Relativistic Effects in Low-energy Electron–Argon Scattering*

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Abstract

We have performed a relativistic treatment at low energy of electron–argon scattering which includes both polarisation and dynamic distortion effects. Our results are in excellent agreement with the experimentally derived momentum transfer cross section and scattering length, as well as with very recent measurements of the elastic differential cross section.

1. Introduction

Although there have been extensive experimental and theoretical investigations of elastic electron scattering from the noble gases for many years, there are still discrepancies between the various theories and experiments for the heavier noble gases, argon through xenon. Recently there have been new experiments and theoretical calculations concerned with low-energy electron–argon collisions. In particular, Petrović *et al.* (1995) have carried out drift velocity measurements of very low energy electrons in an argon–molecular hydrogen mixture. The subsequent analysis of these data yielded a new value for the electron–argon scattering length which the authors feel should be reliable to within 1%. Even more recently, Gibson *et al.* (1996) have performed absolute measurements of the electron–argon differential cross section, using a crossed-beam technique, at energies between 1 and 10 eV for the incident electron. One of the primary aims of this latter investigation was to see if there is any convergence between the various experimental and theoretical predictions. If this were the case, then perhaps argon could also be used as a 'secondary standard' in absolute elastic scattering experiments, using the relative flow technique, for calibrating/checking the operation of a crossed-beam apparatus. Furthermore, argon is an important gas in many aspects of gaseous electronics with applications in particle detectors, lasers and plasma etching and deposition [see for example Petrović *et al.* (1995) and references cited therein]. The momentum transfer cross section for argon has been experimentally determined by Frost and Phelps (1964), Milloy *et al.* (1977), Haddad and O'Malley (1982) and Nakamura and Kurachi (1988), while

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previous measurements of the differential cross section include those of Williams (1979), Srivastava *et al.* (1981), Zhou Qing *et al.* (1982), Weyhreter *et al.* (1988) and Furst *et al.* (1989).

On the theoretical side there have been several recent calculations of low-energy electron–argon cross sections, for example, Nahar and Wadehra (1987) who used a semi-empirical local potential, Sienkiewicz and Baylis (1987) who solved the Dirac scattering equation with a model potential, the multi-configuration Hartree–Fock calculations of Saha (1991, 1993, 1996) and finally that of Mimnagh *et al.* (1993) who included multipole polarisation and dynamic distortion potentials within a non-relativistic polarised-orbital framework. Previous theoretical calculations include those of Fon *et al.* (1983) and Bell *et al.* (1984) based upon *R*-matrix theory, Dasgupta and Bhatia (1985) who used a psuedopotential approach and the polarised-orbital method, Haberland *et al.* (1986) who applied Kohn–Sham density functional theory and the adiabatic exchange approximation of McEachran and Stauffer (1983*b*) which was also within the framework of the polarised-orbital method.

In Section 2 of this paper we outline our general theoretical approach, while in Section 3 we present our results for the momentum transfer cross section, the total elastic cross section and the elastic differential cross section between 0 and 10 eV. In Section 4 we present our conclusions.

2. Theory

The original polarised-orbital approximation (Temkin 1957; Temkin and Lamkin 1961) was derived in order to take into account the dominant long-range adiabatic interaction between the incident electron and the atom. This interaction is usually described in terms of an adiabatic multipole polarisation potential. Previously we have used this basic approach, in conjunction with a first-order perturbation correction to the Hartree–Fock wavefunctions, to calculate these polarisation potentials for the noble gases and have applied them to elastic electron scattering from these atoms with considerable success (McEachran and Stauffer 1983*a*, 1983*b*, 1984). However, it has long been recognised that, in electron scattering, the neglect of the effects of the motion of the incident electron led to an interaction that becomes increasingly too attractive as the incident energy increases. LaBahn and Callaway (1966) and Callaway *et al.* (1968) introduced a formalism whereby such dynamic effects could be incorporated into the polarised-orbital approximation but their results were somewhat mixed. For a review and consistent derivation of these and other polarised-orbital approximations, see Drachman and Temkin (1972).

We have recently determined these dynamic distortion effects, within the general framework of our polarised-orbital method, and have applied them to the elastic scattering of electrons from helium (McEachran and Stauffer 1990) and from neon, argon and krypton (Mimnagh *et al.* 1993). In these calculations, which were carried out within a non-relativistic framework, both the polarisation and distortion potentials included many multipoles, including the monopole terms. With the exception of neon, these new calculations, including multipole polarisation and dynamic distortion effects, gave better agreement with experiment, particularly at the lowest energies, than our previous adiabatic exchange results which included only the dipole polarisation potential.

