Aust. J. Phys., 1992, 45, 591–620

The Physics of Tachyons
I. Tachyon Kinematics

Ross L. Dawe[A] *and Kenneth C. Hines*

School of Physics, University of Melbourne,
Parkville, Vic. 3052, Australia.
[A] Present address: Maritime Operations Division,
Materials Research Laboratory, P.O. Box 1750,
Salisbury, S.A. 5108, Australia.

Abstract

A new formulation of the theory of tachyons is developed using the same two postulates as in special relativity. Use is made of a 'switching principle' to show how tachyons automatically obey the laws of conservation of energy, momentum and electric charge. Tachyonic mechanics is further developed by a consideration of tachyonic rods and clocks. There follows a discussion of the conservation of electric charge and the apparent visual appearance of a tachyonic cube.

1. Introduction

The question of whether or not tachyons can exist has been a subject of considerable theoretical interest in recent decades. The most notable formulations of a theory of tachyons at the classical relativistic level have been provided by Bilaniuk *et al.* (1962, 1969), Recami and Mignani (1974), Recami (1986)* and Corben (1975, 1976, 1978). Recami has built up a detailed formulation based on symmetry arguments within special relativity, while Corben takes a more rigorous approach but has not completed all of the details. Tachyon theories are prone to misinterpretation, and the resulting fallacies and paradoxes have often proved difficult to resolve.

It is the purpose of this and subsequent papers to build upon the work by these authors by presenting a rigorous formulation of the theory of tachyons in which the particles are able to interact with ordinary matter via the laws of mechanics and electrodynamics. This paper will develop tachyon kinematics, while subsequent papers will develop tachyon dynamics, electromagnetism in a vacuum and electrodynamics in dielectric materials. Throughout these papers it will be repeatedly stressed that almost all classical relativistic systems can have a tachyonic analogue.

The formulation presented in this paper will closely follow the ideas set out by Bilaniuk *et al.* and by Corben. The method adopted will follow Recami's suggestion (1986): 'Since *a priori* we know nothing about tachyons, the safest

* The 1986 review article by Recami is a major landmark in the field of tachyon research and contains an excellent summary and a full list of references of the extensive work by Recami and coworkers such as P. Caldirola, A. Castellino, V. De Sabbata, G. D. Maccarrone, R. Mignani, M. Pavsić and W. A. Rodrigues Jnr.

0004-9506/92/050591$05.00

way to build up a theory for them is trying to generalise the ordinary theories (starting with the classical relativistic one, only later passing to the quantum field theory) through *minimal extensions*, i.e. by performing modifications as small as possible.' This indicates that the best method for constructing a theory of tachyons is to extend the corresponding expressions from special relativity into a new domain of allowed particle speeds, with any alterations being kept to a minimum. Recami refers to this new domain as 'extended relativity', so the term will also be used here in order to assist with comparison and cross-referencing of material.

Some terms will be in common usage throughout this work, so they will be defined here. 'Special relativity' (SR) refers to all currently accepted physics for particles which travel more slowly relative to the observer than the speed of light. These particles will henceforth be called 'bradyons'. A 'tachyon' is defined to be a particle which is travelling relative to the observer at a speed greater than the speed of light. 'Extended relativity' (ER) is the theoretical framework which describes the motion and interactions of tachyons. A 'bradyonic observer' travels at a speed less than c, while a 'tachyonic observer' travels at a speed greater than c.

This work is a development of tachyonic kinematics and the basic postulates necessary for the formulation are given in Section 2. The tachyonic equivalent of the Lorentz transformations are then derived from these postulates. This is followed in Section 3 by a discussion of how tachyonic observers regard bradyons and tachyons, while Section 4 gives a consistent method for treating imaginary parts of square roots and also defines the 'γ-rule'. This rule plays a crucial role in the present formulation, as it is the means by which tachyons can be shown later to obey the laws of conservation of energy, momentum and electric charge. Section 5 consists of a development of the γ-rule, and includes a detailed numerical example to illustrate how this rule operates in practice. The γ-rule is the mathematical expression of 'switching', which is a slightly different version of the 'Reinterpretation Principle' expounded by Bilaniuk and Sudarshan (1969) and the 'Stückelberg–Feynman' switching principle expounded by Recami (1986). These mechanisms are similar to each other, and are the means by which each formulation allows tachyons to conserve energy, momentum and electric charge. The remaining sections of this paper contain discussions of tachyonic velocity transformations (Section 6), tachyonic rods and clocks (Section 7), conservation of electric charge (Section 8) and finally a discussion of the visual appearance of a tachyonic cube (Section 9).

The dynamics of tachyons, with particular reference to energy–momentum considerations, will be the subject of the second paper in this series (Paper II). A detailed discussion of tachyon electromagnetism will be the subject of the third paper in this series, and will include a demonstration that tachyons are fully consistent with Maxwell's equations. The fourth paper in this series will develop electrodynamics for tachyons, and will include a discussion of the behaviour of charged tachyons in a dielectric material.

It is implicitly assumed in this theory that tachyons are able to interact with ordinary, i.e. bradyonic, matter. A means of such interaction via mechanics and electrodynamics will be developed by the authors in subsequent papers. Furthermore, it is assumed throughout this work that there are no large gravitating masses present which would deflect the paths of photons. This work will therefore not consider any extension of General Relativity for tachyons.

2. The Tachyonic Transformations

The aim of any approach to the theory of tachyons must be to extend special relativity beyond the light barrier, and so the same postulates are applied to the study of both tachyons and bradyons.

Postulate 1: The laws of physics are the same in all inertial systems.

Postulate 2: The speed of light in free space has the same value c in all inertial systems.

The term 'inertial system' now refers to any system travelling at a constant velocity with respect to an inertial observer, irrespective of whether the system is travelling slower than or faster than the speed of light. The laws of physics that are treated as being the same in all inertial frames are the same as those in special relativity, namely the conservation laws of energy, momentum and electric charge and Maxwell's equations in free space.

If both energy and momentum are to be conserved as a consequence of the first postulate given above, then tachyons must be able to carry real energy and real momentum when they interact with bradyons. The second postulate means that electromagnetic effects travel at the same speed, regardless of whether they were generated by a charged tachyon or a charged bradyon.

Throughout this formulation of the tachyon theory the Euclidean metric will be used, even though Misner *et al.* (1973) consider it to be useless in the context of General Relativity. Firstly, imaginary factors have physical meaning when dealing with tachyons, so it is preferable that a metric be used which keeps all such factors undisguised as an aid to correct interpretation. Secondly, using the Euclidean metric highlights the symmetry between aspects of special relativity (SR) and extended relativity (ER). Of course, a consistent formulation of a theory of tachyons can also be constructed with the more usual Minkowski metric. All results can be converted between the two representations and in the final analysis, it is simply the personal preference of the authors which dictates the choice of representation, with the two reasons given above as added motivation for the present choice.

Henceforth the metric $(+1,+1,+1,+1)$ is used throughout this work, with the conventions that Latin indices i, j, ... run from 1 to 3 and Greek indices λ, μ, ν, ... run from 1 to 4. The position four-vector is written as $X_\lambda = (x_1, x_2, x_3, x_4) = (\mathbf{x}, ict) = (x, y, z, ict)$. Note that the metric $(+1,+1,+1,+1)$ applies to both bradyonic and tachyonic inertial reference frames, as the metric is independent of the observer's relative motion and is the same regardless of whether the observer is dealing with bradyons or tachyons. Furthermore, there is no distinction made in the following work between covariant and contravariant quantities.

For the timelike regions on a Minkowski diagram the square of the interval is written as

$$s^2 = x^2 + y^2 + z^2 + (ict)^2 = -c^2\tau^2\,, \tag{1}$$

where τ is the proper time interval and (x, y, z, ict) are the coordinates used by a bradyonic observer Σ. In bradyonic frame Σ', using coordinates (x', y', z', ict'), this is $c^2\tau^2 = -x'^2 - y'^2 - z'^2 - (ict')^2$, and as $c^2\tau^2$ is a Lorentz invariant then a transformation between two bradyonic frames Σ and Σ' gives

$$-(ict')^2 - x'^2 - y'^2 - z'^2 = -(ict)^2 - x^2 - y^2 - z^2 . \tag{2}$$

In SR the two postulates above lead to the Lorentz transformations:

$$x' = \gamma_u(x - ut), \; y' = y, \; z' = z, \; t' = \gamma_u(t - ux/c^2) , \tag{3}$$

where

$$\gamma_u = (1 - u^2/c^2)^{-1/2} \tag{4}$$

and $u^2 < c^2$. Here u is the relative speed along the common x, x' axes of an inertial frame Σ' with respect to an inertial reference frame Σ. The inverse Lorentz transformations are

$$x = \gamma_u(x' + ut'), \; y = y', \; z = z', \; t = \gamma_u(t' + ux'/c^2) . \tag{5}$$

For spacelike regions on a Minkowski diagram the square of the interval is defined to be

$$s^2 = x'^2 + y'^2 + z'^2 + (ict')^2 , \tag{6}$$

where (x', y', z', ict') are now the coordinates used by a tachyonic observer Σ', and again $s^2 = -c^2\tau^2$ is an invariant. For spacelike regions $c^2\tau^2 < 0$ and $s^2 > 0$, while for timelike regions $s^2 < 0$ and $c^2\tau^2 > 0$. Therefore τ is imaginary for spacelike regions, i.e. proper time for tachyons is imaginary, and so when transforming from a timelike to a spacelike region and vice versa, there is an extra factor of -1 multiplying one side of the equation. This factor of -1 is formally equivalent to either $(+i)^2$ or $(-i)^2$, and so a transformation between a bradyonic frame Σ and a tachyonic frame Σ' is given by

$$-x'^2 - y'^2 - z'^2 - (ict')^2 = (\pm i)^2(-x^2 - y^2 - z^2 - (ict)^2) . \tag{7}$$

Since a factor of $\pm i$ has effectively been inserted with each x, y, z and ict, it can be seen that the tachyonic transformations must have the same form as the Lorentz transformations, but with extra factors of $(\pm i)$:

$$x' = \pm i\gamma_u(x - ut), \; y' = \pm iy, \; z' = \pm iz, \; t' = \pm i\gamma_u(t - ux/c^2) . \tag{8}$$

Here γ_u is again given by (4), but now $u^2 > c^2$ and so γ_u is imaginary. The inverse tachyonic transformations can be shown by substitution to be

$$x = \mp i\gamma_u(x' + ut'), \; y = \mp iy', \; z = \mp iz', \; t = \mp i\gamma_u(t' + ux'/c^2) . \tag{9}$$

The sign indeterminacy must be resolved before further progress can be made. The following derivation of the tachyonic transformations is adapted from the relativistic case, as given for example by Rosser (1964).

It is assumed that there are two inertial reference frames Σ and Σ', each with its own set of rulers and synchronised clocks. Frame Σ is bradyonic, whereas frame Σ' is tachyonic. The clocks in frame Σ are synchronised with each other, keep good time and remain synchronised, while the clocks in frame Σ' are synchronised with each other and also keep good time and remain synchronised. The zero of time in both frames Σ and Σ' is chosen to be the instant when the origins O and O' of Σ and Σ' respectively coincide. Let frame Σ' move with uniform velocity $u > c$ along the common x and x' axes, as shown in Figs 1a and 1b. It is implicitly assumed that the inertial frames Σ and Σ' have been moving, and will continue to move, with a uniform velocity relative to each other.

