CYCLOTRON RADIATION FROM JUPITER

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Summary

Cyclotron radiation from bunches of electrons trapped in a planetary magnetic field is discussed as a possible cause of the decametric wavelength radio emissions of Jupiter. By assuming the existence of an exospheric ionized medium, similar to that of the Earth, many of the observed properties of the radiation may be obtained by this process.

I. Introduction

It has been shown by Field (1960) that the intensity, spectrum, and polarization of the continuous decimetre wavelength radiations from Jupiter are reasonably explained on the basis of cyclotron emission from a diffuse cloud of electrons, trapped in a Jovian magnetic field, like the terrestrial Van Allen particles. The required magnetic field intensity is about 1000 G at the poles. The decametric radiations on the other hand are discontinuous in time, characteristically occurring as sequences of pulses each of about a second's duration. They are therefore apparently caused in some other way, although the basic mechanism may still be cyclotron emission.

It seems likely that a planetary magnetic field of the intensity envisaged by Field will be associated with an extensive ionized hydrogen exosphere like that of the Earth, where the ion density is to a large extent controlled by the magnetic field intensity (Dowden 1961). In looking to an explanation of the Jupiter decametric radiations it is useful first to consider the properties of the radio waves emitted by the terrestrial exosphere.

Here there exists an approximately dipole magnetic field together with a medium of free protons and electrons whose density varies with radial distance nearly as the magnetic field intensity. It is observed that the terrestrial exosphere is a source of electromagnetic radiations which take the form either of band-limited noise outbursts lasting for some hours, or of discrete emissions lasting less than a second. The latter characteristically have a rising or a falling variation of frequency with time. Both types occur mainly in the frequency band from 1 to 10 kc/s. It has been shown recently by Dowden (1962*a*) that the properties of the discrete emissions may be explained in detail if they are caused by cyclotron radiation from isolated bunches of non-relativistic electrons trapped in the geomagnetic field. The former more continuous emissions may originate as amplified Cerenkov radiation from continuous streams of particles (Ellis 1957; Gallet 1959: Piddington 1961; Dowden 1962*b*).

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We shall assume that the properties of any planetary exosphere may be similar to those of the terrestrial one in so far as the acceleration and trapping of isolated bunches of electrons are concerned. A likely explanation of the appearance of bunches in the Earth's exosphere has been given by Piddington (1960). The frequency range and other properties of the electromagnetic radiations emitted by the bunches will vary widely according to the particle speed and trajectory, the magnetic field intensity, and the plasma density.

II. Theory

The theory of radiation by an electron travelling along a helical path in a plasma with superimposed magnetic field has been given by Eidman (1958). He has shown that the power radiated by a single electron at frequency ω, at angle θ to the magnetic field direction, and per steradian is given by

$$W_s=\frac{e^2\omega_s^2 n\left[v_1 J_s'(A)+\left(\dfrac{\alpha c s\,\omega_H(1-v^2/c^2)^{\frac{1}{2}}}{\omega_s n\sin\theta}+\gamma v_2\right)J_s(A)\right]^2}{4\pi c^3\,|\,1-\beta_2\cos\theta(n+\omega_s\partial n/\partial\omega)\,|},\tag{1}$$

where

$$\mathrm{A}=\left[\frac{n\omega_s v_1\sin\theta}{c\omega_H\sqrt{(1-v^2/c^2)}}\right],$$

and v_1, v_2 are the velocity components parallel to and perpendicular to the magnetic field.

$\omega_H=eH/mc=$the cyclotron frequency,
$s=0,\ 1,\ 2,\ 3\ .\ .\ .\ ,$
$\alpha=K\cos\theta+l\sin\theta,$
$\gamma=l\cos\theta-K\sin\theta,$
$\beta_2=v_2/c,$
$K=-2y(1-x)\cos\theta[y^2\sin^2\theta\pm\{y^4\sin^4\theta+4y^2(1-x)^2\cos^2\theta\}^{\frac{1}{2}}]^{-1},$
$l=(-\sin\theta.x.y+ky^2x\cos\theta\sin\theta)(1-y^2-x(1-y^2\cos^2\theta))^{-1},$
n is the Appleton-Hartree refractive index,
$$n^2=1-x(1-x)\{1-x-\tfrac{1}{2}(y^2\sin^2\theta)\mp[\tfrac{1}{4}(y^4\sin^4\theta)+(1-x)^2y^2\cos^2\theta]^{\frac{1}{2}}\}^{-1},\tag{2}$$
$x=4\pi Ne^2m^{-1}\omega^{-2},$
$y=\omega_H/\omega.$

