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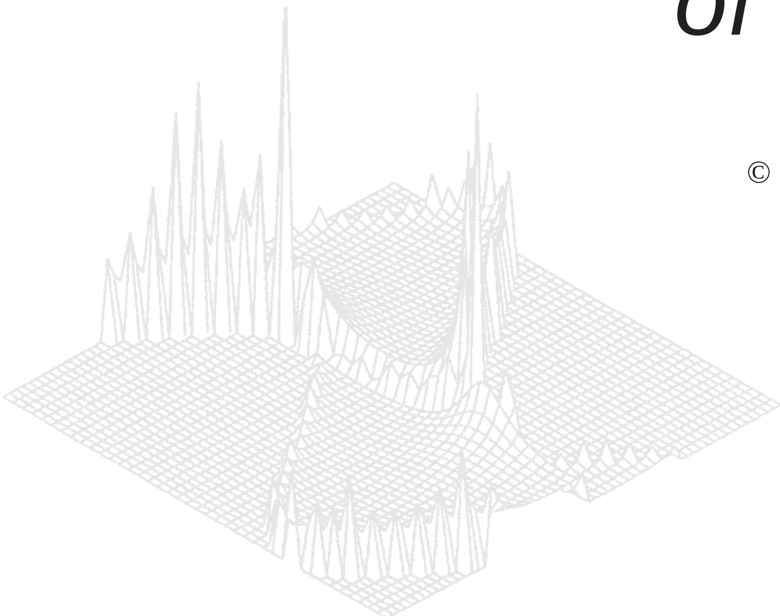

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Some Implications of the Gravitomagnetic Field in Fractal Spacetime Theory

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Abstract

In a fractal spacetime, the absence of a gravitational Meissner effect is thought of as ordering space as a crystal, at both a microscopic and a macroscopic scale. A gravitational Meissner effect keeps a wormhole open and penetrable and, in the same context, a gravitational superconductor levitates in an external gravitomagnetic field, an external gravitomagnetic field induces quantised vortices in a gravitational superconductor and gravitational rotons in a superfluid, and the planetary systems are self-organised as superconducting structures.

1. Introduction

Recent results show that for ordinary matter there is no gravitational Meissner effect. Its absence is interpreted as ordering space as a crystal (Ciubotaru and Agop 1996; Agop *et al.* 1998*a*). A gravitational Meissner effect implies the existence of some 'exotic' matter (Morris *et al.* 1998), i.e. matter that does not satisfy Hawking's theorem about the positiveness of the energy–momentum tensor.

Considering spacetime as being fractal, in the present work we analyse some implications of the gravitomagnetic field. Thus the absence of the gravitational Meissner effect is interpreted as a crystal ordering of space, at both a microscopic and a macroscopic scale. A gravitational Meissner effect may keep a wormhole open and penetrable and, in the same context, a gravitational superconductor levitates in an external gravitomagnetic field, an external gravitomagnetic field induces quantised vortices in a gravitational superconductor and gravitational rotons in a superfluid, and the planetary systems are self-organised as superconducting structures.

2. Fractal Gravitational Meissner Effect: Wormholes and Degenerate Vacuum

If the wavefunction $\Psi = |\Psi|e^{i\varphi}$ is minimally coupled to the gravitational vector potential $\mathbf{A}_g$, the current density expression is

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$$\mathbf{j} = -\mathrm{i}D(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - 2Dg\mathbf{A}_\mathrm{g}|\Psi|^2\,, \tag{1}$$

where $\overline{\nabla} = \nabla - ig\mathbf{A}_\mathrm{g}$ is the covariant derivative, g a coupling constant, D a diffusion coefficient depending on the fractal dimension (Nottale 1996*a*) and m the mass of the particle.

For $\rho = |\Psi|^2 = \text{const.}$, relation (1) becomes

$$\mathbf{J} = 2D|\Psi|^2\nabla\varphi - 2Dg\mathbf{A}_\mathrm{g}|\Psi|^2 \tag{2}$$

from which, applying the curl, one finds

$$\nabla\times\mathbf{j} = -2Dg|\Psi|^2\mathbf{B}_\mathrm{g}\,, \tag{3}$$

with $\mathbf{B}_\mathrm{g}$ the gravitomagnetic field vector. Relation (3) is the London gravitational equation for the fractal spacetime.