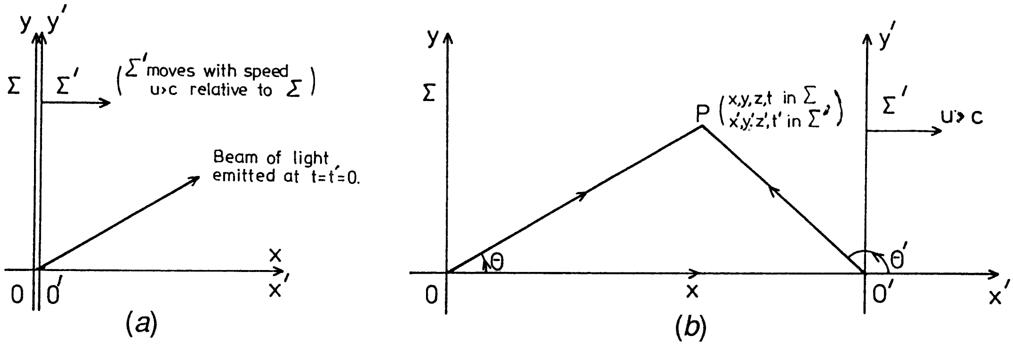

Fig. 1. In diagram (a) a beam of light is emitted at the instant the origins of frames Σ and Σ' coincide, where tachyonic frame Σ' moves with speed $u > c$ along the common x, x' axes relative to a bradyonic frame Σ. In diagram (b) the light reaches a detector at point P along the apparent paths OP in frame Σ and O$'$P in frame Σ'.

Now consider a beam of light emitted at the instant when the origins of frames Σ and Σ' coincide at $t = t' = 0$, as shown in Fig. 1a. In Fig. 1b the light reaches a detector at the point P at time t in frame Σ, and at time t' in frame Σ'. The coordinates of point P are (x, y, z, ict) in frame Σ and (x', y', z', ict') in frame Σ'. The light is propagated rectilinearly in frame Σ, so that an observer at rest in Σ considers the light to travel along the path OP such that

$$OP/t = (x^2 + y^2 + z^2)^{1/2}/t = c\,, \tag{10}$$

and so

$$x^2 + y^2 + z^2 + (ict)^2 = 0\,. \tag{11}$$

An observer at rest in frame Σ' also notes that the light travels rectilinearly and reaches the detector at P, but to this observer the light appears to travel along the path $O'P$ as shown in Fig. 1b. Hence in tachyonic frame Σ' the light travels such that

$$O'P/t' = (x'^2 + y'^2 + z'^2)^{1/2}/t' = c\,, \tag{12}$$

and so

$$x'^2 + y'^2 + z'^2 + (ict')^2 = 0\,. \tag{13}$$

Note that observers in frames Σ and Σ' measure the light as travelling with the same speed c: this follows directly from Postulate 2 above.

As Σ' is a tachyonic frame, both sides of (13) can be multiplied by -1 to give

$$-x'^2 - y'^2 - z'^2 - (ict')^2 = 0\,, \tag{14}$$

and equating (11) with (14) gives

$$x^2 + y^2 + z^2 + (ict)^2 = -x'^2 - y'^2 - z'^2 - (ict')^2\,. \tag{15}$$

This equation is formally equivalent to (7), which described how the square of the interval was transformed between the bradyonic frame Σ and the tachyonic frame Σ'. Hence the mathematical artifice of multiplying zero by -1, which was used to turn (13) into (14), does in fact have physical significance in this particular instance.

Just as in SR, it is assumed that the form of the longitudinal coordinate transformations are

$$x' = K(x - ut) \tag{16}$$

and

$$x = K'(x' + ut')\,, \tag{17}$$

from which we obtain

$$t' = K\left\{t - \frac{x}{u}\left(1 - \frac{1}{KK'}\right)\right\}. \tag{18}$$

Here K and K' are assumed to be independent of x, x', t and t', but they do depend upon the constant relative speed u between the two frames. An observer at rest in frame Σ measures frame Σ' as having speed u, so an observer at rest in frame Σ' should record the speed of frame Σ as $-u$. Substituting for x' and t' from (16) and (18) respectively into (14) and using $y' = \pm iy$, $z' = \pm iz$ leads to

$$\begin{aligned} 0 \;=\; & x^2K^2\left\{\frac{c^2}{u^2}\left(1 - \frac{1}{KK'}\right)^2 - 1\right\} + 2xtuK^2\left\{1 - \frac{c^2}{u^2}\left(1 - \frac{1}{KK'}\right)\right\} \\ & + y^2 + z^2 - (ict)^2K^2(1 - u^2/c^2)\,. \end{aligned} \tag{19}$$

Now, x, y, z and ict are the coordinates of the event in frame Σ corresponding to the detection of the light at P (see Fig. 1*b*), and so must be related as in (11). Thus the coefficients of x^2, xt and t^2 must be the same in (11) and (19). Equating the coefficients of t^2 gives

$$K = (u^2/c^2 - 1)^{-1/2} = i(1 - u^2/c^2)^{-1/2} \tag{20}$$

and equating the coefficients of xt leads to

$$K' = -i(1 - u^2/c^2)^{-1/2}\,. \tag{21}$$

Equations (16) and (17) now become

$$x' = i\gamma_u(x - ut) \tag{22}$$

and

$$x = -i\gamma_u(x' + ut') \tag{23}$$

where γ_u is given by (4). Substitution back into (18) gives

$$t' = i\gamma_u(t - ux/c^2) \tag{24}$$

and also

$$t = -i\gamma_u(t' + ux'/c^2)\,. \tag{25}$$

The sign of the transverse transformations must now be chosen to match the sign of the longitudinal transformations in x and t, in order to be consistent with the metric adopted for this work. Hence $y' = +iy$ and $z' = +iz$.

Collecting these results shows that the tachyonic transformations are

$$x' = i\gamma_u(x - ut),\; y' = iy,\; z' = iz,\; t' = i\gamma_u(t - ux/c^2)\,, \tag{26}$$

and the inverse tachyonic transformations are

$$x = -i\gamma_u(x' + ut'),\; y = -iy',\; z = -iz',\; t = -i\gamma_u(t' + ux'/c^2)\,. \tag{27}$$

These tachyonic transformations, which are also known as 'superluminal Lorentz transformations' or 'SLTs', are to be used when transforming between inertial frames on opposite sides of the light barrier, i.e. timelike to spacelike frames and vice versa. Note that the SLTs given by (26) are the same for both of the metrics discussed above.

These transformations are similar in form to those given by Corben (1975), although he used the opposite sign for the transformations of the transverse components and a real γ such that

$$\gamma = \pm(\beta^2 - 1)^{-1/2}, \text{ where } \beta c = u\,. \tag{28}$$

In the Recami (1986) formulation of extended relativity the SLTs are

$$x' = \mp\frac{x - Ut}{(U^2 - 1)^{1/2}},\quad y' = \pm iy,\quad z' = \pm iz,\quad t' = \mp\frac{t - Ux}{(U^2 - 1)^{1/2}}, \tag{29}$$

where $U^2 > 1$ and dimensionless units have been used so that $c = 1$. Apart from the sign indeterminacy, the other difference between Recami's SLTs and the ones derived here turns out to be a minus sign multiplying the transformations of x and t.

The tachyonic transformations in Antippa's (1975) formulation are

$$x' = \mu\gamma(x - \beta ct),\quad y' = y,\quad z' = z,\quad ct' = \mu\gamma(ct - \beta x)\,, \tag{30}$$

where

$$\beta = v/c, \quad \gamma = (| 1 - \beta^2 |)^{-1/2} \tag{31}$$

and $\mu = 1$ for subluminal transformations, $\mu = \beta/|\beta|$ for superluminal transformations.

Antippa's formulation omits the imaginary factor in the transverse transformations because he postulates a 'tachyon corridor' which effectively singles out a preferred direction of motion for tachyons. Such a corridor has no place in the formulation presented in this paper as the common x, x' axes are defined by the motion of one inertial observer relative to another such observer, and not by a special motion of the particle being considered.

3. Tachyonic Worldview

Consider the inertial frame used by a bradyonic observer Σ. In such a frame the observer considers himself to be at rest and measures all other objects relative to this coordinate system. The same is true for the inertial frame used by a tachyonic observer Σ'. In this frame the observer considers himself to be at rest and so measures all other objects relative to his own coordinate system, regardless of whether they are travelling faster than or slower than the speed of light. As light travels at speed c relative to the coordinate system used by Σ', then Σ' is able to calibrate his rods and synchronise his clocks in exactly the same manner as any other relativistic observer.

Now suppose an observer Σ sees a tachyon travelling with speed $+2c$ along the x axis. An observer Σ' also travelling at speed $+2c$ relative to Σ along this same axis will see the tachyon as being at rest relative to his own coordinate system, and will see the observer Σ travelling at speed $-2c$ along the x' axis. Hence a tachyonic observer Σ' sees other tachyons as bradyons, while bradyons appear to Σ' as tachyons. It is therefore reasonable that the transformations between two tachyonic observers will simply be the ordinary Lorentz transformations, since these observers think they are transforming between two bradyonic inertial reference frames.

Fig. 2 is a Minkowski diagram with the axes $(x_o, ict_o), (x', ict')$ and (x'', ict'') used by bradyonic observers Σ_o, Σ' and Σ'' respectively, while the axes (x'_T, ict'_T) and (x''_T, ict''_T) are used by tachyonic observers Σ'_T and Σ''_T respectively. The worldline of object A appears to observers Σ_o, Σ' and Σ'' as spacelike so they consider A to be a tachyon, while observers Σ'_T and Σ''_T see the worldline of A as timelike, so they consider A to be a bradyon. Likewise object B appears to Σ_o, Σ' and Σ'' as a bradyon but appears to Σ'_T and Σ''_T as a tachyon, which is in agreement with the work of Corben (1976).

Although it is possible in theory to transform between reference frames on opposite sides of the light barrier, in a given inertial reference frame a material object cannot actually cross the light barrier and change from being a bradyon to a tachyon and vice versa. In a given reference frame, an object which begins existence as a bradyon will always be a bradyon, while an object which begins existence as a tachyon will always remain a tachyon. Once a tachyon is created it always moves with a speed greater than c, even if it is interacting with other particles or fields.

Fig. $3a$ is a Minkowski diagram showing several sets of axes used by bradyonic observers Σ_o, Σ', Σ'' and Σ'''. Observer Σ_o uses axes (x_o, ict_o), with each of the other three observers having a speed u relative to Σ_o of $0{\cdot}2c$, $0{\cdot}5c$ and $0{\cdot}8c$ respectively. Fig. 3b shows the axes used by tachyonic observers Σ_T', Σ_T'' and Σ_T''' moving with speed u_T relative to bradyonic observer Σ_o of $5c$, $2c$ and $1{\cdot}25c$ respectively. Comparison between these two figures shows that pairs of observers whose axes are inverted, e.g. (x', ict') and (x_T', ict_T'), obey the condition $u.u_T = c^2$. Pairs of inertial frames which obey this condition are called 'dual

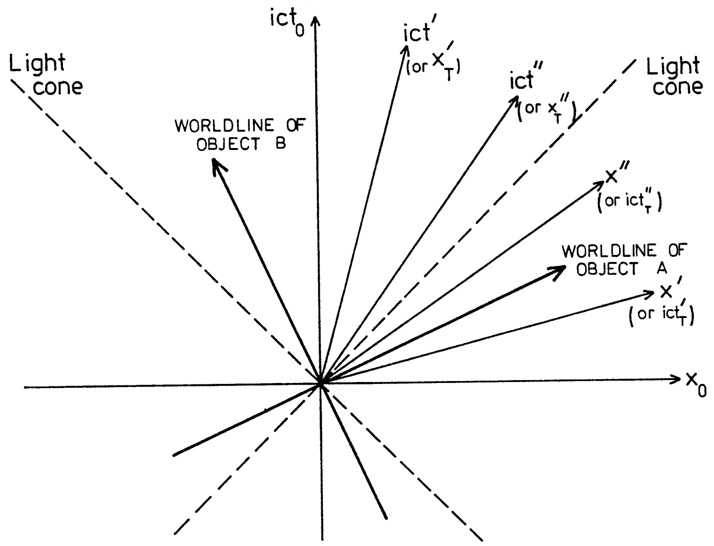

Fig. 2. Minkowski diagram showing how bradyonic observers Σ_o, Σ' and Σ'' consider object A to be a tachyon and object B to be a bradyon, while tachyonic observers Σ_T' and Σ_T'' consider object A to be a bradyon and object B to be a tachyon.