The wave frequency ω and the direction of emission θ are given by the equation

$$\omega_s=\frac{s.\omega_H\sqrt{(1-v^2/c^2)}}{1-\beta_2 n\cos\theta}.\tag{3}$$

Eidman's theory contains as special cases the theory of Cerenkov radiation ($s=0$, $1-\beta_2 n\cos\theta=0$), the theory of cyclotron radiation ($s=1$), and the theory of synchrotron radiation ($s\gg 1$), but in addition permits the calculation of the radiation properties in situations not dealt with earlier.

If the electrons are non-relativistic the radiation at the higher harmonics of the cyclotron frequency may be neglected. Also, emission in the Cerenkov mode ($s=0$) is possible only if the refractive index of the medium is greater than

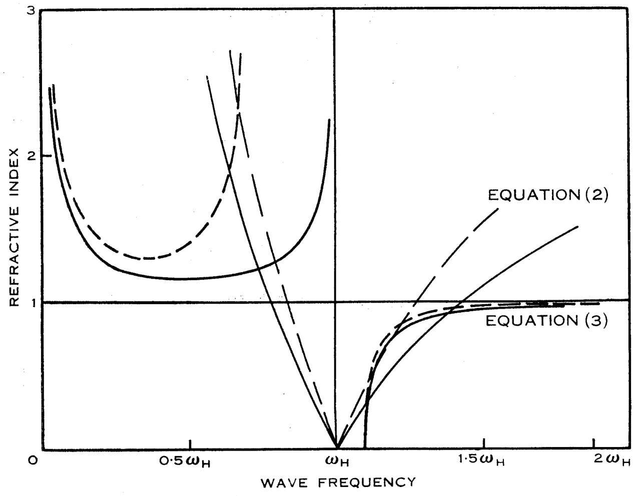

Fig. 1.—Behaviour of equations (2) and (3), $\omega_0/\omega_H=0 \cdot 3$, $\beta_2=0 \cdot 3$.
——— $\theta=0°$, 180° ; — — $\theta=45°$, 135°.

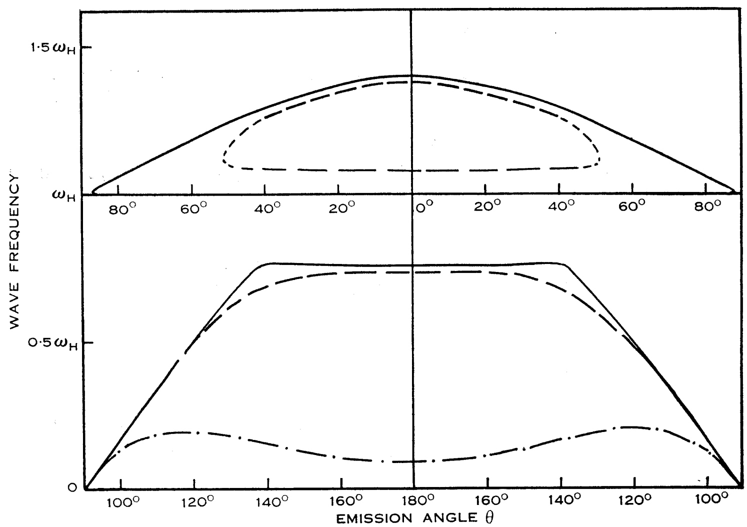

Fig. 2.—Variation of emission angle θ with frequency from equations (1) and (2), $\beta_2=0 \cdot 3$.
——— $\omega_0/\omega_H=0 \cdot 04$; — — $\omega_0/\omega_H=0 \cdot 3$; —·— $\omega_0/\omega_H=4$.

unity, which in an exosphere normally occurs for the extraordinary mode where the wave frequency is less than the local cyclotron frequency ω_H. This radiation cannot escape outwards from the exosphere and will not be considered here. For cyclotron radiation ($s=1$) the situation is different. Equation (3) shows that the frequency of the emitted wave will be Doppler shifted to a frequency greater than the local cyclotron frequency in the forward direction ($0<\theta<\pi/2$) and to a lower frequency in the backward direction ($\pi/2<\theta<\pi$). Again the latter radiation cannot escape from the exosphere but the forward radiation will escape.