On the other hand, taking the curl of the equation (Peng 1990)

$$\nabla\times\mathbf{B}_\mathrm{g} = -\frac{16\pi G}{c^2}\mathbf{j} + \frac{1}{c^2}\frac{\partial\mathbf{g}}{\partial t}\,, \tag{4}$$

where one neglects the gravitational displacement currents, i.e. $\partial_t\mathbf{g} = 0$, and imposing $\mathrm{div}\mathbf{B}_\mathrm{g} = 0$ (Peng 1990) one gets

$$\nabla\mathbf{B}_\mathrm{g} = \frac{16\pi G}{c^2}\nabla\times\mathbf{j}\,. \tag{5}$$

Therefore, by substituting relation (3) in (5), one finds

$$\nabla\mathbf{B}_\mathrm{g} + \frac{32\pi gGD}{c^2}|\Psi|^2\mathbf{B}_\mathrm{g} = 0\,. \tag{6}$$

This means that the fractal spacetime is structured by the gravitomagnetic field as a crystal, the fractal gravitational lattice constant being

$$\lambda_\mathrm{g} = \left(\frac{c^2}{32\pi gGD|\Psi|^2}\right)^{1/2}. \tag{7}$$

In relations (4)–(7), G is Newton's constant and c the vacuum speed of light. Relation (6) is a generalisation of some well known results. Thus, with $g = 1/2D$, this relation becomes

$$\nabla\mathbf{B}_\mathrm{g} + \frac{1}{\lambda_\mathrm{g}^2}\mathbf{B}_\mathrm{g} = 0\,, \tag{8}$$

where

$$\lambda_\mathrm{g} = \left(\frac{c^2}{16\pi G\rho}\right)^{1/2}. \tag{9}$$

The fractal scale is involved through the coupling constant g so that for $D = \hbar/2m$ (Nottale 1996a) the microspace, and for $D = Gm/2\alpha_{\mathrm{g}}c$ (Nottale 1996b), where α_{g} is the gravitational structure constant, the macrospace is structured as a crystal.

Assuming in equation (6) negative energy densities, $\epsilon = -|\Psi|^2c^2$, one gets

$$\nabla \mathbf{B}_{\mathrm{g}} - \frac{1}{\Lambda^2}\mathbf{B}_{\mathrm{g}} = 0\,, \tag{10}$$

with

$$\Lambda = \left(\frac{c^4}{32\pi g G D \epsilon |\Psi|^2}\right)^{1/2}. \tag{11}$$

In particular, for a homogenous and infinite gravitational superconductor, located in the half space $x > 0$ of a Cartesian system of coordinates, the equation becomes

$$d_{xx}B_{\mathrm{g}} - \frac{1}{\Lambda^2}B_{\mathrm{g}} = 0 \tag{12}$$

and allows the solution

$$B_{\mathrm{g}}(x) = B_{\mathrm{g}}(0)\mathrm{e}^{-x/\Lambda}\,. \tag{13}$$

Relation (10) or (12) with $g = 1/2D$ defines the fractal gravitational Meissner effect, and Λ, through (13), the fractal gravitational penetration depth. Hence, for $D = \hbar/2m$ the gravitomagnetic field is expelled from the microspace (quantum gravitational Meissner effect), and for $D = Gm/2\alpha_{\mathrm{g}}c$ from the space at a cosmological scale (cosmological gravitational Meissner effect).

Table 1. Fractal gravitational depths for some gravitational superconductors

Type of gravitational superconductor	ρ (kg m^{-3})	Λ (m)
Cosmological background	5×10^{-28}	$2{\cdot}3 \times 10^{26}$
Cosmic dust	10^{-25}	$1{\cdot}6 \times 10^{24}$
Neutron stars	5×10^{17}	$7{\cdot}3 \times 10^{3}$

In Table 1 we list the fractal gravitational penetration depths calculated from equation (11) for various gravitational superconductors. It results that even the neutron stars have a commensurate fractal gravitational Meissner effect. In essence this implies that the gravitomagnetic field caused by the huge angular momentum of the star will be expelled from the centre of the neutron star [probably from its hyperonic nucleus, taking into account the dimensions of the area where the gravitomagnetic field is completely absent, i.e. $L \sim (10{-}7{\cdot}3) \times 10^3 = 2{\cdot}7 \times 10^3$ (for more details see Ureche 1987)]. This could be accomplished through matter-induced supercurrents in the outer layers of the neutron star creating counter-gravitomagnetic fields to expel the gravitomagnetic field from its interior, in analogy to the electromagnetic case. In such a context one can select even the type of gravitational Meissner effect imposed by the structure of the neutron star by evaluating its kinetic momentum. Thus, if a quantum Meissner effect could exist, its kinetic momentum would be