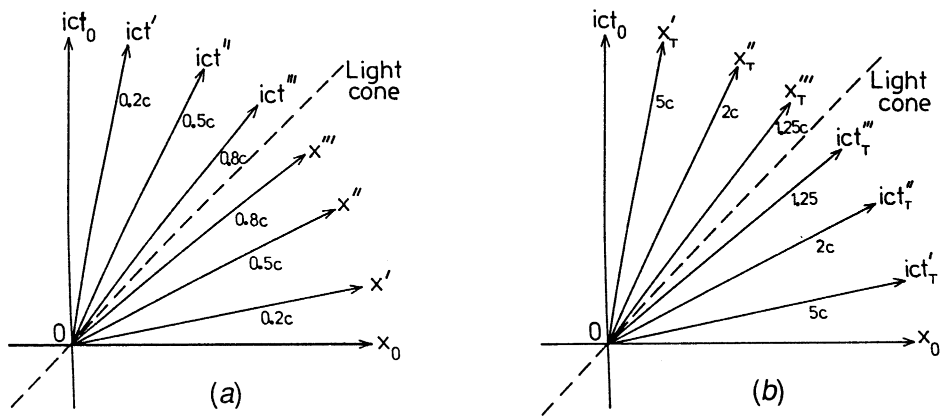

Fig. 3. Two Minkowski diagrams showing sets of axes used by various observers and their speeds relative to frame Σ_o. Pairs of observers whose axes satisfy the condition $u.u_T = c^2$ effectively have their space and time axes inverted, which is an important feature in ER.

frames', as introduced by Recami (1975) and Garuccio *et al.* (1980). When u_T is the speed of a tachyon the condition $u.u_T = c^2$ becomes a special case of some importance throughout the theory of tachyons.

Further inspection of Figs 3*a* and 3*b* shows that for both bradyonic and tachyonic observers, changing the speed further away from c relative to frame Σ_o moves the coordinate axes further away from the light cone. The angle on a Minkowski diagram between an observer's x or x' axis and the x_o-axis is given by $\tan\theta = u/c$, where $u^2 < c^2$ for bradyonic observers and $u^2 > c^2$ for tachyonic observers.

Returning to (7) it can be seen that the appearance of the factor $(\pm i)^2$ for tachyons has destroyed the Lorentz invariance of the space–time four-vector $X_\lambda = (\mathbf{x}, ict)$, $\lambda = 1,2,3,4$. When transforming between two inertial frames in ER the space–time four-vectors X_λ and X'_λ are related by

$$\sum_{\lambda=1}^{4}(X'_\lambda)^2 = \pm\sum_{\lambda=1}^{4}(X_\lambda)^2\,, \tag{32}$$

where the upper (+) sign refers to transformations between two bradyonic frames or two tachyonic frames. The lower (−) sign applies when transforming between a bradyonic frame and a tachyonic frame, and is due to the appearance of factors of $+i$ and $-i$ when transforming across the light barrier. While SR requires Lorentz invariance to hold, ER requires only that the magnitude of the four-vector squared is a constant.

The appearance of factors of i has resulted in a complex space–time for tachyons instead of the real space–time for bradyons. When transforming to tachyonic frames, the longitudinal space axis x' (parallel to the boost) is real since γ_u is imaginary for $u^2 > c^2$, while the space axes y' and z' transverse to the boost are both imaginary. The time axis ict' is imaginary for both bradyons and tachyons. Throughout this and following papers it will be seen that for tachyonic transformations all longitudinal variables are real, while all transverse variables are imaginary. Examples of these include velocities, momenta, forces, accelerations, electromagnetic fields and potentials and also current densities. However, while the tachyonic y' and z' axes are imaginary for us as bradyonic observers, tachyonic observers will regard their y' and z' axes as being real. Conversely, the y and z axes are real for us but are imaginary for tachyonic observers.

4. The Gamma Factor

Throughout the rest of this work the gamma factor will be given by (4) for both $u^2 < c^2$ and $u^2 > c^2$, even though it is imaginary in the latter case. In contrast to this choice, Corben (1975) has

$$\gamma_u = \pm(u^2/c^2 - 1)^{-1/2} \quad \text{for } u^2 > c^2\,, \tag{33}$$

and Recami (1986) has

$$\gamma_u = (u^2 - 1)^{-1/2} \quad \text{for } u^2 > 1\,, \tag{34}$$

where $c = 1$. The different choices in the form of γ_u represent different approaches to the SLTs in each of the formulations of the theory of tachyons. Choosing a different form of γ_u other than (4) has the effect of changing the positions of factors of $+i$ and $-i$ throughout the intermediate steps during calculations, but does not change any of the results. In this work and in following papers it will be seen that the imaginary factors usually cancel out during the course of calculations, so that there is no overall change in the results and conclusions of ER if γ_u is real for $u^2 > c^2$, as long as the SLTs are changed accordingly. The form of γ_u given by (4) has been adopted for convenience, as it is no longer necessary to distinguish between the bradyonic γ_u and the tachyonic γ_u.

During the derivation of expressions describing various tachyonic effects, it will at times be necessary to remove the imaginary part of γ_u, or to combine $|\gamma_u|$ with factors of i. In order to ensure an internally consistent set of signs is always obtained when decomposing γ_u into its component parts, or to recombine components to complete a γ_u, the following '$i - \gamma$ convention' will be used throughout ER. In an imaginary square root the factor of $+i$ is taken outside and the square root itself becomes real. For example, when $u^2 > c^2$ then

$$(1 - u^2/c^2)^{1/2} = i(u^2/c^2 - 1)^{1/2}\,, \tag{35}$$

so that

$$\gamma_u = (1 - u^2/c^2)^{-1/2} = -i(u^2/c^2 - 1)^{-1/2}\,. \tag{36}$$

When dealing with quantities such as length and volume which must be positive and real, it will be necessary to use modulus signs with intrinsically imaginary factors. In order to allow consistent decomposition and recombination of γ's with modulus signs, the following 'γ-rule' will be used:

$$\gamma_u = \mathrm{sign}(\gamma_u)|\gamma_u| \quad \text{for } u^2 < c^2\,, \tag{37}$$

$$\gamma_u = -i.\mathrm{sign}(\gamma_u)|\gamma_u| \quad \text{for } u^2 > c^2\,, \tag{38}$$

where

$$\mathrm{sign}(\gamma_u) = +1 \tag{39}$$

for all bradyons and unswitched tachyons, and

$$\mathrm{sign}(\gamma_u) = -1 \tag{40}$$

for switched tachyons. The terms 'unswitched tachyons' and 'switched tachyons' will be explained fully in the next section: only mathematical definitions are given here.

The $i - \gamma$ convention and the γ-rule are used extensively in ER, and as will be seen in a later paper in this series, they are especially important in the development of electromagnetism for tachyons. The origin of the γ-rule, as expressed by (37) and (38), will be shown in the next section when considering how a tachyon obeys the laws of conservation of energy and momentum as required by Postulate 1.

5. The Switching Principle

What is a Switched Tachyon?

In order to develop kinematics for tachyons, it is first necessary to show how they obey the laws of conservation of energy and momentum through the use of a 'switching principle' (Bilaniuk and Sudarshan 1969; Recami 1986, chapter 6). Fig. 4 is a Minkowski diagram showing the worldline of a tachyon T and several sets of axes, each used by a different bradyonic observer. The tachyon has speed $v > c$ in the reference frame of observer Σ_o, who uses axes (x_o, ict_o), and appears to travel forwards in time via the sequence $0 \to 1 \to 2$. Observer Σ', using axes (x', ict'), has a speed relative to Σ_o of $u' < c^2/v$ and hence sees T travel forwards in t'-time via the sequence $0 \to 1 \to 2$. Observer Σ'', using axes (x'', ict''), has a speed of $u'' = c^2/v$ relative to Σ_o and sees T as being stationary in t''-time, as events 0, 1 and 2 all appear to occur at the same time in this particular reference frame. Hence Σ'' considers T to have an infinite speed and indeterminate position. Observer Σ''', using axes (x''', ict'''), has a speed of $u''' > c^2/v$ relative to Σ_o. Since all inertial observers move forwards in time along their respective time axes, then Σ''' sees T travel forwards in t'''-time via

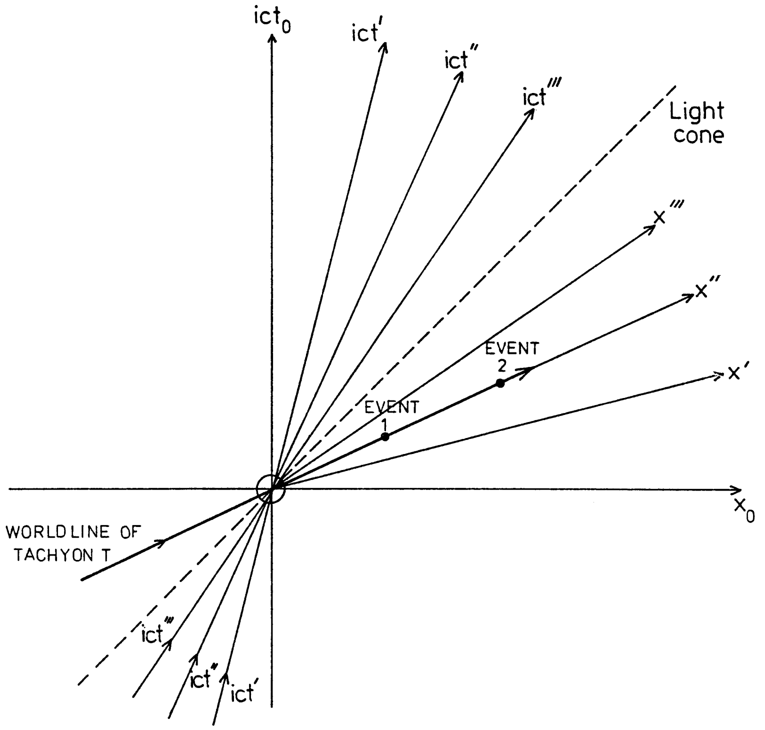

Fig. 4. Minkowski diagram demonstrating the apparent sequence of events involving a tachyon T at events 0, 1 and 2 according to various bradyonic observers. For observers Σ_o and Σ' the apparent sequence of events is $0 \to 1 \to 2$, for observer Σ'' the events 0, 1 and 2 all appear to occur at the same t''-time, while observer Σ''' sees the order of events in t'''-time as being $2 \to 1 \to 0$.

the sequence $2 \rightarrow 1 \rightarrow 0$. This means that for bradyonic observers using frames in which $u''' > c^2/v$ the tachyon T has reversed its apparent direction of motion and undergone '*switching*'. The condition $u'' = c^2/v$ is the boundary between bradyonic frames in which the tachyon appears to be 'switched' or 'unswitched'.