To obtain the radiated frequency and angle of emission θ it is necessary to solve simultaneously equations (2) and (3). Because equation (2) for the refractive index is complicated it is convenient to illustrate the solutions graphically (Figs. 1 and 2). In general for a given value of θ, there are two possible frequencies in the forward direction. In the backward direction, ($\frac{1}{2}\pi<\theta<\pi$), there is one frequency of emission. Considering the case of forward emission, it is seen that as θ is increased from zero, the two frequencies of the radiation approach each other and become equal at a limiting maximum angle θ_m. No radiation is possible if $\theta>\theta_m$. This behaviour should be contrasted with cyclotron radiation in a vacuum where there are no forbidden directions of emission.

Radiation in the forward direction is possible only when solutions to equations (2) and (3) exist, the limiting circumstances being that the values of the frequency are equal when $\theta=0$. This condition is obtained simply. When $\theta=0$ equation (2) becomes

$$n^2=1-\frac{\omega_0^2}{\omega^2-\omega_H.\omega}, \qquad \omega_0^2=\frac{4\pi Ne^2}{m},$$

substituting in (3) gives

$$n^3-n-\frac{\omega_0^2}{\omega^2\beta_2}=0. \tag{4}$$

Two roots of (4) are the same and equal to 1/3 when

$$\beta_2=\frac{\omega_0^2}{\omega^2}\sqrt{(27/4)}, \tag{5}$$

or, eliminating ω in equation (3) and (5) we have

$$\omega_0=\left(\frac{\omega_H}{1-\beta_2/3^{\frac{1}{2}}}\right)\frac{(2\beta_2)^{\frac{1}{2}}}{27^{\frac{1}{4}}}. \tag{6}$$

Inspection of (2) and (3) shows that this value of ω_0 is the maximum for which radiation is possible. Since $\beta_2>1$, equation (6) implies that for non-relativistic particles, cyclotron radiation in the forward direction is possible in an exosphere only where the plasma frequency is relatively small compared with the cyclotron frequency. In the terrestrial exosphere, for example, ω_0 is everywhere greater than about $\frac{1}{2}\omega_H$ and forward radiation will occur only for electrons with energy greater than 60 keV. The Earth is unlikely to be a strong external emitter of

Doppler-shifted cyclotron radiation. Backward radiation is emitted everywhere, on the other hand, and accounts for the terrestrial V.L.F. emissions.

To obtain an expression for the power radiated by a number of electrons travelling in helices along the magnetic field lines we consider an element of length $\mathrm{d}l$ of the field line.

A single electron will traverse $\mathrm{d}l$ in time $\mathrm{d}l/V_2$ and the energy radiated into solid angle $\mathrm{d}\Omega$, and in particular direction θ at a particular frequency ω will be

$$E = W\frac{\mathrm{d}l}{V_2}\mathrm{d}\Omega.$$

If in length $\mathrm{d}l$ there are $M\mathrm{d}l$ electrons emitting in random phases the energy radiated will be

$$E_m = \frac{W\mathrm{d}lM\mathrm{d}l}{V_2}\mathrm{d}\Omega.$$

Since the magnetic field intensity varies over $\mathrm{d}l$ the instantaneous frequency range which the emissions in a particular direction of all the electrons will occupy will be $\mathrm{d}\omega$. The power radiated by the electrons per steradian per cycle per second will therefore be

$$P_m = W.M.\frac{\mathrm{d}l}{\mathrm{d}\omega}.$$

If there are density fluctuations in the electron stream then the intensity of the radiation may be different from that obtained by summing the radiation in random phases from $M\mathrm{d}l$ electrons. Where the scale of the irregularities is much less than the gyro radius of the electrons, for example, the intensity may be enhanced, depending on the number of electrons in each irregularity.

