

For a charge Q moving with velocity v in the electrostatic or non-retarded approximation we have

$$\phi_{\text{ext}}(\mathbf{r}, t) = \frac{Q}{|\mathbf{r} - \mathbf{r}_0 + \mathbf{v}t|} = \frac{Q}{2\pi^2} \int \frac{d^3k}{k^2} \exp\{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_0 + \mathbf{v}t)\}. \quad (21)$$

Proceeding as before we get

$$s(\boldsymbol{\rho}, t) = \frac{Qe}{m} \frac{1}{2\pi^2} \int \frac{d^3k}{k^2} i k_3 \exp\{i\boldsymbol{\kappa} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}_0) - i k_3 z_0\} \\ \times \exp\{i(\mathbf{k} \cdot \mathbf{v})t\} / \{\omega_s^2 - (\mathbf{k} \cdot \mathbf{v})^2\}, \quad (22)$$

$$\phi_I(\mathbf{r}, t) = -\frac{Qe^2 n_0}{m} \frac{1}{\pi} \int \frac{d^3k}{k^2} i k_3 \frac{\exp\{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_0 + \mathbf{v}t)\}}{\omega_s^2 - (\mathbf{k} \cdot \mathbf{v})^2} \\ \times \{\exp(-\kappa|z| - i k_3 z)\} / \kappa. \quad (23)$$

The force on the moving charge is then (for $\mathbf{r} \rightarrow \mathbf{r}_0 - \mathbf{v}t$)

$$\mathbf{F} = Q(-\nabla\phi_I) = -\frac{Q^2 e^2 n_0}{m} \frac{1}{\pi} \int \frac{d^3k}{\kappa k^2} \frac{k_3 \mathbf{k} \exp(-\kappa|z| - i k_3 z)}{\{\omega_s^2 - (\mathbf{k} \cdot \mathbf{v})^2\}}. \quad (24)$$

The integral can be done and the resulting functions are complicated. In the case when v_3 vanishes, i.e. the motion of the charged particle is parallel to the jellium surface, we get the results obtained earlier by Muscat and Newns (1977) from the non-dispersive surface plasmon response

$$F_{\parallel}(z_0) = -\frac{Q^2 e^2 n_0}{m} \frac{1}{\pi} \int \frac{d^2\kappa dk_3}{\kappa(\kappa^2 + k_3^2)} \frac{k_3 \boldsymbol{\kappa} \exp(-\kappa|z_0| - i k_3 z_0)}{\{\omega_s^2 - (\boldsymbol{\kappa} \cdot \mathbf{v})^2\}} \\ = -(Q^2/4z_0)(2\kappa_c z_0)^2 K_0(2\kappa_c z_0), \quad (25)$$

$$F_{\perp}(z_0) = -\frac{Q^2 e^2 n_0}{m} \frac{1}{\pi} \int \frac{d^2\kappa dk_3}{\kappa(\kappa^2 + k_3^2)} \frac{k_3^2 \exp(-\kappa|z_0| - i k_3 z_0)}{\{\omega_s^2 - (\boldsymbol{\kappa} \cdot \mathbf{v})^2\}} \\ = -Q^2 \kappa_c^2 [1 + \frac{1}{2}\pi\{L_1(2\kappa_c z_0) - I_1(2\kappa_c z_0)\}]. \quad (26)$$

where L_1 is the Struve function, K_0 and I_1 are the modified Bessel functions, and $\kappa_c = \omega_s/v$. Here $F_{\perp}(z_0)$ goes to the image force limit $-Q^2/4z_0^2$ for $v \rightarrow 0$ or $\kappa_c \rightarrow \infty$, and goes to zero for $v \rightarrow \infty$. The damping force $F_{\parallel}(z_0)$ as a function of v has a maximum for $2\kappa_c z_0 \approx 1.6$, or for $v \approx 1.25\omega_s z_0$, and the value at maximum is 0.481 times the image force.