It must be stressed that switching does not affect the actual properties of the tachyon, but merely affects the apparent properties as seen by the observer. Henceforth any tachyons which have been switched will be denoted by a subscript minus sign, while ordinary, i.e. unswitched, tachyons will be denoted by a subscript plus sign.

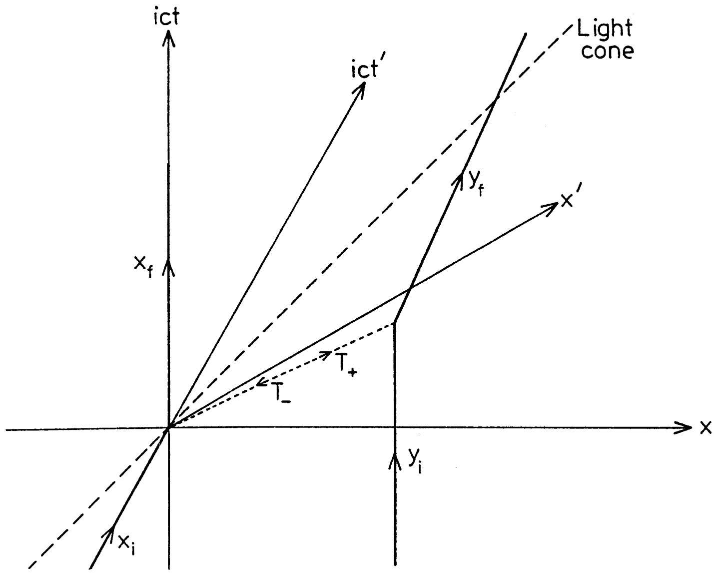

Fig. 5. Minkowski diagram of the exchange of a tachyon T between two bradyonic objects X and Y, as seen by bradyonic observers Σ and Σ'. Observer Σ sees T as an unswitched tachyon T_+ travelling from X to Y, while observer Σ' sees T as a switched tachyon T_- travelling from Y to X. At the X-vertex Σ sees the reaction $X_i \rightarrow X_f + T_+$ but Σ' sees the reaction $X_i + T_- \rightarrow X_f$, while at the Y-vertex Σ sees the reaction $Y_i + T_+ \rightarrow Y_f$ but Σ' sees the reaction $Y_i \rightarrow Y_f + T_-$.

Consider the exchange of a tachyonic particle T between two bradyonic objects X and Y, as shown in Fig. 5. An observer Σ using axes (x, ict) sees an unswitched tachyon T_+ carry positive energy, positive momentum and electric charge +Q from X to Y, while an observer Σ' using axes (x', ict') sees a switched tachyon T_- travel from Y to X. Observer Σ measures X as having lost positive energy, lost positive momentum and lost charge +Q, and at a later time measures Y as having gained a corresponding amount of energy, momentum and charge. In order to determine the properties of T_+ in the reference frame used by Σ, an appeal is made to the first postulate of ER: 'The laws of physics are the same in all inertial systems.' This means that tachyons must also conserve energy, momentum and charge in a given reference frame, just as bradyons always do. Therefore in Fig. 5 observer Σ will measure T_+ as having positive energy, positive momentum and charge +Q.

The question can now be asked: 'what are the properties of the exchanged tachyon as measured by observer Σ'?' The complication here of course is that in the reference frame used by Σ', the tachyon is switched and appears to travel from Y to X. From conservation of electric charge it is known that the tachyon T_- must be carrying charge $-Q$ from Y to X: this will be discussed in Section 8 and again in the third paper of this series. Observer Σ' measures X as initially being at rest and then gaining negative momentum when it 'absorbs' T_-, while Y initially has negative momentum but gains positive momentum by 'emitting' T_-. Therefore conservation of momentum indicates that T_- must in effect be carrying negative momentum from Y to X, which agrees with the apparent direction of motion of T_-.

To determine what energy is carried by T_- from Y to X, the initial and final energies of both X and Y must be considered. As both Σ and Σ' are bradyonic frames, the ordinary Lorentz transformations can be used to calculate both the net change in energy of X after 'absorbing' T_-, and of Y after 'emitting' T_-.

Numerical Example

To facilitate the discussion of the apparent properties of the exchanged tachyon, a numerical example based upon Fig. 5 will now be discussed in detail. Such a discussion serves several useful purposes. First, it will demonstrate the necessity of the γ-rule in order to explain how tachyons always obey the conservation laws, and also to show how the rule automatically comes into force. The example will further clarify the apparent properties of a switched tachyon, and finally, it will also demonstrate the internal consistency of this formulation of tachyon theory.

Each of the interactions at the X and Y vertices in Fig. 5 are assumed to be elastic. Let the incoming bradyons X_i and Y_i have proper masses of $10m_o$ and $12m_o$ respectively, and the outgoing bradyon X_f a proper mass of $9m_o$. In bradyonic frame Σ object X_i has speed $c/\sqrt{3}$, while both X_f and Y_i have speed zero. A second bradyonic observer Σ' has a speed of $u = c/\sqrt{3}$ relative to Σ, so that $\gamma_u = \sqrt{3/2}$. The speeds of various particles as measured by Σ' are $v_{X_i} = 0, v_{X_f} = v_{Y_i} = -c/\sqrt{3}$. Quantities in the following calculations have been rounded off to the fourth decimal place where appropriate.

First the properties of the tachyon as seen by observer Σ will be determined. The incoming bradyon X_i has momentum of $\gamma_u m_{X_i} v_{X_i} = 7\cdot0711 m_o c$, so that conservation of momentum indicates that T_+ must have momentum $7\cdot0711 m_o c$. From conservation of energy it can be seen that $E_{T_+} = E_{X_i} - E_{X_f} = 3\cdot2474 m_o c^2$. For tachyons the energy–momentum relation is

$$E^2 = p^2c^2 + m_*^2c^4\,, \tag{41}$$

where m_* is imaginary (this will be discussed in detail in the next paper in this series), so that T_+ has a proper mass of $m_* = 6\cdot2813 i m_o$. Its gamma factor is given by

$$\gamma_{T_+} = E_{T_+}/(m_*c^2) = -0\cdot5170 i\,, \tag{42}$$

and as $\gamma_{T_+} = (1 - v_{T_+}^2/c^2)^{-1/2}$ then the tachyon's speed v_{T_+} is $2\cdot1774c$. As a check of these calculations the tachyon's momentum is given by $p_{T_+} = \gamma_{T_+} m_* v_{T_+} =$

$7{\cdot}0710 m_o c^2$, which confirms the above result to within rounding error. Note that the factors of i not only cancelled out, but also helped to give the correct sign for real quantities. The angle made by the tachyon's worldline and the x axis is given by $\arctan(c/v_{T_+}) = 24°40'$, and as this is smaller than $\arctan(1/\sqrt{3}) = 30°$ then the tachyon will be switched when viewed by observer Σ' using axes (x', ict').

Both Σ and Σ' are bradyonic observers, and so to determine the energies of the various particles in an elastic collision, the ordinary Lorentz energy–momentum transformations are used. These transformations are

$$p_{x'} = \gamma_u(p_x - uE/c^2), \; p_{y'} = p_y, \; p_{z'} = p_z, \; E' = \gamma_u(E - up_x)\,. \tag{43}$$

This yields the energy of each bradyon in frame Σ' as $E'_{X_i} = 10 m_o c^2$, $E'_{X_f} = 11{\cdot}0227 m_o c^2$, $E'_{Y_i} = 14{\cdot}6969 m_o c^2$ and $E'_{Y_f} = 13{\cdot}6742 m_o c^2$. The tachyon's energy is clearly the difference between the initial and final energy of each bradyon. Hence the magnitude of the energy of the switched tachyon T_- is $1{\cdot}0227 m_o c^2$, but to determine its sign consider the following argument.

In frame Σ the tachyon is emitted by X_i, so that conservation of energy is written as $E_{X_i} = E_{X_f} + E_{T_+}$. However, observer Σ' sees T_- as an incoming particle at the X-vertex instead of the outgoing particle T_+ seen by Σ. (Remember that T_+ and T_- represent the same object viewed by different inertial observers.) Therefore Σ' measures the energies as

$$E'_{X_i} + E'_{T_-} = (10 + 1{\cdot}0227) m_o c^2 = E'_{X_f}\,. \tag{44}$$

At the Y-vertex the tachyon is absorbed by Y_i in frame Σ and emitted by Y_i in frame Σ', so that conservation of energy at the Y-vertex is expressed as

$$E_{Y_i} + E_{T_+} = (12 + 3{\cdot}2474) m_o c^2 = E_{Y_f} \tag{45}$$

in frame Σ, and

$$E'_{Y_i} = 14{\cdot}6969 m_o c^2 = E'_{Y_f} + E'_{T_-} \tag{46}$$

in frame Σ'. Therefore the observer Σ' treats the switched tachyon T_- in the same way as any other particle with regard to its collisions: when it appears to be emitted by Y_i it is a product and when it appears to be absorbed by X_i it is an incoming particle. Thus the energy of T_- is $+1{\cdot}0227 m_o c^2$, whereas in the Recami formulation of tachyonic theory a switched tachyon always has negative energy (Recami 1986, chapter 6). This is just one of many subtle differences between the present approach and that of Recami. In the present formulation switched tachyons can at times appear to have negative energies, but this depends upon the observer's reference frame and the relative velocities of the particles.

As both frames Σ and Σ' are bradyonic frames, then the energy of the switched tachyon in frame Σ' should also be given by the ordinary Lorentz energy transformation in (43). This gives

$$E'_{T_-} = \gamma_u(E_{T_+} - up_{T_+}) = -1{\cdot}0228 m_o c^2\,. \tag{47}$$

Since the energy of T_- has already been determined to be $+1\cdot0227m_oc^2$ (again note the rounding error), then the following 'γ-rule' will henceforth be used to give the correct sign for the energy.

For switched tachyons the negative root of γ_u is used and for unswitched tachyons the positive root of γ_u is used.

Written explicitly, this rule is

$$\gamma_u = \mathrm{sign}(\gamma_u)(1 - u^2/c^2)^{-1/2}\,, \tag{48}$$

where

$$\mathrm{sign}(\gamma_u) = +1 \tag{49}$$

if the particle appears to that observer to be an unswitched tachyon or a bradyon, and

$$\mathrm{sign}(\gamma_u) = -1 \tag{50}$$

if the particle appears to that observer to be a switched tachyon. Note that as there is no switching for a particle viewed by an observer to be a bradyon, then $\mathrm{sign}(\gamma_u)$ is always $+1$ and the standard result of SR is automatically recovered. Here the speed u is the relative speed between two inertial reference frames, and is distinct from v, the speed of the particle in the observer's inertial reference frame. The γ-rule gives

$$E'_{T_-} = \gamma_u(E_{T_+} - up_{T_+}) = +1\cdot0228m_oc^2 \tag{51}$$

so that the negative root of γ_u gives the correct result for energy conservation for a switched tachyon. The γ-rule is the mechanism by which switched tachyons always automatically conserve energy, momentum and electric charge, regardless of the observer's inertial reference frame.

