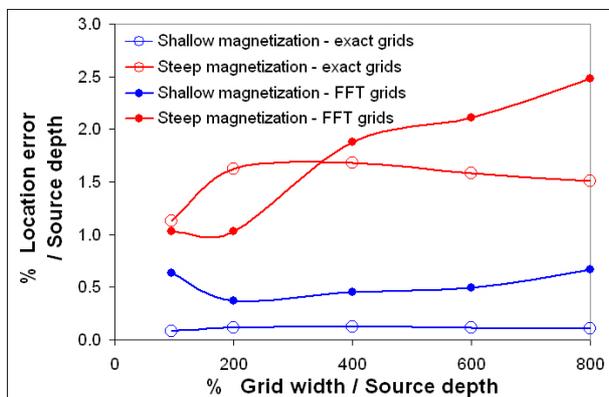


Figure 4. Error in source location using FFT-derived grids

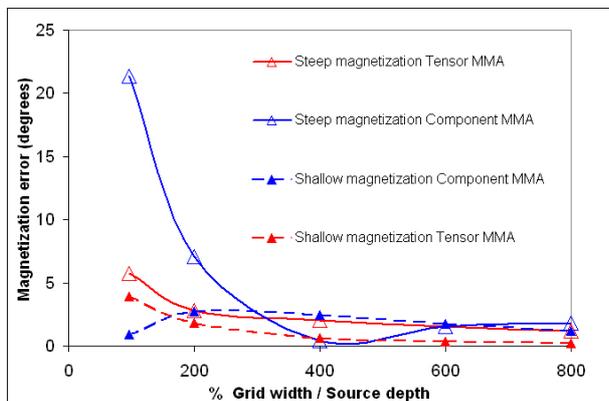


Figure 5. Error in magnetization direction using FFT-derived grids

Figure 5 shows the error in magnetization direction using FFT grid inputs to both component and tensor MMA techniques. We have previously (Foss and McKenzie, 2011b) reported errors in recovering magnetization directions from FFT derived grids over monopole-type sources (vertical pipes) of up to 40° from component MMA and 5° for the tensor MMA at a grid width to source depth ratio of 1 (in that case measuring depth to the top of the pipe). As shown in Figure 4 at the same grid width to source depth ratio of 1 we have a direction of magnetization error of slightly over 20° from the dipole source estimated from component MMA for the steep

inclination magnetizations (a similar inclination of magnetization used in the pipe study). For a low inclination magnetization (not tested in the previous pipe study) the equivalent error in the component MMA at a grid size to source depth ratio of 1 is only 5° . As reported from the previous pipe study, the tensor MMA provides substantially superior results for small grids. Magnetization directions estimated by tensor MMA from small grids also show larger errors for steep compared to shallow magnetization directions, but this difference is much more subdued and the maximum error is only 6° . The major finding of this new study with respect to the influence of grid size is that the sharp increase in error of magnetization directions estimated by component MMA of small grids (which is introduced by the FFT process) increases significantly with increase in inclination of the magnetization. Error in the FFT process for small grids substantially arises from the procedures used to pad the grids. In this study we used 50% minimum curvature padding which is added for the FFT processes and then subtracted prior to the numeric integration of the MMA computations. An improved padding method is clearly required to recover reliable estimates of magnetization direction from component MMA of grids with side length to source depth ratio of 1 and less. Caratori Tontini and Pedersen (2008) recommend the use of an equivalent dipole source to generate padding but we have not as yet incorporated this into our analysis.

MMA results from off-centred analyses

Having established that, with care, grids of side length down to as low as twice the source depth can be used to recover reasonable estimates of magnetization direction and horizontal centre of magnetization, we investigated the consequence of starting an analysis off-centred by 40% of the source depth. To establish the fundamental sensitivity of the analysis we again started with exact component and tensor gradient grids forward computed from a source model and then repeated the analysis with grids derived from FFT of TMI. The results of offsetting MMA from above the horizontal centre of magnetization are also dependent on direction of magnetization. For low inclination magnetizations the location and magnetization direction errors are highly sensitive to the direction of offset. Offsets of sources with steep magnetization are less sensitive to offset direction but are larger in amplitude, with offsets of 75° inclination magnetizations of both polarities causing approximately three times the error in magnetization direction than equivalent offsets of 15° inclination magnetizations.