We have also previously shown, within the adiabatic exchange approximation, that the low-energy total and momentum transfer cross sections for krypton (McEachran and Stauffer 1988) and xenon (McEachran and Stauffer 1987) are in much better agreement with experiment, particularly in the vicinity of the Ramsauer–Townsend minimum, if the phase shifts are determined from the solution of the relativistic Dirac scattering equations rather than from the equivalent non-relativistic Schrödinger equation. In this paper, we employ the same polarisation and dynamic distortion potentials as used by Mimnagh *et al.* (1993), however, we now determine the phase shifts relativistically from the solution of the Dirac scattering equations. As will be shown in the following section, the resulting momentum transfer cross section, total elastic cross section and elastic differential cross sections are in considerably better agreement with experiment than either of our previous calculations (McEachran and Stauffer 1983*b*; Mimnagh *et al.* 1993).

We now present a brief outline of the theory for the relativistic formulation of electron scattering from the noble gases, including polarisation and dynamic distortion effects. In particular, the phase shifts are determined from the solution of the coupled first-order differential equations

$$f'_\kappa(r) + \frac{\kappa}{r} f_\kappa(r) - \frac{1}{c} \left[2c^2 + U(r) + \epsilon \right] g_\kappa(r) = \frac{1}{c} W_Q(\kappa; r), \tag{1a}$$

$$g'_\kappa(r) - \frac{\kappa}{r} g_\kappa(r) + \frac{1}{c} \left[U(r) + \epsilon \right] f_\kappa(r) = -\frac{1}{c} W_P(\kappa; r). \tag{1b}$$

Here $f_\kappa(r)$ and $g_\kappa(r)$ are the large and small components of the scattering wavefunction respectively, $U(r)$ is the total scattering potential and $W_P(r)$ and $W_Q(r)$ are the exchange kernels involving the large and small components of the relativistic Dirac–Fock bound state wavefunctions respectively. Furthermore $(1+\gamma)\epsilon = k^2$ with $\gamma = \sqrt{1 - v^2/c^2}$, where v is the speed of the incident electron and k is the magnitude of its momentum. The quantum number κ is defined in terms of the orbital and total angular momentum quantum numbers of the incident electron according to

$$\kappa = \begin{cases} j + \frac{1}{2}, & \text{if } j = l - \frac{1}{2} \\ -(j + \frac{1}{2}), & \text{if } j = l + \frac{1}{2}. \end{cases} \tag{2}$$

The relativistic scattering phase shifts $\delta_\kappa(k)$ can be determined from the asymptotic form of the large component of the scattering wavefunction, i.e.

$$f_\kappa(r) \xrightarrow[r \to \infty]{} A_\kappa \sin\left(kr - \frac{l\pi}{2} + \delta_\kappa(k) \right). \tag{3}$$

Alternatively, these phase shifts can be written as $\delta_l^\pm$ where the spin-up phase shifts (+) correspond to $j = l + \frac{1}{2}$, while the spin-down phase shifts (−) correspond to $j = l - \frac{1}{2}$.

We express the total scattering potential $U(r)$ as the sum of a static potential $U_{st}(r)$, a polarisation potential $U_p(r)$ and a dynamic distortion potential $U_d(r)$ as follows:

$$U(r) = U_{st}(r) + U_p(r) + \tfrac{1}{2} U_d(r) \,. \tag{4}$$

The static potential vanishes exponentially at infinity while the polarisation and distortion potentials behave according to

$$U_p(r) \xrightarrow[r \to \infty]{} \sum_\nu U_p^\nu(r) = -\sum_\nu \frac{\alpha_\nu}{2r^{2\nu+2}} \,, \tag{5}$$

$$U_d(r) \xrightarrow[r \to \infty]{} \sum_\nu U_d^\nu(r) = +\sum_\nu \frac{6\beta_\nu}{r^{2\nu+4}} \,. \tag{6}$$

Here the constants α_ν and β_ν are the static and dynamic multipole polarisabilities respectively of the atom and can be found from the asymptotic form of the $U_p^\nu(r)$ and $U_d^\nu(r)$. For more details on the derivation of these polarisation and distortion potentials, the reader is referred to the papers of McEachran *et al.* (1977) and McEachran and Stauffer (1990) respectively.

Once the phase shifts have been determined, the elastic differential cross section can be found from the expression

$$\sigma^{el}(\theta) = |f(\theta)|^2 + |g(\theta)|^2 \,, \tag{7}$$

where

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} \{(l+1)\, T_l^+(k) + l\, T_l^-(k)\} P_l(\cos\theta) \,, \tag{8}$$

$$g(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} \{T_l^-(k) - T_l^+(k)\} P_l^1(\cos\theta) \tag{9}$$

are the direct and spin-flip scattering amplitudes respectively and the T-matrix elements, $T_l^\pm(k)$, are given by

$$T_l^\pm(k) = \exp\left(i\delta_l^\pm(k)\right) \sin\left(\delta_l^\pm(k)\right) \,. \tag{10}$$