III. Discussion

In applying this theory to a Jovian exosphere we will assume the conditions proposed by Field (1960) to explain the high frequency radiations in the 300 Mc/s to 2000 Mc/s band, namely, a dipole magnetic field with a polar intensity of 1000 gauss. In addition Jupiter is assumed to have an ionosphere with a maximum critical frequency of 10 Mc/s and the electron density in the exosphere is assumed initially to vary as the magnetic field intensity. Estimates of the maximum critical frequency of the Jupiter ionosphere based on the intensity of the solar radiation vary from 7·5 Mc/s (Field 1960) to 20 Mc/s (Rishbeth 1959).

Field has suggested that the magnetic field extends outwards for a distance of 67 radii to a region where the magnetic energy density is commensurate with the energy density of the solar wind. The latitude at which the corresponding field lines intersect the planet surface is 84°. As for the terrestrial exosphere the magnetic disturbance phenomena and hence particle acceleration would be expected to be most pronounced in the region.

Over a major part of such an exosphere the cyclotron frequency is much greater than the plasma frequency, unlike the terrestrial case. Equation (4) shows immediately that forward cyclotron radiation by non-relativistic trapped electrons ($\beta_2 < \frac{1}{2}$ say) is possible in a frequency range from a few tens of kc/s to over 2000 Mc/s. Whether or not the radiation would be observable from the Earth depends on the geometry of the electron trajectory and the allowable directions of radiation. Here we consider the likely properties of the radiation in the frequency range below 40 Mc/s.

For the purposes of illustration the electromagnetic radiation by electrons travelling along the 84° field line is calculated. An electron kinetic energy of 15 keV is chosen, commensurate with the energy of the trapped electrons in the Earth's exosphere. The mirror point of the electrons is arbitrarily taken to be at the level where the cyclotron frequency is 37 Mc/s. The actual value is unimportant providing it is somewhat greater than the required wave frequency.

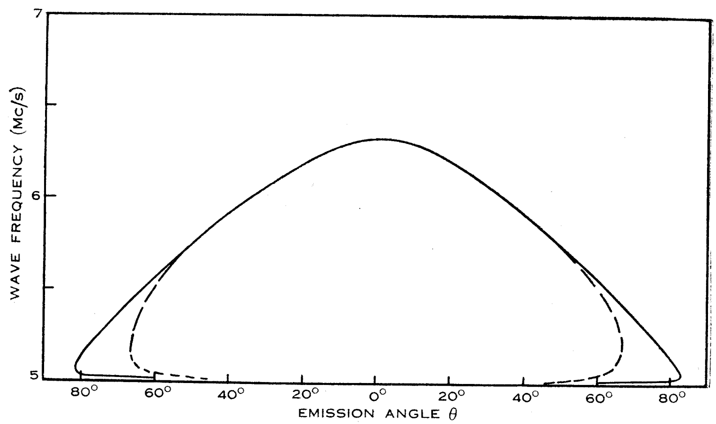

Fig. 3.—Variation of emission angle with frequency for 15 keV electrons at a distance of 8 radii. Assumed trajectory along the 84° field line with mirror points at 4 radii. $\omega_0=0\cdot43$ Mc/s. Polar magnetic field intensity 1000 G. The dotted line shows the final direction of emission after allowing for refraction by the exospheric medium.

As the electron travels inward along the field line it will radiate at increasing frequencies determined by equations (2) and (3) until it is almost at the mirror point. The forward radiation will also travel inwards until reflected outwards at the appropriate extraordinary mode reflection level. After the electron is reflected at the mirror point it will emit descending frequency radiation in an outward direction.

Figure 3 shows the variation of the frequency of the radiation with emission angle θ at a level where the cyclotron frequency is 5 Mc/s. Refraction of the waves by the medium after emission will occur, the amount of refraction being given very nearly by Snell's law with the assumption that surfaces of constant refractive index are plane and perpendicular to the field direction. The dotted line in Figure 3 shows the final direction of the radiation. The translation of the electron through the magnetic field causes all the emitted frequencies to change with time as shown in Figure 4.

Observations have shown that over a frequency range from $4\cdot8$ to 30 Mc/s the occurrence of radiation from Jupiter varies cyclically with a period of nearly

9 h 55 m 28·8 s, that is approximately the rotational period of the surface markings of the planet (Shain 1958 ; Smith and Carr 1959 ; Ellis 1962). There is also strong evidence, particularly at 4·8 and above 20 Mc/s, for a period of half this value. It is observed in addition that between 20 and 30 Mc/s the radiation becomes increasingly difficult to detect and that the periodic variation becomes more pronounced (Gardner and Shain 1958).