In this type of system an observer can always tell whether to use a plus sign or a minus sign for the gamma factor, because the observer can determine whether the bradyons X and Y increase or decrease their proper masses when they absorb or emit the tachyon. Furthermore, it will be seen in a later section that the velocity transformations always indicate when a tachyon appears to a specific observer to be switched or unswitched. In the current example, X_i has a larger proper mass than X_f, so in frame Σ the positive root of γ_u applies and the negative root applies in frame Σ'. Similarly, Y_f has a larger proper mass than Y_i and so in frame Σ the positive root of γ_u is appropriate, while in frame Σ' the tachyon appears to be switched and the negative root of γ_u is appropriate.

The first test of the γ-rule is to determine whether it automatically gives conservation of momentum in frame Σ'. The Lorentz momentum transformation given by (43) gives the following values for the momentum of the bradyons in frame Σ': $p'_{X_i} = 0$, $p'_{X_f} = -6\cdot3640m_oc$, $p'_{Y_i} = -8\cdot4853m_oc$ and $p'_{Y_f} = -2\cdot1212m_oc$. Conservation of momentum at the X-vertex in frame Σ is expressed as $p_{X_i} = 7\cdot0711m_oc = p_{X_f} + p_{T_+}$, and in frame Σ' as $p'_{X_i} + p'_{T_-} = p'_{X_f} = -6\cdot3640m_oc$, so that $p'_{T_-} = -6\cdot3640m_oc$. Conservation of momentum at the Y-vertex in Σ is given by $p_{Y_i} + p_{T_+} = 7\cdot0711m_oc = p_{Y_f}$, and in frame Σ' it is given by

$p'_{Y_i} = -8\cdot 4853 m_o c = p'_{Y_f} + p'_{T_-}$ and so $p'_{T_-} = -6\cdot 3641 m_o c$. Using the negative root of γ_u in the Lorentz momentum transformation given by (43) gives the momentum of the switched tachyon in Σ' as

$$p'_{T_-} = \gamma_u (p_{T_+} - u E_{T_+}/c^2) = -6\cdot 3640 m_o c\,. \tag{52}$$

This agrees, to within a rounding error, with the tachyon's momentum as deduced from the law of conservation of momentum at the X-vertex and Y-vertex. Just as with the energy, it can be seen that using the negative root of γ_u for the switched tachyon automatically allowed the transformation to give the correct momentum.

The remaining details of the system can now be calculated as a check of the internal consistency of this formulation. The apparent energy of T_- in frame Σ' is $1\cdot 0227 m_o c^2$, so its gamma factor is $\gamma_{v'} = E'_{T_-}/m_* c^2 = -0\cdot 1628i$. As $\gamma_{v'} = (1 - v'^2_{T_-}/c^2)^{-1/2}$, the apparent speed of T_- is $-6\cdot 2234c$ and its momentum is $p'_{T_-} = \gamma_{v'} m_* v'_{T_-} = -6\cdot 3640 m_o c$. Thus the factors of $+i$ and $-i$ cancel out to give the correct sign, which shows this formulation has the necessary internal consistency.

The relativistic velocity transformation along the common x, x' axis is given by

$$v_{x'} = \frac{v_x - u}{1 - u v_x/c^2}\,, \tag{53}$$

where u is again the relative speed between the two reference frames. Using $u = c/\sqrt{3}$ gives a speed of the tachyon which agrees, to within rounding errors, with the speed of T_- calculated above.

For a bradyon both γ_v and the proper mass m_o are always positive and real, so that the bradyon's momentum is $\mathbf{p} = \gamma_v m_o \mathbf{v}$. The tachyon's momentum is given by $\mathbf{p} = \gamma_v m_* \mathbf{v}$, and so effectively the momentum is $\mathbf{p} = m\mathbf{v}$ for all particles in all frames, where m is the relativistic mass given by

$$m = \gamma_v m_o \quad \text{for bradyons,} \tag{54}$$

$$m = \gamma_v m_* \quad \text{for tachyons}\,, \tag{55}$$

with $\gamma_v = (1 - v^2/c^2)^{-1/2}$ for both $v^2 < c^2$ and $v^2 > c^2$.

Since X emits T_+ and absorbs T_- while Y absorbs T_+ and emits T_- depending on the reference frame, it is now obvious that the normal definitions of *source* and *detector* must be qualified when dealing with tachyons. This is done using the following definitions.

Sources emit unswitched tachyons and appear to absorb switched tachyons.

Detectors absorb unswitched tachyons and appear to emit switched tachyons.

In the numerical example above, bradyon X_i is a source which *intrinsically emits* the tachyon, while bradyon Y_i is a detector which *intrinsically absorbs* the tachyon. A more detailed discussion of intrinsic emission and intrinsic absorption has been given by Recami (1986, chapter 6).

6. Transformation of Velocity

The tachyonic transformations (SLTs) given by (26) can be written in differential form as $dx' = i\gamma_u(dx - u.dt)$, $dy' = i.dy$, $dz' = i.dz$, $dt' = i\gamma_u(dt - u.dx/c^2)$, with the inverse transformations being given by $dx = -i\gamma_u(dx' + u.dt')$, $dy = -i.dy'$, $dz = -i.dz'$, $dt = -i\gamma_u(dt' + u.dx'/c^2)$. Here u is the speed of tachyonic frame Σ' relative to bradyonic frame Σ. These lead straight to the ER velocity transformations:

$$\frac{dx'}{dt'} = v_{x'} = \frac{v_x - u}{1 - uv_x/c^2}, \quad \frac{dy'}{dt'} = v_{y'} = \frac{v_y}{\gamma_u(1 - uv_x/c^2)},$$
$$\frac{dz'}{dt'} = v_{z'} = \frac{v_z}{\gamma_u(1 - uv_x/c^2)}, \tag{56}$$

and the inverse ER velocity transformations:

$$\frac{dx}{dt} = v_x = \frac{v_{x'} + u}{1 + uv_{x'}/c^2}, \quad \frac{dy}{dt} = v_y = \frac{v_{y'}}{\gamma_u(1 + uv_{x'}/c^2)},$$
$$\frac{dz}{dt} = v_z = \frac{v_{z'}}{\gamma_u(1 + uv_{x'}/c^2)}. \tag{57}$$

Equations (56) and (57), which are valid for $u^2 > c^2$, have exactly the same form as the corresponding relativistic velocity transformations for $u^2 < c^2$. As γ_u also has the same form in both cases, then (56) and (57) are valid for $-\infty < u < \infty$. Note that $v_{x'}$ is always real, while $v_{y'}$ and $v_{z'}$ are real for $u^2 < c^2$ and imaginary for $u^2 > c^2$. When allowance is made for the $i - \gamma$ convention, these expressions differ from the velocity transformations given by Maccarrone and Recami (1984) and Recami (1986) by a factor of -1 in the transverse components. This is due to the slightly different form for the SLTs in the two formulations.

Equation (57) leads to the useful equality

$$\gamma_v = \gamma_u \gamma_{v'}(1 + uv_{x'}/c^2), \tag{58}$$

where $v^2 = v_x^2 + v_y^2 + v_z^2$ and $v'^2 = v_{x'}^2 + v_{y'}^2 + v_{z'}^2$ and $\gamma_u = (1 - u^2/c^2)^{-1/2}$, $\gamma_v = (1 - v^2/c^2)^{-1/2}$, $\gamma_{v'} = (1 - v'^2/c^2)^{-1/2}$. This expression and its inverse

$$\gamma_{v'} = \gamma_u \gamma_v (1 - uv_x/c^2) \tag{59}$$

are valid in both SR and ER.

Two numerical examples will now be discussed in order to demonstrate how the velocity transformations automatically indicate whether or not a tachyon appears to be switched relative to an observer. In the first example an observer Σ sees a particle travel with speed $v_x = 1{\cdot}25c$ along the x axis. A second observer Σ' moves with constant speed u along the common x, x' axes relative to Σ and sees the particle travel along the x' axis with speed $v_{x'}$. Fig. 6a is a graph of the observed speed of the particle $v_{x'}$ as a function of the relative speed u between the two frames. For $u < 0{\cdot}8c$ (the dual speed to $1{\cdot}25c$), it can be seen that $v_{x'} > 1{\cdot}25c$ and the particle appears to be an unswitched

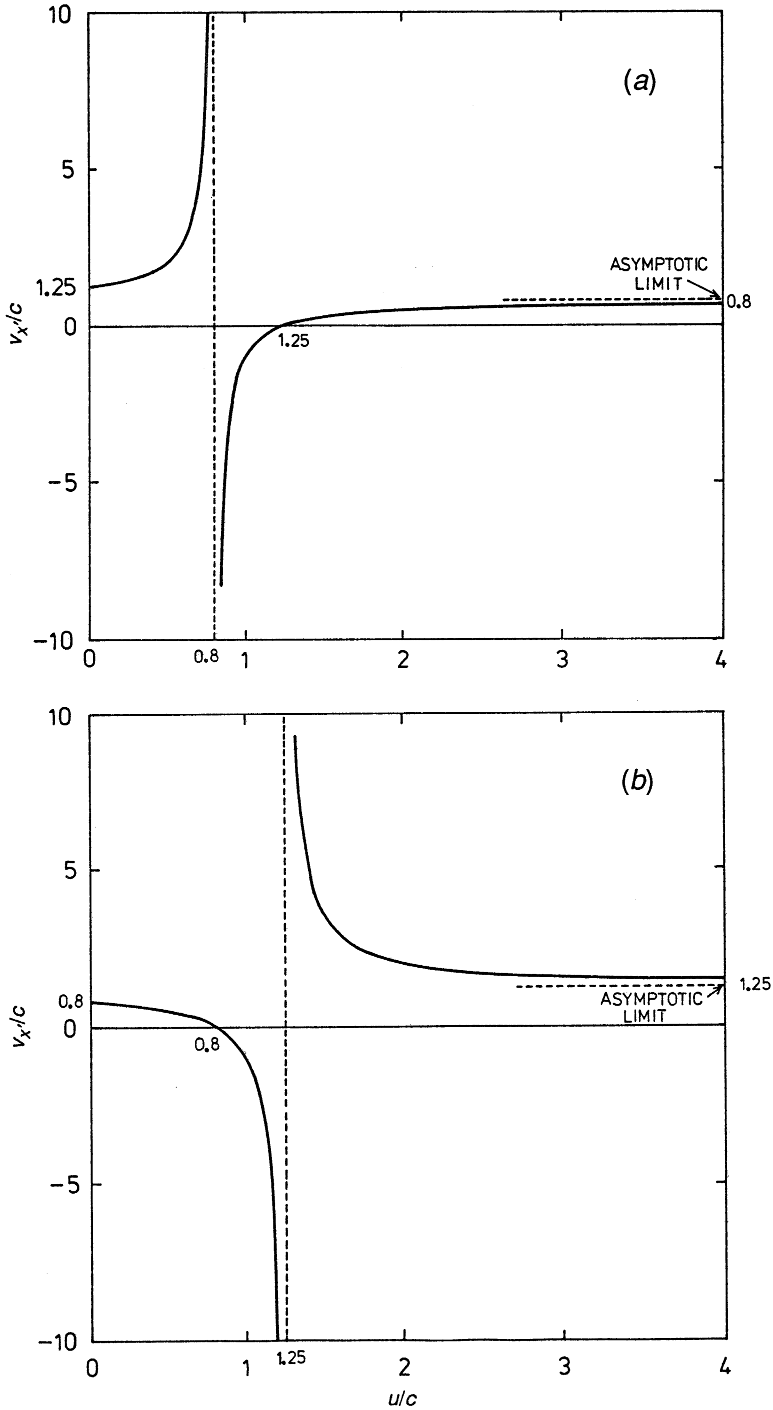

Fig. 6. Observed speed $v_{x'}$ as a function of the relative speed u/c between two inertial reference frames for a particle speed of (a) $v_x = 1 \cdot 25c$ and (b) $v_x = 0 \cdot 8c$.

tachyon. At $u = 0\cdot 8c$ the particle appears to be a tachyon with infinite speed. For $0\cdot 8c < u < c$ the observed speed $v_{x'}$ is negative, which indicates that the particle's apparent direction of motion has reversed and so it appears to Σ' as a switched tachyon. For $c < u < 1\cdot 25c$ the particle still appears to Σ' to have negative speed, but now $|v_{x'}| < c$ so that the particle appears to Σ' to be a bradyon. For $u = 1\cdot 25c$ the particle appears to Σ' to be a bradyon at rest, while for $u > 1\cdot 25c$ the particle appears to Σ' to be a bradyon with a positive relative speed. This indicates that an observer Σ', moving with speed $u > c$ relative to Σ, will see other particles moving with speeds greater than c (relative to Σ) as bradyons. This agrees with the discussion in Section 3, in which it was argued that tachyons would see other tachyons as bradyons, and illustrates in detail the general result given by Corben (1976). It also demonstrates how the velocity transformations automatically show the bradyonic frames in which the tachyon appears to the observer to have undergone switching, and that these frames are the ones obeying the condition $c > u > c^2/v_x$. This same condition was deduced in Section 5 using Minkowski diagrams, showing that these methods are consistent with each other in ER.