Here $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre and associated Legendre polynomials respectively. The total elastic cross section and the momentum transfer cross section can, in turn, be expressed in terms of the T-matrix elements according to

$$\sigma_{tot}^{el}(k^2) = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} \left\{(l+1)\, \sin^2\left(\delta_l^+(k)\right) + l\, \sin^2\left(\delta_l^-(k)\right)\right\} \,, \tag{11}$$

$$\sigma^{\mathrm{mt}}(k^2) = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} \left\{ \frac{(l+1)(l+2)}{2l+3} \, \sin^2\left(\delta_l^+(k) - \delta_{l+1}^+(k)\right) \right.$$

$$+ \frac{l(l+1)}{2l+1} \, \sin^2\left(\delta_l^-(k) - \delta_{l+1}^-(k)\right)$$

$$\left. + \frac{(l+1)}{(2l+1)(2l+3)} \, \sin^2\left(\delta_l^+(k) - \delta_{l+1}^-(k)\right) \right\}. \qquad (12)$$

The relativistic ground state wavefunction of argon was determined using the Dirac–Fock computer code of Grant *et al.* (1980). This wavefunction was then used to determine the static potential as well as in the evaluation of the exchange kernels. The polarisation and distortion potentials, on the other hand, were determined from the first-order correction to the non-relativistic Hartree–Fock wavefunction for argon and are thus identical to those used by Mimnagh *et al.* (1993). In these potentials all multipole components up to and including those which vanish at infinity as r^{-12} were included.

3. Results

In Table 1 we present our relativistic phase shifts as a function of k, the magnitude of the momentum of the incident electron. We note that for $l \geq 3$ the spin-up and spin-down phase shifts are effectively equal to each other for this range of energies of the incident electron. Also included in this table are the corresponding values of the elastic total and momentum transfer cross sections.

In Table 2 we make a detailed comparison of theoretical and experimental values of the argon scattering length. The theoretical values include those derived in our previous non-relativistic adiabatic exchange and dynamic distortion approximations as well as our present value. Also included is the multi-configuration Hartree–Fock result of Saha (1993). On the experimental side there is the swarm derived value of Haddad and O'Malley (1982), which was later revised by Petrović and Crompton (1987), as well as the values of Ferch *et al.* (1985), Buckman and Lohmann (1986) and Buckman and Mitroy (1989), all of which were deduced from modified effective range theory (MERT) fits to elastic total cross section measurements at low energies. The experimental value of Weyhreter *et al.* (1988) was deduced from low energy measurements of the elastic differential cross section and it would now appear to be slightly too large in magnitude. Finally, there is the recent value for the scattering length of Petrović *et al.* (1995) which was determined from drift velocity measurements in an argon–molecular hydrogen mixture. It would now appear that there is basic overall agreement between the theoretical and experimental determinations of this quantity, at least to the 1 to 2% level, i.e. $-1 \cdot 45 \pm 0 \cdot 01 \, a_0$.

Robertson (1977) and Milloy and Crompton (1977) made high precision measurements of drift velocities as well as of the ratio of the transverse diffusion coefficient to the mobility, D_{T}/μ, in low-energy swarm experiments in argon. These measurements were then used by Milloy *et al.* (1977) and by Haddad and O'Malley (1982) to determine the momentum transfer cross section for

Table 1. Relativistic phase shifts for argon as a function of the momentum k and the total elastic and momentum transfer cross sections in units of 10^{-16} cm²