If the magnetic dipole axis is inclined to the rotation axis, the restriction in the allowable directions of radiation by electrons travelling along a line of force can cause a considerable variation in the intensity with rotation as seen from the Earth. Using the above model for example, at 5 Mc/s there is a maximum final direction of emission θ_m of 66° with respect to the local field

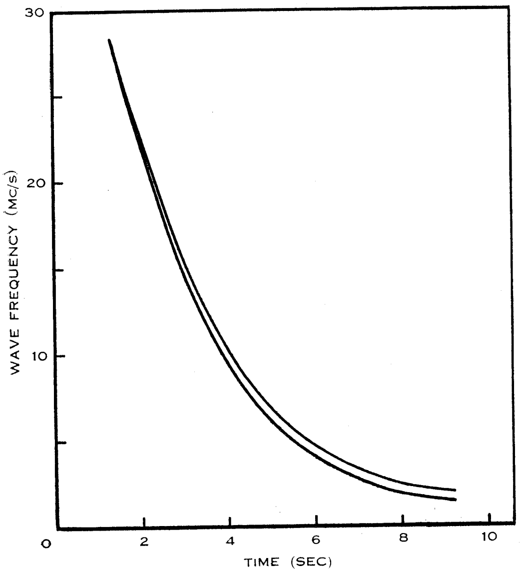

Fig. 4.—Variation of emitted frequencies with time for 15 keV electrons trapped on the 84° field line and moving outwards. Mirror point at 4 radii.

direction. This latter direction at the point of emission is inclined 61° to the magnetic equator. A change of the assumed electron kinetic energy from 15 to 4 keV or of the local plasma frequency from 0·4 to 0·6 Mc/s would reduce θ_m to 60° and no radiation would be emitted in the magnetic equatorial direction. With an inclined magnetic dipole axis, therefore, the observed radiation would fall to zero twice during each rotation of the planet. Figure 5 shows the geometry of emission at 5 Mc/s.

If we assume that the average radiation intensity observed at the Earth results from electrons which may have trajectories in all magnetic longitudes, then the intensity may be calculated by using equations (1), (2), (3), and (5) and integrating over all longitudes. Figure 6 shows how the relative average intensity would change with rotation, using the above model for a wave frequency of 5 Mc/s. At higher frequencies the same electrons would radiate lower in the exosphere where their translational velocity v_2 is less. As a result the maximum radiation angle θ_m may be less. In addition the field line is more inclined to the magnetic equator at the emission point. As the frequency is increased these two factors first cause the rotational minima to become zero and finally cause the radiation to be cut off entirely from the direction of the Earth at all times. The change in the rotation pattern with frequency for electrons which have mirror points at $\omega_H = 37$ Mc/s is shown in Figure 6. If electrons with mirror points lower in the exosphere (higher ω_H) are considered the observed spectrum cuts off at correspondingly higher frequencies.

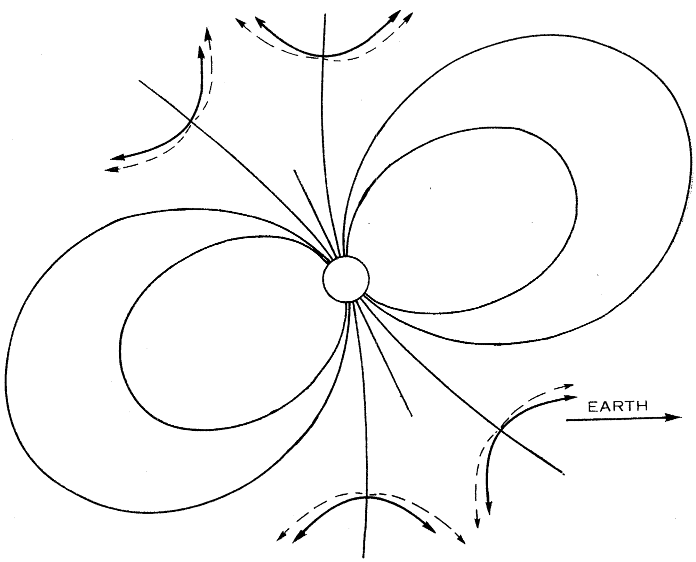

Fig. 5.—Geometry of emission with the dipole axis inclined 30° to the rotation axis. The solid lines show the limits of the cones of radiation for outward-moving electrons, and the dotted lines for inward motion.