$$L \sim \tfrac{1}{2} N\hbar \sim \frac{M_{\mathrm{NS}}}{m_{\mathrm{N}}}\hbar = 0.6 \times 10^{23}\,\mathrm{J\,s}\,,$$

where N is the total particle number, M_{NS} the mass of the neutron star and m_{N} the mass of a neutron, and for the cosmological gravitational Meissner effect $L \sim \frac{1}{2} G M_{\mathrm{NS}}^2/\alpha_{\mathrm{g}} c = 8 \cdot 9 \times 10^{44}\,\mathrm{J\,s}$. Since the last value is closer to the experimental value, e.g. for the Crab pulsar $L\ 10^{40}\,\mathrm{J\,s}$, it results that the neutron stars are very good relativistic laboratories for the cosmological gravitational Meissner effect.

The fractal gravitational Meissner effect relates two seemingly very different concepts. One of these implies the existence of some distinct spacetime structures, very special in regard to their topologies, which directly connect, along extremely short trajectories through quite extradimensional tunnels (with respect to the hyperplane of the metagalaxy), what are usually very remote areas of the universe. Such structures, which alter the simply connected characteristics of the universe, are named 'wormholes'. Recent results (Morris *et al*. 1998) indicate that for a wormhole to remain open and penetrable, one must inject into it some 'exotic' matter, namely matter with a negative energy density, just like the one described by the component $T_{44} = \Lambda/k_0$ (< 0 for $\Lambda < 0$ of the conservative energy–momentum tensor $T_{ab} = -g_{ab}(\Lambda/k_0)$ in a rigid pseudo-orthonormalised frame, where $g_{ab} = \eta_{ab}$. In the previous relations Λ is the cosmological constant and k_0 is Einstein's constant. In our opinion it is not necessary to inject exotic matter into the wormhole to keep it open; it is sufficient for the wormhole's matter to become a gravitational superconductor (Agop *et al*. 1998*b*). The wormhole then remains penetrable by a gravitational Meissner effect. We mention that, in agreement with the wormhole's definition (Visser and Hochberg 1997), the gravitational Meissner effect violates the null energy condition. Indeed, as results from the previous observations, for the gravitational Meissner effect to exist it is necessary to admit in equation (8) negative densities of energy ($T_{44} = -c^2|\Psi|^2 < 0$). At a cosmic scale such a situation is accomplished by the cosmic dust which, as it constitutes the matter of the wormhole, behaves in rotation as a gravitational superconductor. Consequently, if the universe is open, its rotation keeps the wormholes open, any use of an inflationary model being futile (Kolb and Turner 1989).

A second concept is the degenerate vacuum. As a typical example, let us consider the free complex scalar field ϕ. To the Lagrangian density

$$L = \partial^a \overline{\Phi} \partial_a \Phi + m_0^2 \overline{\Phi}\Phi\,, \tag{14}$$

where m_0 is the mass of the quanta, corresponds the energy–momentum tensor

$$T_{ab} = \partial_a \overline{\Phi} \partial_b \Phi + \partial_b \overline{\Phi} \partial_a \Phi - \eta_{ab}(\partial^c \overline{\Phi} \partial_c \Phi + m_0^2 \overline{\Phi}\Phi) \tag{15}$$

and hence the Hamiltonian density

$$H = T_{44} = \delta^{ab} \partial_a \overline{\Phi} \partial_b \Phi + m_0^2 \overline{\Phi}\Phi\,. \tag{16}$$

Here T_{44} is positively defined and achieves its minimum for $\overline{\Phi} = \Phi_0 = 0$. Therefore the vacuum state is well defined, being nondegenerate. Such a situation is typical for all non-self-interacting free fields.

If the free field is self-interacting, to the Lagrangian density

$$L = \partial^a \overline{\Phi} \partial_a \Phi - \mu^2 \overline{\Phi} \Phi + \frac{\lambda}{2} (\overline{\Phi} \Phi)^2 \,, \tag{17}$$

where m_0^2 is replaced by $-\mu^2$ and the self-interaction is achieved through the coupling constant λ, corresponds the energy–momentum tensor

$$T_{ab} = \partial_a \overline{\Phi} \partial_b \Phi + \partial_b \overline{\Phi} \partial_a \Phi - \eta_{ab} (\partial^c \overline{\Phi} \partial_c \Phi - \mu^2 \overline{\Phi} \Phi + \frac{\lambda}{2} (\overline{\Phi} \Phi)^2) \tag{18}$$

and hence the Hamiltonian density is

$$H = T_{44} = \delta^{ab} \partial_a \overline{\Phi} \partial_b \Phi - \mu^2 \overline{\Phi} \Phi + \frac{\lambda}{2} (\overline{\Phi} \Phi)^2 \,. \tag{19}$$