k	δ_0^+	δ_1^+ / δ_1^-	δ_2^+ / δ_2^-	δ_3^+ / δ_3^-	δ_4^+ / δ_4^-	δ_5^+ / δ_5^-	Energy (eV)	σ^{el}	σ^{mt}
0·0271	0·030425	0·001500	0·000237	0·000079	0·000036	0·000019	0·01	4·4679	4·0231
		0·001502	0·000237	0·000079	0·000036	0·000019			
0·0470	0·042173	0·004128	0·000707	0·000237	0·000108	0·000058	0·03	2·9180	2·3425
		0·004143	0·000707	0·000237	0·000108	0·000058			
0·0606	0·045611	0·006423	0·001189	0·000395	0·000179	0·000097	0·05	2·1193	1·5247
		0·006456	0·001188	0·000395	0·000179	0·000097			
0·0717	0·045956	0·008499	0·001667	0·000552	0·000251	0·000135	0·07	1·6052	1·0261
		0·008553	0·001667	0·000552	0·000251	0·000135			
0·0813	0·044637	0·010385	0·002148	0·000710	0·000323	0·000174	0·09	1·2481	0·6998
		0·010463	0·002148	0·000710	0·000323	0·000174			
0·0977	0·039274	0·013637	0·003118	0·001026	0·000466	0·000251	0·13	0·7972	0·3291
		0·013772	0·003118	0·001026	0·000466	0·000251			
0·1118	0·031885	0·016359	0·004105	0·001345	0·000610	0·000329	0·17	0·5428	0·1599
		0·016558	0·004105	0·001345	0·000610	0·000329			
0·1181	0·027843	0·017513	0·004594	0·001501	0·000681	0·000367	0·19	0·4621	0·1198
		0·017745	0·004593	0·001501	0·000681	0·000367			
0·1242	0·023527	0·018549	0·005095	0·001661	0·000753	0·000406	0·21	0·4002	0·0983
		0·018847	0·005094	0·001661	0·000753	0·000406			
0·1301	0·018995	0·019520	0·005607	0·001823	0·000827	0·000445	0·23	0·3548	0·0917
		0·019826	0·005606	0·001823	0·000827	0·000445			
0·1356	0·014471	0·020367	0·006109	0·001981	0·000898	0·000483	0·25	0·3242	0·0969
		0·020710	0·006108	0·001981	0·000898	0·000483			
0·1460	0·005186	0·021793	0·007125	0·002298	0·001042	0·000560	0·29	0·2934	0·1316
		0·022214	0·007123	0·002298	0·001042	0·000560			
0·1509	0·000511	0·022376	0·007634	0·002455	0·001113	0·000599	0·31	0·2898	0·1573
		0·022837	0·007632	0·002455	0·001113	0·000599			
0·1534	−0·001936	0·022650	0·007902	0·002538	0·001150	0·000619	0·32	0·2904	0·1724
		0·023132	0·007900	0·002538	0·001150	0·000619			
0·1736	−0·023988	0·024213	0·010271	0·003255	0·001474	0·000793	0·41	0·3504	0·3396
		0·024883	0·010267	0·003255	0·001474	0·000793			

Table 1. *Continued*

k	δ_0^+	δ_1^+ δ_1^-	δ_2^+ δ_2^-	δ_3^+ δ_3^-	δ_4^+ δ_4^-	δ_5^+ δ_5^-	Energy (eV)	σ^{el}	σ^{mt}
0.1936	−0.047095	0.024470 0.025358	0.012991 0.012983	0.004055 0.004055	0.001834 0.001834	0.000986 0.000986	0.51	0.4756	0.5349
0.2117	−0.070527	0.023455 0.024564	0.015802 0.015791	0.004857 0.004857	0.002194 0.002194	0.001179 0.001179	0.61	0.6403	0.7381
0.2284	−0.093462	0.021389 0.022721	0.018727 0.018711	0.005664 0.005664	0.002556 0.002556	0.001373 0.001373	0.71	0.8240	0.9318
0.2440	−0.115853	0.018439 0.019994	0.021769 0.021747	0.006469 0.006469	0.002919 0.002919	0.001568 0.001568	0.81	1.0184	1.1139
0.2586	−0.137540	0.014769 0.016542	0.024909 0.024880	0.007279 0.007279	0.003281 0.003281	0.001762 0.001762	0.91	1.2176	1.2835
0.2711	−0.156600	0.010925 0.012892	0.027845 0.027809	0.008012 0.008012	0.003608 0.003608	0.001937 0.001937	1.00	1.4002	1.4280
0.3320	−0.254463	−0.016817 −0.013850	0.045973 0.045880	0.012124 0.012123	0.005425 0.005425	0.002910 0.002910	1.50	2.4307	2.1438
0.3834	−0.341482	−0.050871 −0.047040	0.067477 0.067299	0.016336 0.016334	0.007255 0.007254	0.003886 0.003886	2.00	3.4680	2.8318
0.4696	−0.492050	−0.125298 −0.120589	0.121484 0.121055	0.025087 0.025079	0.010945 0.010945	0.005845 0.005845	3.00	5.5581	4.4046
0.5422	−0.620472	−0.200570 −0.194301	0.191082 0.190312	0.034317 0.034298	0.014688 0.014687	0.007814 0.007814	4.00	7.7317	6.3502
0.6062	−0.733308	−0.271998 −0.264993	0.276770 0.275610	0.044061 0.044028	0.018493 0.018491	0.009795 0.009796	5.00	10.0665	8.6317
0.7424	−0.969243	−0.433011 −0.424798	0.551955 0.550029	0.070529 0.070447	0.028306 0.028300	0.014805 0.014805	7.50	16.7176	14.6559
0.8573	−1.160734	−0.569394 −0.561349	0.863342 0.861677	0.099496 0.099354	0.038575 0.038561	0.019915 0.019914	10.00	22.4036	17.8325

Table 2. Theoretical and experimental values for the argon scattering length *A* in atomic units