The observed gradual decrease in the number of noise bursts above 20 Mc/s may be caused by a decrease in the number of bunches with low mirror points. Alternatively a stronger variation of the electron density with radius than that assumed here would produce a geometrical cut-off in the observed radiation by all bunches at the higher frequencies. Existing data are inadequate to distinguish between these alternatives. It is highly desirable that measurements of the variation of the intensity of the bursts with longitude and frequency, as well as their probability of occurrence, should be made, particularly above 20 Mc/s.

The radiation intensity from a single bunch of electrons may be calculated using equations (1) and (4). If the bunch is in the most favourable magnetic longitude and the magnetic dipole axis is inclined 30° to the rotation axis, the flux density received at the Earth at 5 Mc/s on the above model is $M \times 10^{-44}$ W m^{-2} (c/s)$^{-1}$, where M is the number of electrons in 1 cm length of the bunch. The reported peak observed intensities of the bursts range from 10^{-22} to 10^{-20} W m^{-2} (c/s)$^{-1}$ (Ellis 1962). If the radiation from all the electrons is summed in random phases it would appear that a bunch with somewhat improbable dimensions and density is required to explain the observed intensity. However, because of the possibility of large enhancement of the radiation by coherent emission it is considered that the cyclotron theory should be tested by its other predictions.

Since we have considered here the emission and propagation of cyclotron radiation, the wave polarization will be elliptical. However, the observed axial ratio of the polarization ellipse will depend in part on the propagation properties of the exospheric medium. In addition if successive noise bursts are caused by bunches occurring at random in different magnetic longitudes the axial ratio

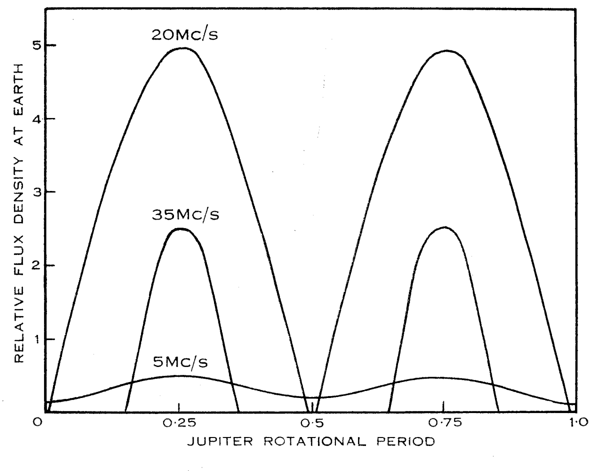

Fig. 6.—Variation of the integrated flux density of the radiation with rotation of Jupiter for different frequencies. Inclination of dipole axis 30°. Assumed mirror points of electrons 4 radii.

will vary considerably from burst to burst. The minimum values occur when the longitude of the bunch is near the Earth meridian. As Jupiter rotates the sense of the polarization may reverse twice for each rotation, depending on the inclination of the dipole axis. It would seem advisable in any observations of the way the wave polarization changes with rotation to use only the minimum values of the observed axial ratios in successive time intervals.

With the above model, radiation below about 15 Mc/s would be observable at all times and, near the rotational minima, from either polar region (Fig. 6). A bunch of electrons, trapped in the magnetic field and travelling along a field line, would therefore be observed to radiate first from near one magnetic pole and then a short time later, from near the opposite pole. Two noise bursts would be observed of opposite polarization and separated by the travel time of the bunch along the line of force. Using the parameters considered earlier, the time interval between the successive bursts would be approximately 4 minutes.

IV. Conclusions

Cyclotron radiation from bunches of electrons trapped in a Jovian exosphere may have the following observable properties : (i) A rising or a falling frequency time characteristic ; (ii) a periodic variation in average intensity with planetary rotation ; (iii) an increase in the amplitude of this variation with wave frequency ; (iv) a high frequency spectral cut off ; (v) elliptical wave polarization with reversal of the polarization sense with planetary rotation ; and (vi) the peak observed intensity and the duration of the noise bursts depend on the density and size of the bunches.

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