In the second example an observer Σ sees a particle travelling along the x axis with speed $v_x = 0\cdot 8c$. Observer Σ' again travels along the common x, x' axes with speed u and sees the particle as having speed $v_{x'}$. A plot of $v_{x'}$ as a function of u for this system is given in Fig. 6*b*. For $0 < u < c$ it can be seen that $v_{x'}$ exhibits all the correct behaviour according to special relativity: $v_{x'}$ is positive for $u < 0\cdot 8c$, zero for $u = 0\cdot 8c$ and negative for $0\cdot 8c < u < c$. In each of these cases Σ' sees the particle as a bradyon. For $c < u < 1\cdot 25c$ (here $1\cdot 25c$ is the dual speed) observer Σ' sees the particle as a switched tachyon. At $u = 1\cdot 25c$ the particle appears to Σ' as a tachyon with infinite speed and zero energy, while for $u > 1\cdot 25c$ the particle appears to Σ' to be an unswitched tachyon. Hence an observer travelling faster than the speed of light sees bradyons as tachyons and, depending upon the relative speed, even sees some of the bradyons as switched tachyons.

These two examples have demonstrated the mathematical condition for switching. Here v_x is the speed of the particle in the initial frame Σ, while u is the speed of the final frame Σ' relative to Σ. The particle will appear to Σ' to be switched if

$$c > u > c^2/v_x \text{ for } v_x > c \text{ and } |u| < c, \text{ or} \tag{60}$$

$$c < u < c^2/v_x \text{ for } v_x < c \text{ and } |u| > c\,. \tag{61}$$

The velocity transformations automatically showed whether the particle is switched or unswitched relative to a particular observer. However, in both examples the particle had to appear to the observer as a tachyon to be switched, even though $v_x = 1\cdot 25c$ in the first example and $v_x = 0\cdot 8c$ in the second example. The particle appeared to behave normally according to SR when its apparent speed made it appear to the final observer as a bradyon.

The velocity transformations agree with the second postulate of ER given in Section 2. Putting $v_x = c$ into (56) gives $v_{x'} = c$, regardless of whether Σ' is a bradyonic or tachyonic observer. Thus tachyonic observers will measure the speed of photons in a vacuum as being c, even though those tachyonic observers are travelling at speeds far greater than c relative to bradyonic observers. The velocity transformations can also be used to prove Corben's (1975) result that for any three inertial reference frames, the relative speeds between them are either all less than c, or two are greater than c and one is less than c.

As a tachyon appears to have infinite speed in the dual frame of a bradyonic observer, then such a tachyon may instantaneously transfer momentum and charge between two objects. This is a distinct property of tachyons and so it would be a definitive test for their existence. Further discussion of how tachyons could possibly be involved in instantaneous transfers between particles can be found in the review paper by Recami (1986). Of course, in all other bradyonic frames the tachyon has a finite transit time between two objects.

7. Rods and Clocks

Introduction

Having developed the switching principle and the γ-rule, it is now possible to examine the behaviour of tachyonic rods and clocks. As the tachyonic transformations have a similar form to the Lorentz transformations, it is expected that contraction and dilation effects also apply to tachyonic rods and clocks. Time dilation effects are apparent in Fig. 4, in which it is clear that the measured time interval between the events involving the tachyon is different for each of the observers. However, the new range of speeds means that the magnitude of γ_u is greater than 1 for $u^2 < 2c^2$ and less than 1 for $u^2 > 2c^2$. Therefore it is anticipated that some apparent differences in behaviour between tachyonic and bradyonic rods and clocks will occur. There is of course a further complication due to the possibility that the tachyonic rod or clock appears to undergo switching in some reference frames.

Both bradyonic and tachyonic observers must still use light signals to perform synchronisation of clocks and calibration of rods. This is a direct consequence of the second postulate, which states that the speed of light in free space is constant for all inertial observers. These observers cannot use tachyons to synchronise clocks for the same reason that bradyons cannot be used: the apparent velocity of the particle depends upon the velocity of the observer relative to a fixed inertial reference frame. As tachyonic observers consider themselves and each other to be bradyons which travel more slowly than the speed of light, they can use photons to communicate information to each other (Corben 1976). This means that a pair of tachyonic observers investigating tachyonic rods and clocks is equivalent to a pair of bradyonic observers investigating bradyonic rods and clocks. Therefore it only remains to determine what happens when a bradyonic observer investigates tachyonic rods and clocks.

Rods

Imagine a rod lying at rest along the x' axis of frame Σ'. The ends of the rod are at x_1' and x_2' so that its rest length is $x_2' - x_1' > 0$. Now suppose that the rod is moving with speed $u > c$ along the x-axis relative to an observer in frame Σ, so that Σ considers the rod to be a tachyonic object. The SLTs give $x_1' = i\gamma_u(x_1 - ut_1)$, $x_2' = i\gamma_u(x_2 - ut_2)$, so that $x_2' - x_1' = i\gamma_u(x_2 - x_1)$, where it is assumed that the clocks in frame Σ are synchronised so that $t_1 = t_2$ when x_1 and x_2 are measured. In the complex plane the distance between any two points z and a is $|z - a|$ (Kreyszig 1983). The modulus signs are necessary as length is always a positive quantity, regardless of the actual coordinates (real or imaginary) of the points being measured. Hence the apparent length of the tachyonic rod as viewed in bradyonic frame Σ must be $|x_2 - x_1|$, so that

$$|x_2 - x_1| = |(u^2/c^2 - 1)^{1/2}|(x_2' - x_1') . \tag{62}$$

The SR equivalent for a bradyonic rod is

$$x_2 - x_1 = (1 - u^2/c^2)^{1/2}(x_2' - x_1') . \tag{63}$$

For $c^2 < u^2 < 2c^2$ the length of the rod measured in frame Σ is shorter than its rest length in Σ', so the rod is contracted, just as it is for $u^2 < c^2$. For $u^2 = 2c^2$ the rod appears to have the same length in both frames, while for $u^2 > 2c^2$ the length of the rod appears to be dilated so that the rod is longer in Σ than it is in Σ'.

If the rod is at rest along one of the transverse axes in Σ', i.e. the y' or z' axes, then the rod's apparent length in frame Σ is the same as in Σ'. For example, if the rod is at rest along the y' axis then its length as measured by Σ' is $y_2' - y_1' > 0$. (Remember that Σ' considers the y' and z' axes to be real, even though they are imaginary for Σ.) The apparent length of the rod as measured by Σ is $y_2 - y_1 = |iy_2' - iy_1'| = y_2' - y_1'$.

Fig. 7 contains worldlines representing a rod moving with speed $v > c$ relative to a bradyonic observer Σ_o who uses axes (x_o, ict_o). The rest frame of the tachyonic rod is Σ_T'' who uses axes (x_T'', ict_T''). In such a frame the end of the rod

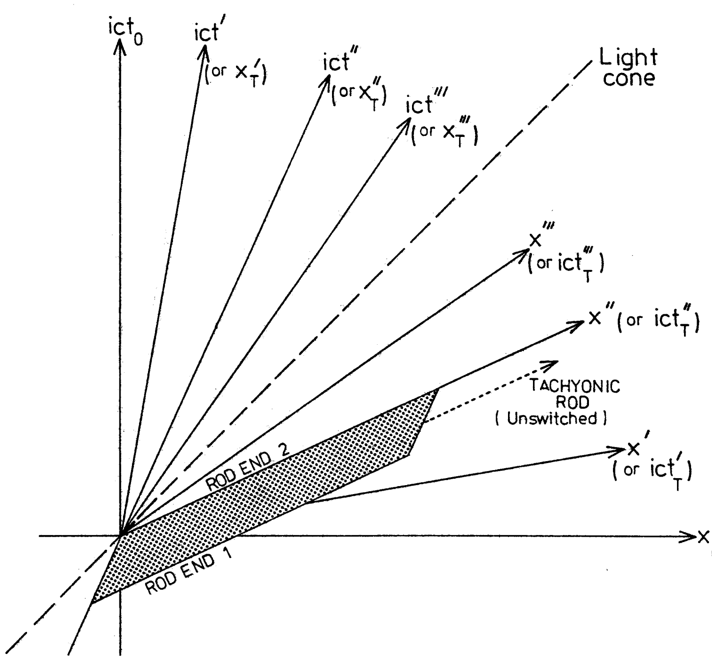

Fig. 7. Minkowski diagram showing the axes used by various observers and the worldlines of a tachyonic rod, whose rest frame is Σ_T'' in which the rod's proper length is $x_{T_2}'' - x_{T_1}'' > 0$.

labelled '2' leads the end of the rod labelled '1' so that $x''_{T_2} - x''_{T_1} > 0$. Another tachyonic observer Σ'_T using axes (x'_T, ict'_T) sees end 2 lead end 1 in the apparent direction of motion, which in this frame is in the positive x'_T direction. A third tachyonic observer Σ'''_T using axes (x'''_T, ict'''_T) considers the rod to be moving in the negative x'''_T direction, as the rod has a negative velocity in this frame due to the observer's relative speed. Observer Σ'''_T still measures $x'''_{T_2} - x'''_{T_1} > 0$, but now end 1 leads end 2 in the apparent direction of motion.

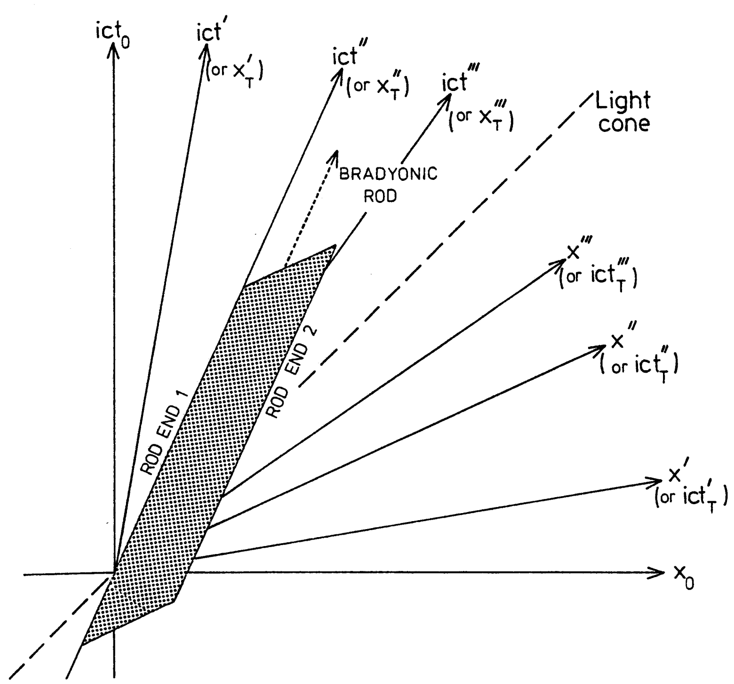

Fig. 8. Minkowski diagram showing the axes used by various observers and the worldlines of a bradyonic rod, whose rest frame is Σ''.