Theory	A	Experiment	A
McEachran and Stauffer (1983)	-1.506	Haddad and O'Malley (1982)	$-1 \cdot 492$
Mimnagh *et al.* (1993)	-1.386	Ferch *et al.* (1985)	$-1 \cdot 449$
Saha (1993)	$-1 \cdot 486$	Buckman and Lohmann (1986)	$-1 \cdot 492$
Present work	$-1 \cdot 441$	Petrović and Crompton (1987)	$-1 \cdot 48$
		Weyhreter *et al.* (1988)	$-1 \cdot 593$
		Buckman and Mitroy (1989)	$-1 \cdot 442$
		Petrović *et al.* (1995)	$-1 \cdot 459$
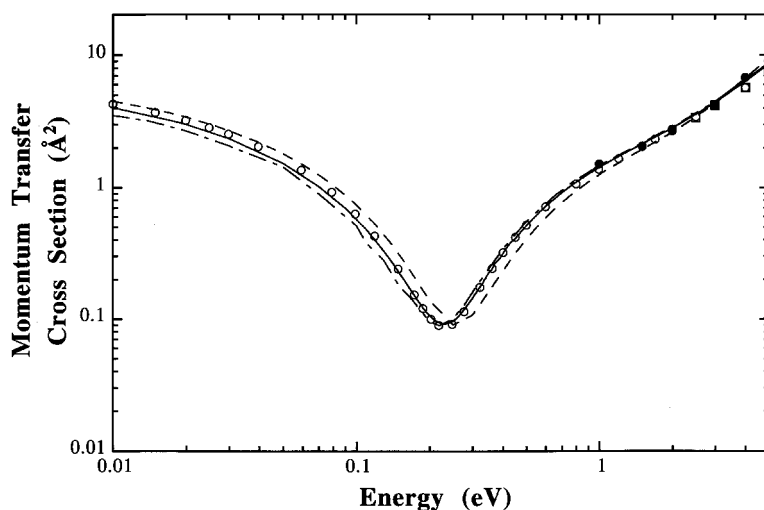
Fig. 1. Momentum transfer cross section in units of Å^2. Theory: - - -, McEachran and Stauffer (1983*b*); — · —, Mimnagh *et al.* (1993); ——, present results. Experiment: ○, Haddad and O'Malley (1982); □, Nakamura and Kurachi (1988); ●, Gibson *et al.* (1996).

argon. In Fig. 1 we present our momentum transfer cross section, in three separate approximations, together with the swarm derived results of Haddad and O'Malley (1982) and Nakamura and Kurachi (1988). Also included are the cross sections of Gibson *et al.* (1996) which were determined from a phase shift analysis of their differential cross section measurements. We see that our present calculation, determined from a relativistic formulation of the scattering equations, and including dynamic distortion effects, is in excellent agreement with the results of Haddad and O'Malley at all energies and particularly throughout the entire Ramsauer–Townsend region. The magnitude of our momentum transfer cross section at the minimum is $0 \cdot 0917 \times 10^{-16}$ cm^2 and occurs for an incident electron energy of $0 \cdot 23$ eV. Thus, our position for the minimum is in complete accord with experiment although our magnitude is slightly larger ($\sim$5%) than the experimental value of $0 \cdot 0870 \times 10^{-16}$ cm^2.

In Fig. 2 we present our results for the elastic total cross section together with the theoretical results of Saha (1996) and the direct experimental measurements of
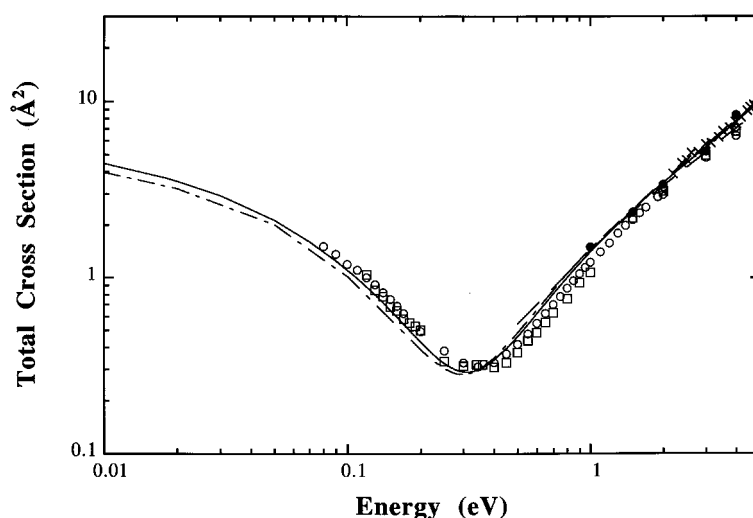
Fig. 2. Total elastic cross section in units of Å^2. Theory: — · —, Mimnagh *et al.* (1993); — —, Saha (1996); ——, present results. Experiment: ×, Charlton *et al.* (1980); ○, Ferch *et al.* (1985); □, Buckman and Lohmann (1986); ◇, Nickel *et al.* (1985); ●, Gibson *et al.* (1996).