4. Conclusions

It is shown here that the charge response behaviour of the jellium surface due to ripples on it leads to the image force on a stationary external charge, and a damping force on a moving external charge. The expression for the force is identical with what one gets by considering the response due to the surface plasmons in the non-dispersive limit, although the boundary conditions in the two situations are radically different.

Since the free surface of the jellium electron gas is likely to respond through ripple-like displacements more easily than through the rigid boundary condition corresponding to surface plasmons, it would perhaps be appropriate to regard the image potential and associated effects as arising out of the ripplon modes.

As seen in equation (19), the image of a stationary point charge formed due to ripplon response is a point image, so that there will be a divergence in the self-energy of the charge as $z_0 \rightarrow 0$. This, of course, is a spurious divergence since for very small values of z_0 an upper cut-off must be used in the κ integrals that arise through the Fourier transforms (such as in equation 16), beyond which the continuum hydrodynamic model is not valid. The image due to the response through surface plasmons has a finite extension for non-zero β , so that the self-energy does not diverge for $z_0 \rightarrow 0$ even without any cut-off in κ space.

A complete analysis of the charge oscillation modes of the jellium surface must take into consideration the equilibrium density profile $n_0(z)$ which is not a step function. This has been done in detail for the surface plasmon modes (Equiluz *et al.* 1975, and references cited therein). In principle, the solution of equation (2) with the appropriate density profile $n_0(z)$ will give the density oscillation modes. The method of solution indicated in Section 2 is based on specific assumptions on the form of n_1 for the surface plasmon and the ripplon modes—the actual modes may be expected to be a mixture of both. For the electron gas in jellium, however, the frequencies of surface plasmons and ripples at small wave numbers are essentially the same and so are the associated density fluctuations. Hence, either of them could be used to work out the surface response due to an external charge, particularly at larger values of its distance from the surface.

An analysis of ripplon dispersion including a realistic equilibrium density profile can be made by assuming that the entire surface profile undergoes transverse displacements, that is,

$$n_0(\mathbf{r}) = n_0 T(-z), \quad (27)$$

where $T(-z)$ is a function that falls to zero from unity rapidly across the boundary, and $z = 0$ corresponds to the point at which $T''(-z)$ vanishes. The above form of the ripple implies

$$n(\mathbf{r}) = n_0 T\{s(\boldsymbol{\rho}, t) - \tau\} \approx n_0 T(-z) + n_0 s(\boldsymbol{\rho}, t) T'(-z) + \dots,$$

so that

$$n_1 = n_0 T'(-z) s(\boldsymbol{\rho}, t). \quad (28)$$

Equation (2) along with the boundary condition (11) leads to

$$mn_0 \omega_R^2(\boldsymbol{\kappa}) = n_0^2 e^2 \frac{\partial}{\partial z} \int \frac{T'(-z') \exp\{i\boldsymbol{\kappa} \cdot (\boldsymbol{\rho}' - \boldsymbol{\rho})\} d^2 \rho' dz'}{|\mathbf{r} - \mathbf{r}'|} \Big|_{z=0} \\ + m\beta^2 n_0 T''(-z)|_{z=0},$$

or

$$\omega_R^2(\boldsymbol{\kappa}) = \omega_s^2 \int_{-\infty}^{\infty} T'(z') \exp(-\kappa|z'|) dz'. \quad (29)$$

We obviously have $\omega_R^2(\kappa) \rightarrow \omega_s^2$ for $\kappa \rightarrow 0$, while for higher values of κ the value of $\omega_R^2(\kappa)$ diminishes. Although $\omega_R^2(\kappa)$ for these ripples appears to diminish to zero for $\kappa \rightarrow \infty$, there would be a practical upper limit for κ beyond which, as pointed out above, the hydrodynamic model breaks down. Since $T'(-z)$ is almost a δ function, the previous analysis is useful as a semi-quantitative description of phenomena associated with ripples.

When the external charged particle is an electron, then for sufficiently large z_0 the above theory is valid. But for small z_0 exchange corrections become important—these corrections would reduce its self-energy from its value when exchange is neglected. Incorporation of this correction in the hydrodynamic model is difficult.

References

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