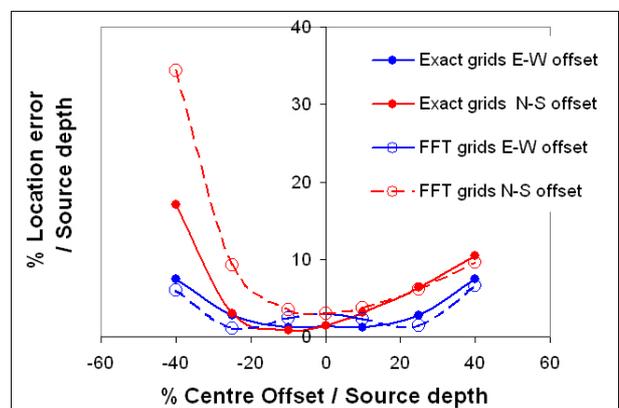


Figure 6. Error in source location for a steep (75°) magnetization and W-E offset

Figure 6 shows the error in estimated horizontal centre of magnetization from our implementation of Caratori Tontini and Pedersen's method for magnetizations of $\pm 75^\circ$ inclination across a range of S-N and W-E offset analyses. The results for west-east offsets are symmetric as expected from the W-E symmetry of the magnetic field, but north-south offsets produce asymmetric errors, with larger errors for southerly offsets seen both in the fundamental analysis with exact grids and then amplified in the analysis using FFT derived grids. Figure 6 is strategically significant because it indicates the convergence for an iterative analysis, with the offset error of the input centre estimate read off the X-axis and the resulting output error read from the Y-axis. If an analysis starts with a horizontal offset of up to 40% of source depth then in most cases a single iteration, and in all cases two iterations, would reduce the location error to less than 5% of source depth.

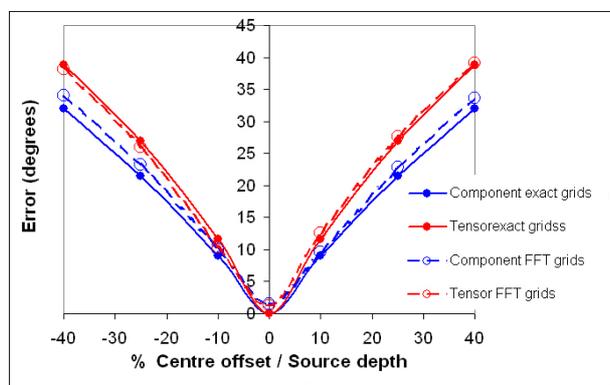


Figure 7. Error in magnetization direction for a steep (75°) magnetization and W-E offset

Figure 7 plots the angular error in the estimated magnetization direction as function of the centre offset. If the centre of magnetization offset is reduced to no more than 5% of source depth then the resulting error in magnetization direction should be less than 5° . This section of the analysis was performed with grid widths of eight times the source depth and in this case use of FFT derived grids and exactly computed grids gave similar errors. The amplitude of the errors for low inclination magnetizations were between two and three times less (so that those magnetization directions should be resolved to better than 2°).

CONCLUSIONS

We have mapped the fundamental capabilities of MMA in recovering magnetization direction and centre of magnetization from exact component and tensor grids computed from a source model, and also the practical capabilities of MMA in using grids derived from FFT of TMI.

We have shown that a simple one or two step iterative analysis should be sufficient to refine an analysis starting from a horizontal offset of 40% of source depth to within 5% of source depth. We have also shown that the magnitude of errors in estimating magnetization direction and centre of magnetization are dependent on direction of magnetization with higher amplitude errors for steep inclination magnetizations (this result may be different for geomagnetic inclinations different to the -60° value used throughout this study).

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