Charlton *et al.* (1980), Nickel *et al.* (1985), Ferch *et al.* (1985) and Buckman and Lohmann (1986), as well as the total cross sections derived from the differential cross section measurements of Gibson *et al.* (1996). Total cross sections have also been deduced from differential cross section measurements and a subsequent phase shift analysis by Srivastava *et al.* (1981) and by Furst *et al.* (1989). Both of these sets of data are very close to those already presented in Fig. 2 and have been omitted simply for the sake of clarity. Our present cross section, determined within a relativistic framework, is in much better agreement with experiment than our comparable non-relativistic result (Mimnagh *et al.* 1993); this is particularly true on the low-energy side of the Ramsauer–Townsend minimum. However, all three of the theoretical curves lie somewhat higher than the experiments of Ferch *et al.* (1985) and Buckman and Lohmann (1986) on the high-energy side of the minimum. The magnitude of our total elastic cross section at the minimum is $0 \cdot 290 \times 10^{-16}$ cm^2 and occurs for an incident electron energy of $0 \cdot 31$ eV.

In Figs 3–8 we present our values for the elastic differential cross section for incident electron energies of 1, 1·5, 2, 3, 5 and 10 eV respectively. Other recent theoretical calculations included in these figures are the local potential results of Nahar and Wadehra (1987), the relativistic model potential work of Sienkiewicz and Baylis (1987), the non-relativistic dynamic distortion approximation of Mimnagh *et al.* (1993) and the multi-configuration Hartree–Fock calculation of Saha (1996) which included dipole and quadrupole polarisation terms. The experimental work presented includes the differential cross section measurements of Srivastava *et al.* (1981) at 3, 5 and 10 eV, of Weyhreter *et al.* (1988) at 1 and 1·5 eV and of Gibson *et al.* (1996) at all energies. In the case of the latter experiment, measurements were made on two different spectrometers, one at the Australian National University and the other at the University of Nebraska. All of these experiments employed the relative flow method for normalisation to helium cross
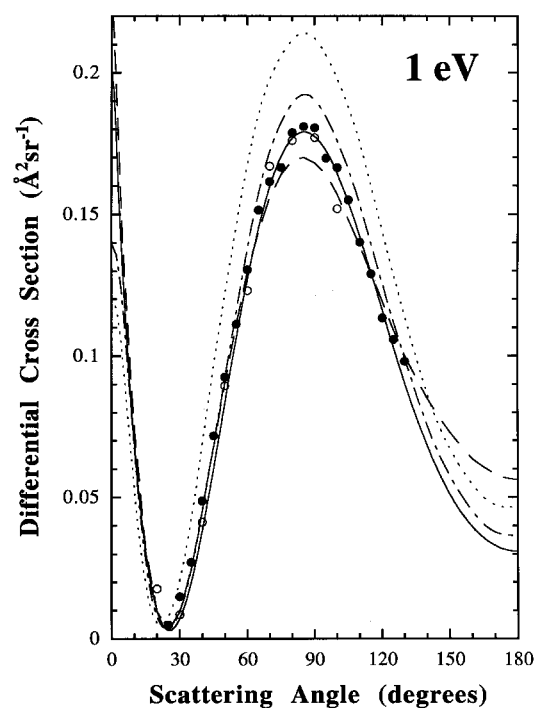
Fig. 3.   Differential cross section at $1 \cdot 0$ eV in units of $\text{Å}^2 \, \text{sr}^{-1}$. Theory: $\cdots\cdots$, Sienkiewicz and Baylis (1987); $-\cdot-$, Mimnagh *et al.* (1993); $--$, Saha (1996); ——, present results. Experiment: $\circ$, Weyhreter *et al.* (1988); $\bullet$, Gibson *et al.* (1996).
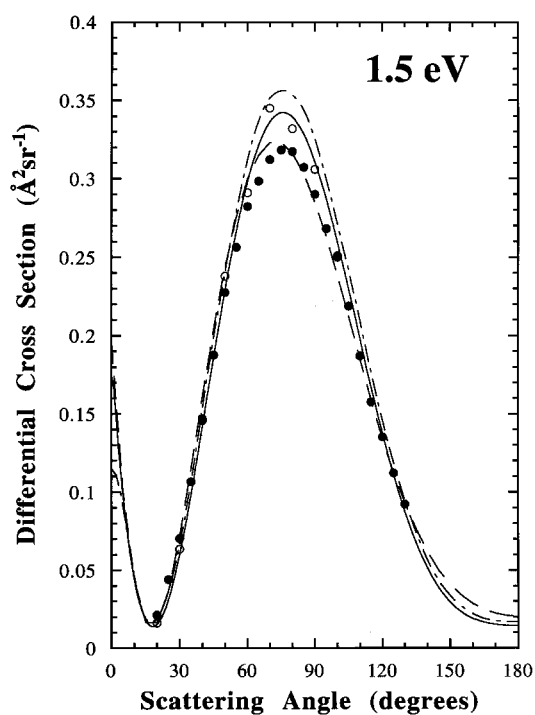