Now consider the bradyonic reference frames used by observers Σ', Σ'' and Σ''', who use the coordinate axes (x', ict'), (x'', ict'') and (x''', ict''') respectively. Observers Σ_o and Σ' view the tachyonic rod such that end 1 leads end 2 in the apparent direction of motion, and so $x_{o_2} - x_{o_1} < 0$ and $x'_2 - x'_1 < 0$. For observer Σ'' the length $|x''_2 - x''_1|$ is indeterminate, while observer Σ''' measures $x'''_2 - x'''_1 > 0$. Note that the tachyonic rod is unswitched in frames Σ_o and Σ', but appears to be switched in frame Σ'''. The rod appears to observers Σ_o and Σ' to be travelling with positive velocity and with end 1 leading end 2, but for the switched frame used by observer Σ''' the rod appears to be travelling in the negative x''' direction and so has negative velocity, but with $x'''_2 - x'''_1 > 0$. Hence for all bradyonic observers end 1 appears to be leading end 2 in the apparent direction of motion. Combining this result with the discussion of the apparent behaviour of the rod in tachyonic frames leads to the following conclusion: end 1 appears to lead end 2 in the apparent direction of motion in all reference frames in which the observer has a relative speed less than that of the tachyonic rod. In all frames in which the observer has a greater relative speed than that of the rod, end 2 appears to lead end 1 in the apparent direction of motion.

If the rod in Fig. 7 were travelling such that its worldlines appeared in the fourth octant of the Minkowski diagram, then the relevant observer's axes should be reflected about the ict_o axis. This gives the same results as for the tachyonic rod travelling through the first octant. If the modulus signs are removed from (62) it can be seen that the resultant sign is opposite to what one would expect from the above discussion of the tachyonic rod. This means that the SLTs should not be used just to determine which end leads the other one in any particular reference frame: a Minkowski diagram is adequate for this task.

Fig. 8 shows the worldlines of the ends of a bradyonic rod, along with the axes used by the same set of observers as in the previous figure. In this case the bradyonic observers Σ_o and $\Sigma^{'}$ see that end 2 leads end 1 such that $x_{o_2} - x_{o_1} > 0$ and $x_2^{'} - x_1^{'} > 0$, and that the rod moves in the positive x_o and $x^{'}$ directions. In frame $\Sigma^{''}$ the rod is at rest with $x_2^{''} - x_1^{''} > 0$. In frame $\Sigma^{'''}$, which moves faster than the rod relative to frame Σ_o, the rod has negative relative speed but still has $x_2^{'''} - x_1^{'''} > 0$. In this frame the relative speed causes end 1 to appear to lead end 2 in the motion along the negative $x^{'''}$ direction.

Now consider the motion of the rod in Fig. 8 as viewed by the tachyonic observers $\Sigma_T^{'}$, $\Sigma_T^{''}$ and $\Sigma_T^{'''}$. In frame $\Sigma_T^{'}$ the rod appears to have positive speed, end 1 leads end 2 and $x_{T_2}^{'} - x_{T_1}^{'} < 0$. In frame $\Sigma_T^{''}$ the rod appears to have infinite speed and so its length is indeterminate. In the tachyonic frame $\Sigma_T^{'''}$ the rod has undergone switching and appears to move in the negative $x_T^{'''}$ direction. In this case $x_{T_2}^{'''} - x_{T_1}^{'''} > 0$ and end 1 leads end 2 in the apparent direction of motion. Hence for all frames with a relative speed greater than that of the bradyonic rod, it appears to the observer that end 1 leads end 2 in the apparent direction of motion. Conversely, in all frames in which the observer has relative speed less than that of the bradyonic rod, end 2 appears to lead end 1 in the apparent direction of motion.

Clocks

Now suppose there is a clock at rest in frame $\Sigma^{'}$, and that $\Sigma^{'}$ moves with speed $u > c$ relative to frame Σ. The time interval in $\Sigma^{'}$ is $t_2^{'} - t_1^{'} > 0$. Using the SLTs gives the corresponding times recorded in Σ as $t_1 = -i\gamma_u(t_1^{'} + ux_1^{'}/c^2)$ and $t_2 = -i\gamma_u(t_2^{'} + ux_2^{'}/c^2)$ so that $t_2 - t_1 = -i\gamma_u(t_2^{'} - t_1^{'})$, where the clocks have been arranged so that $x_1^{'} = x_2^{'}$. All observers move forwards in time along their respective time axes: $ict^{'}$ for observer $\Sigma^{'}$, ict for observer Σ. Therefore the time interval between two events must be positive for each observer. However, due to switching the apparent order of the two events may be reversed. Hence the elapsed time interval as measured by Σ is given by

$$|t_2 - t_1| = |-i\gamma_u|(t_2^{'} - t_1^{'}) = (t_2^{'} - t_1^{'})|(u^2/c^2 - 1)^{-1/2}|\,. \qquad (64)$$

The equivalent expression in SR for the time interval is

$$t_2 - t_1 = (t_2^{'} - t_1^{'})(1 - u^2/c^2)^{-1/2}\,. \qquad (65)$$

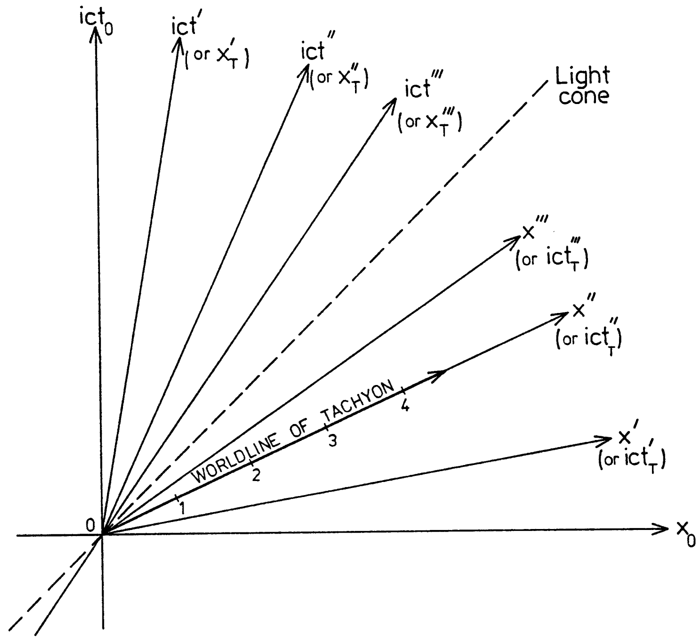

Fig. 9. A tachyonic clock is at rest in frame Σ_T'', with the points marked 0, 1, 2, 3 and 4 representing ticks of the clock. In tachyonic frames Σ_T' and Σ_T''' and bradyonic frames Σ_o and Σ' the clock appears to tick in the sequence 0, 1, 2, 3, 4. In bradyonic frame Σ'' the ticks occur at the same t''-time, while in bradyonic frame Σ''' the ticks occur in the sequence 4, 3, 2, 1, 0.

Note that (64) only gives the elapsed time between events in frame Σ: it does not indicate which event appears to occur first in that particular frame. For $c^2 < u^2 < 2c^2$ the clock appears to Σ to be slowed down, just as it would be for $u^2 < c^2$. For $u^2 = 2c^2$ the clock appears to run at the same rate in both frames, while for $u^2 > 2c^2$ the clock as seen by Σ will appear to run fast.

Fig. 9 shows a Minkowski diagram of a tachyonic clock passing successively through the points 0, 1, 2, 3 and 4. The coordinate axes used by bradyonic observers Σ_o, Σ', Σ'' and Σ''' and tachyonic observers Σ_T', Σ_T'' and Σ_T''' are the same as those in the previous figure. In all of the tachyonic frames Σ_T', Σ_T'' and Σ_T''' the clock appears to travel forwards in time via the sequence 0, 1, 2, 3, 4. The bradyonic observers Σ_o and Σ' also see the clock travel through the sequence 0, 1, 2, 3, 4 and so $t_{o_2} - t_{o_1} > 0$ and $t_2' - t_1' > 0$. For observer Σ'' the clock appears to have infinite speed and the points 0, 1, 2, 3, 4 all occur at the same t''-time. As Σ'' is the dual frame to Σ_T'' the apparent time interval between the points is zero. For observer Σ''' the tachyonic clock appears to have undergone switching, so that it travels forwards along the ict'''-axis via the sequence 4, 3, 2, 1, 0. (see also Fig. 4). Hence the correct time ordering and the apparent time interval in any bradyonic frame is given by

$$t_2 - t_1 = (t_2' - t_1')(u^2/c^2 - 1)^{-1/2}\,, \tag{66}$$

where $u^2 > c^2$ and the sign of the square root indicates whether the tachyonic clock is unswitched (+ root) or switched (− root). For $t_2' - t_1' > 0$ in the tachyonic clock's rest frame, equation (66) gives $t_2 > t_1$ for unswitched frames and $t_2 < t_1$ for switched frames.

8. Conservation of Electric Charge

The first postulate of ER that tachyons must obey the laws of physics means that in a given inertial reference frame there must be conservation of electric charge. While a detailed discussion of this topic will be given in the third paper of this series, a derivation will be given here to show how the γ-rule automatically allows charge to be conserved in each frame.

Consider what happens to the electric charge carried by the exchanged tachyon in Fig. 5. Observer Σ sees T_+ carry charge +Q from X to Y, while conservation of charge in frame Σ' however, indicates that T_- must carry charge −Q from Y to X. As T_+ and T_- are in fact the same particle viewed from two separate bradyonic reference frames, then the apparent disparity in the electric charge they carry disagrees with the result of SR, according to which electric charge is an invariant. In order to resolve this difficulty, it is necessary to digress briefly and discuss the volume of a tachyon.

Consider a cube which has sides of length l_o in its own rest frame. If the cube has speed $|u| < c$ relative to the observer, it will appear to have a volume of $(l_o/\gamma_u)(l_o)(l_o) = l_o^3/\gamma_u$, where γ_u is real. The volume of the cube when $|u| > c$ and γ is imaginary is given by

$$|il_o/\gamma_u|.|il_o|.|il_o| = l_o^3/|i\gamma_u|\,, \tag{67}$$

where each side has a length which is positive and real, even though the transverse dimensions are imaginary. Therefore by extension all tachyonic objects will have real, positive volumes, regardless of the observer's reference frame.