**Fig. 4.**   Differential cross section at $1 \cdot 5$ eV in units of $\text{Å}^2 \, \text{sr}^{-1}$. Theory: $-\cdot-$, Mimnagh *et al.* (1993); $--$, Saha (1996); ——, present results. Experiment: $\circ$, Weyhreter *et al.* (1988); $\bullet$, Gibson *et al.* (1996).
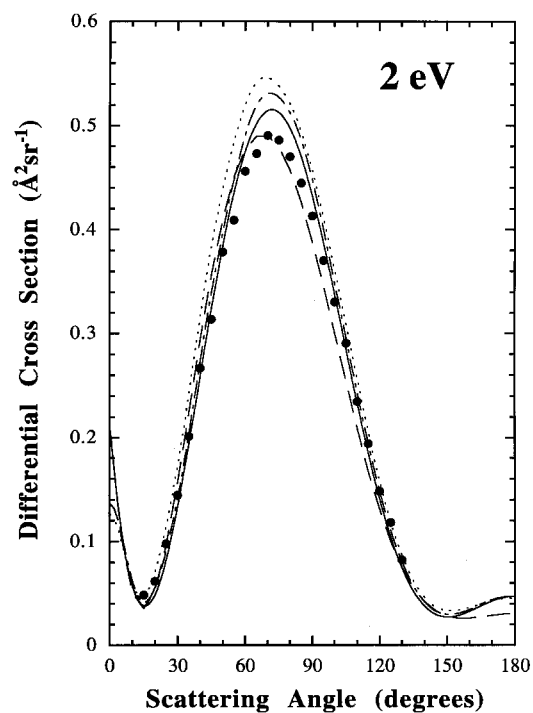
Fig. 5. Differential cross section at $2 \cdot 0$ eV in units of $\text{Å}^2 \text{sr}^{-1}$. Theory: $\cdots\cdots$, Sienkiewicz and Baylis (1987); $-\cdot-$, Mimnagh *et al.* (1993); $--$, Saha (1996); $-\!\!-$, present results. Experiment: •, Gibson *et al.* (1996).
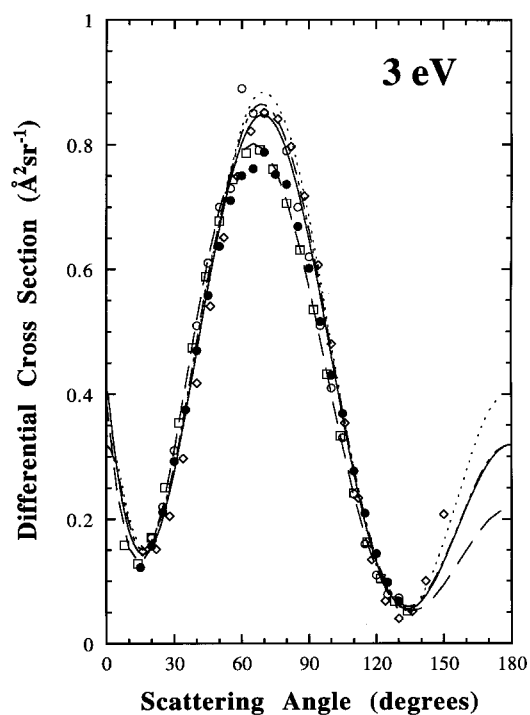
Fig. 6. Differential cross section at $3 \cdot 0$ eV in units of $\text{Å}^2 \text{sr}^{-1}$. Theory: $\cdots\cdots$, Nahar and Wadehra (1987); $-\cdot-$, Mimnagh *et al.* (1993); $--$, Saha (1996); $-\!\!-$, present results. Experiment: ◇, Williams (1979); ○, Srivastava *et al.* (1981); □, Furst *et al.* (1989); •, Gibson *et al.* (1996).
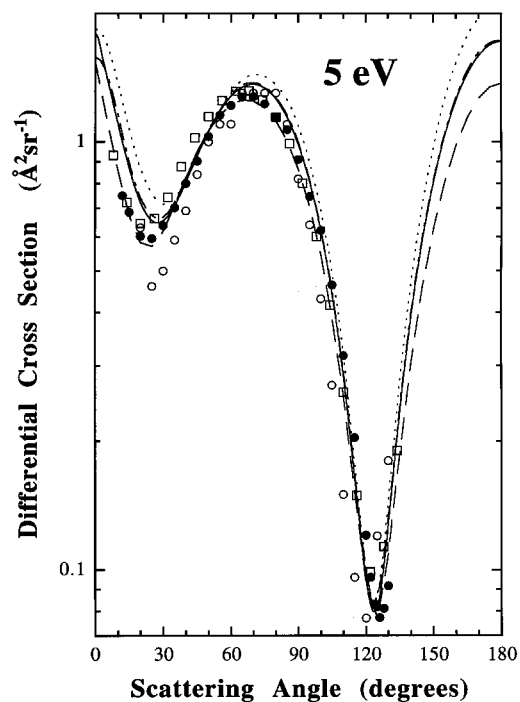
Fig. 7. Differential cross section at $5 \cdot 0$ eV in units of $\text{Å}^2 \text{sr}^{-1}$. Theory: $\cdots\cdots$, Nahar and Wadehra (1987); $—\cdot—$, Mimnagh *et al.* (1993); $——$, Saha (1996); $———$, present results. Experiment: $\circ$, Srivastava *et al.* (1981); $\square$, Furst *et al.* (1989); $\bullet$, Gibson *et al.* (1996).
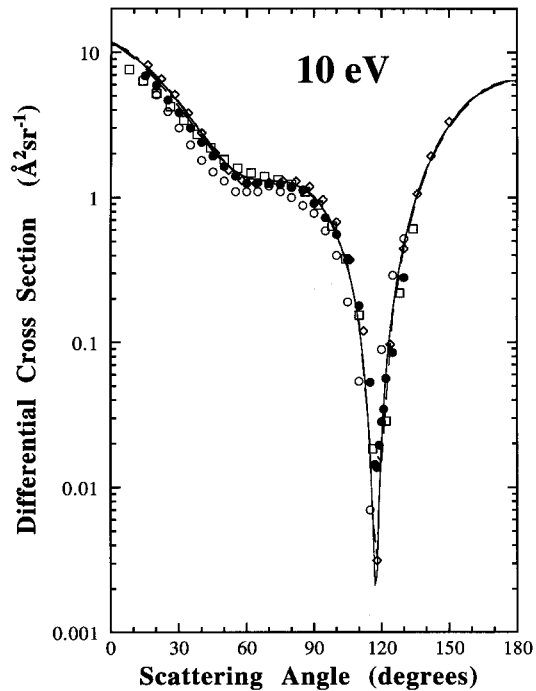

Fig. 8. Differential cross section at $10 \cdot 0$ eV in units of $\text{Å}^2 \text{sr}^{-1}$. Theory: $—\cdot—$, Mimnagh *et al.* (1993); $——$, Saha (1996); $———$, present results. Experiment: $\diamond$, Williams (1979); $\circ$, Srivastava *et al.* (1981); $\square$, Furst *et al.* (1989); $\bullet$, Gibson *et al.* (1996).

sections in order to put their argon results on an absolute scale. The experimental error quoted by Srivastava *et al.* is 20% while, except where the differential cross sections are very small, the typical experimental errors given by both Weyhreter *et al.* and Gibson *et al.* are around 7%. Also included in some of these figures are differential cross sections deduced from the phase shift analyses of Williams (1979) and Furst *et al.* (1989).

At all the energies presented the present work and that of Saha (1996) are in best overall agreement with experiment. In particular, with regard to the maxima in the 60°–90° region in the differential cross sections of Gibson *et al.* (1996), their magnitudes are predicted best by Saha while their locations are somewhat better determined by our present calculations. The cross sections of Sienkiewicz and Baylis (1987) at 1 and 1·5 eV as well as those of Nahar and Wadehra (1987) at 3 eV are too large in the intermediate angular range between 30 and 120°. This is also true, to a somewhat lesser extent, for the differential cross sections determined by Mimnagh *et al.* (1993). The polarised-orbital calculation of Dasgupta and Bhatia (1985) as well as the *R*-matrix calculations of Fon *et al.* (1983) and Bell *et al.* (1984) are also in very good agreement with experimental measurements of Gibson *et al.* (1996), but have not been shown for reasons of clarity.

4. Conclusions

There is now very good agreement between theory and experiment in the low energy region for electron–argon scattering. This is particularly true in the vicinity of the Ramsauer–Townsend minimum of the momentum transfer cross section and to a slightly lesser extent for the minimum in the total elastic cross section. Furthermore, it would appear that the electron–argon scattering length has been determined to within an accuracy of about one percent. There is also very good overall agreement between theory and experiment with respect to the elastic differential cross section between 1 and 10 eV.

As was the case in the low energy regime with krypton and xenon, the determination of the phase shifts within a relativistic formulation of the scattering problem significantly improved the agreement between theory and experiment for argon, vis-à-vis the comparable calculation carried out within a non-relativistic framework. This is particularly true around and below the Ramsauer–Townsend minimum in both the momentum transfer and the total elastic cross section.

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