Let $d\omega_o$ be the volume of a small element of charge as measured from an inertial frame Σ_o, relative to which the charge is instantaneously at rest. The total charge within the element is equal to $\rho_o d\omega_o$, where ρ_o is the proper density of proper charge. In a second frame Σ' travelling with speed $v' > c$ with respect to Σ_o the charge density is $\rho' = i\gamma_{v'}\rho_o$. The factor i has appeared because both charge and volume are always real regardless of the observer's inertial reference frame, and so the charge density must also be real: this will be proved in Paper III. The volume of this element as measured by Σ' is $d\omega' = d\omega_o/|i\gamma_{v'}|$, which is real and positive. Therefore the total charge within the element as measured by Σ' is

$$\rho' d\omega' = \frac{i\gamma_{v'}\rho_o d\omega_o}{|i\gamma_{v'}|} = \pm\rho_o d\omega_o\,, \tag{68}$$

where the + sign applies if the positive root of $\gamma_{v'}$ is used (tachyon is unswitched) and the − sign applies if the negative root of $\gamma_{v'}$ is used (tachyon is switched). As the positive root of $\gamma_{v'}$ is always used for bradyons, then the standard result from SR is obtained: electric charge is an invariant for bradyons viewed by bradyonic observers.

In the example above of tachyon exchange, observer Σ sees the unswitched tachyon T_+ carry charge $+Q$ from X to Y as Σ uses the positive root of γ_u. Observer Σ' however must use the negative root of γ_u as he sees a switched tachyon T_-, and so (68) indicates that Σ' sees T_- carry charge $-Q$ from Y to X.

Switching is purely an artefact of the observer's motion relative to the viewed object. The object itself does not change in any way because of switching, only the observer's perception of the object changes. For example, a tachyonic electron will always be an electron, even though observers in switched frames will measure it as having a positive charge. Hence the Feynman picture of a positron as being an electron going backwards in time is not immediately applicable to the case of a switched tachyon.

The sign change due to switching will have subtle but far reaching effects in any tachyonic system. For example, the sign change on the electric charge will carry through any calculations involving electromagnetism, which will be considered in detail in Paper III. Examples in that paper will include a calculation of the electric and magnetic fields generated by a charged tachyon, as well as investigations of the tachyonic Doppler effect and retarded potentials.

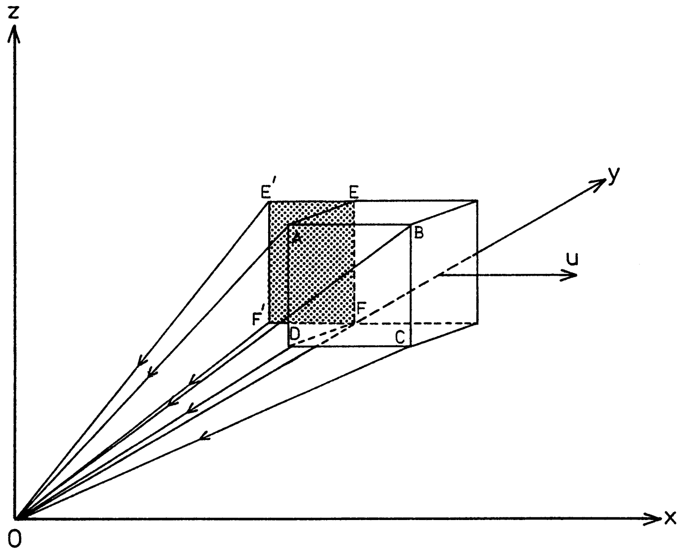

Fig. 10. A demonstration of the apparent rotation of a tachyonic cube as seen by an observer at O.

9. Visual Appearance of a Tachyonic Cube

Any object which is moving at a relativistic speed relative to the observer appears to undergo a rotation, and so a tachyonic object should also appear to be rotated. The following discussion is adapted from the relativistic case given by Rosser (1964).

Consider a cube which has edge length l_o in its rest frame. The cube is moving with a uniform velocity v relative to an inertial frame Σ, as shown in Fig. 10. Let the cube be viewed from a large distance in a direction perpendicular to its direction of motion, so that the angle subtended by the cube at the position of the observer situated at O, the origin of Σ, is very small. When the cube is moving the light quanta from the four corners A, B, C and D, which reach the observer's eye at the same time, form a rectangle of height l_o and apparent length l given by

$$l = l_o(1 - v^2/c^2)^{1/2} \quad \text{for } v^2 < c^2\,, \tag{69}$$

$$l = l_o|(v^2/c^2 - 1)^{1/2}| \quad \text{for } v^2 > c^2\,. \tag{70}$$

The apparent length along the x-axis is contracted for $v^2 < c^2$ and $c^2 < v^2 < 2c^2$, but is dilated for $v^2 > 2c^2$.

When the cube is moving relative to the observer, light quanta from the corners E and F can also reach the observer's eye at the same time as the quanta from A, B, C and D, as shown in Fig. 10. The light quanta from these corners leave the cube at an earlier time, when the corners E and F are at the positions E' and F' respectively. Therefore the side ADFE of the moving cube is visible to the observer and appears to be a rectangle. If the observer is far away from the cube then, to a first approximation, the light from E travels the extra distance l_o in the time that E' goes to E. Hence the distance EE' is equal to $l_o v/c$, so that for $c^2 < v^2 < 2c^2$ the face ABCD appears to be contracted and the face ADFE is dilated. For $v^2 > 2c^2$ both faces ABCD and ADFE are dilated, and so as $v \to \infty$ the cube appears to become enormously elongated. (The tachyonic rod discussed in Section 7 had an infinite apparent length in the frame in which it appeared to have infinite speed.) Of course, for speeds such that $v \gg c$ the approximation that the distance EE' is given by $l_o v/c$ is no longer valid, as the observed angle subtended by the face of the cube is then quite large. In this case a far more detailed analysis is required, which will not be attempted here.

10. Conclusion

The overall framework of special relativity can be extended to include particles having speeds greater than c, simply by using the postulates of special relativity and allowing the existence of inertial reference frames travelling at a constant speed greater than c. The only other requirements necessary to allow tachyons to behave in a logical and consistent manner are the *switching principle* developed in Section 5 (expressed mathematically as the γ-rule), a standard convention for decomposing imaginary square roots, and the minor modification of some familiar definitions. Even so, the results and modified definitions in ER automatically reduce to the standard ones of SR as soon as the objects appear to the observer to be bradyons. This formulation does not change SR in any way and automatically accounts for familiar results, such as sources and detectors being fixed due to bradyons never appearing to be switched relative to a bradyonic observer.

The switching principle may appear to be a mere mathematical artifice, but the fact that it automatically allows tachyons to obey the laws of conservation of energy, momentum and electric charge in a given reference frame shows that it has deep physical significance. In the third paper of this series it will be shown that the theory of tachyons as described here is completely consistent with electromagnetism, to the point where Maxwell's equations in a vacuum apply for all speeds from 0 to ∞. It is not necessary to make any changes to Maxwell's equations to accommodate charged tachyons as has been done by Recami and Mignani (1974) and by Mignani and Recami (1975).

As stated by Imaeda (1979) it is not possible to retain all three of the following: (i) the Minkowskian (in our case Euclidean) nature of space–time, (ii) the reality of the extended transformation (i.e. in four dimensions) and (iii) the group character of the transformations for all particles. Clearly the present formulation falls into Imaeda's case A in which the transformation formulae contain imaginary quantities as well as real ones. The appearance of imaginary factors in the SLTs does not lead to any major problems, except that of interpretation of intermediate steps in calculations. The fact that the imaginary factors cancelled out when necessary, as demonstrated in the numerical example of Section 5, means that tachyons will have real and detectable properties such as energy, momentum and electric charge.

Imaeda suggests that the introduction of either a complex space–time or an increase in the number of dimensions of space–time would enable the coordinates of an event to be maintained as real quantities. He goes on to formulate for tachyons a quarternionic approach. We have not yet investigated the implications of this sort of approach to our own work.

In this paper there is no detailed discussion of tachyons and their consequences for causality. The reader is referred to an excellent review of these considerations by Recami (1987).

Switching has been discussed from a different point of view in an interesting paper by Schwartz (1982). By studying in detail the integration of the four-divergence of a conserved quantity over the three-dimensional surface bounding a region of interaction containing both bradyons and tachyons, he comes to the conclusion that the momentum of a particle and the question of whether it enters or leaves the interaction region are not to be treated as separate aspects if the particle is a tachyon. It is the 'product' of these two concepts or what Schwartz refers to as the 'momentum flow' which is significant. The new ideas of Schwartz seem of particular importance in a quantum formulation of tachyon properties, although they do result in the switching entering in an automatic way. A later paper in the present series will investigate the implications of Schwartz's approach for our own work based upon the Klein–Gordon equation.

Further discussion of such quantum aspects is not appropriate for the present paper where purely classical considerations are involved. Thus at no stage in this work has ER indicated how to create a tachyon: this is equivalent to the fact that SR does not say exactly how to create a bradyon. An attempt to determine how to create tachyons must be made at the quantum level and would require the development of a tachyonic analogue of relativistic quantum mechanics.

Corben (1978) has argued that tachyons, should they exist, 'are basically the same objects as ordinary particles (they just look different because they are moving so fast).' This is certainly the case in the present formulation, where the emphasis is on rigorous derivations of tachyonic transformations with worked examples used to check for internal consistency and the sensibleness of results. All of the tachyonic transformations and expressions for various quantities given in this series of papers can be derived using similar steps to the corresponding relativistic analogue. Finally, one of our referees has drawn our attention to an unpublished paper by Finch (1990) in which the result for the tachyonic transformations (equation 26) in this paper is studied carefully in the light of the group properties of the transformations.

Acknowledgments

This work was completed with the assistance of an ARC grant. One of the authors (RLD) wishes to acknowledge the receipt of a University of Melbourne Postgraduate Research Scholarship.

References

Antippa, A. F. (1975). *Phys. Rev.* D **11**, 724.

Bilaniuk, O. M. P., Deshpande, V. K., and Sudarshan, E. C. G. (1962). *Am. J. Phys.* **30**, 718.

Bilaniuk, O. M. P., and Sudarshan, E. C. G. (1969). *Phys. Today* **22** (5), 43.

Corben, H. C. (1975). *Nuovo Cimento* A **29**, 415.

Corben, H. C. (1976). *Int. J. Theor. Phys.* **15**, 703.

Corben, H. C. (1978). *In* 'Erice-1976: Tachyons, Monopoles and Related Topics' (Ed. E. Recami), p. 31 (North-Holland: Amsterdam).

Finch, P. D. (1990). Preprint, Dept of Mathematics, Monash University, Victoria, Australia.

Garuccio, A., Maccarrone, G. D., Recami, E., and Vigier, J. P. (1980). *Lett. Nuovo Cimento* **27**, 60.

Imaeda, K. (1979). *Nuovo Cimento* B **50**, 271.

Kreyszig, E. (1979). 'Advanced Engineering Mathematics', fourth edition, p. 573 (Wiley: New York).

Maccarrone, G. D., and Recami, E. (1984). *Found. Phys.* **14**, 367.

Mignani, R., and Recami, E. (1975). *Nuovo Cimento* A **30**, 533.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). 'Gravitation' (Freeman: San Francisco).

Recami, E. (1975). *Scientia (Milan)* **109**, 721.

Recami, E. (1986). *Riv. Nuovo Cimento* **9** (4), 1.

Recami, E. (1987). *Found. Phys.* **17**, 239.

Recami, E., and Mignani, R. (1974). *Riv. Nuovo Cimento* **4**, 1.

Rosser, W. G. V. (1964). 'An Introduction to the Theory of Relativity' (Butterworths: London).

Schwartz, C. (1982). *Phys. Rev.* D **25**, 356.

Manuscript received 5 July 1991, accepted 23 June 1992