If one defines the vacuum in this theory as the state that achieves the minimum of H, then from the extremum condition

$$\frac{\partial H}{\partial \Phi} = \frac{\partial H}{\partial \overline{\Phi}} = 0 \tag{20}$$

we get the algebraic equation

$$(\overline{\Phi} \Phi)_m = \frac{\mu^2}{\lambda} \tag{21}$$

with the roots

$$\Phi_m = \frac{\mu}{\sqrt{\lambda}} \mathrm{e}^{\mathrm{i}\alpha}, \quad \Phi_m = \frac{\mu}{\sqrt{\lambda}} \mathrm{e}^{-\mathrm{i}\alpha} \,, \tag{22}$$

with $\alpha \in R$. Therefore, the vacuum states of this theory are degenerate. Obviously, compared to the first situation, the modulus of the expectation value of the field in its vacuum state is positive, namely

$$|\Phi_m| = \frac{\mu}{\sqrt{\lambda}} \,. \tag{23}$$

Calculating the components of the energy–momentum tensor of the degenerate vacuum, one finds

$$T_{ab}[(\overline{\Phi} \Phi)_m] = \eta_{ab} \frac{\mu^4}{2\lambda} \tag{24}$$

and the resemblance with $T_{ab}(\Lambda)$ is remarkable for $\Lambda = -k_0 \mu^4 / 2\lambda < 0$ with $\lambda > 0$.

Considering that Φ characterises a gravitational superconductor, in situations where the dependence $\mu_{\rm eff}^2 = -\mu^2(1 - T/T_{\rm c})$ applies, with T and $T_{\rm c}$ the temperature and the critical temperature associated with the transition from normal matter to gravitational superconductor, the negative energy states are obtained for $T < T_{\rm c}$ (Agop *et al.* 1996). In this case a micro-wormhole is kept open by the quantum gravitational Meissner effect. This effect, if it exists, would imply a density of the gravitational superconductor of $\rho \sim 10^{96}$ kg m^{-3} and, by relation (11), a gravitational penetration depth of $\Lambda \sim 10^{-35}$ m, which is equal to the Planck length. Only at this scale does the linear approximation of the gravitational field become invalid.

3. Gravitational Levitation

A consequence of the gravitational Meissner effect is that any gravitational superconductor placed in an external gravitomagnetic field levitates. We name such a phenomenon 'gravitational levitation'. The levitation length z is obtained following the method given in Burns (1992). Thus, a gravitational superconductor of volume V and density ρ has in the gravitomagnetic field $B_{\rm g}$ the energy

$$W = \frac{B_{\rm g}^2(z)c^2}{8\pi G}V\,. \tag{25}$$

At equilibrium, the force $F = -\partial W/\partial z$ corresponds to the gravitational force $F = \rho g(z)\,V$, i.e.

$$F = -\frac{\partial W}{\partial z} = \rho g(z)V\,. \tag{26}$$

If one considers the source of the gravitoelectromagnetic field to be a rigid sphere of mass M and radius R, spinning uniformly with the angular speed Ω, then

$$B_{\rm g} = \frac{GM}{(R+z)c^2}\Omega, \quad g = \frac{GM}{(R+z)^2} \tag{27}$$

or, with the notation $B_{\rm g0} = GM\Omega/Rc^2$, $g_0 = GM/R^2$, for $R \ll z$,

$$B_{\rm g}(z) \approx B_{\rm g0}\frac{R}{z}, \qquad g(z) \approx g_0\frac{R^2}{z^2}\,. \tag{28}$$

Under these circumstances, from (26) and (28) one gets

$$z = \frac{M}{4\pi\rho c^2}\Omega^2 = \frac{\pi M}{\rho c^2}\frac{1}{T^2}, \qquad \Omega = \frac{2\pi}{T}\,, \tag{29}$$

where T is the period of rotation of the source body. Thus the levitation length is directly proportional to the mass of the source body and inversely proportional to its rotation period and to the density of the gravitational superconductor.

Table 2. Levitation lengths

Structure type	M (kg)	T (s)	ρ (kg m^{-3})	z (m)
Typical neutron star (pulsar)	10^{30}	1	10^{17}	$0{\cdot}85 \times 10^{-4}$
Earth	6×10^{24}	86164	10^{17}	$0{\cdot}7 \times 10^{-19}$
Sun	2×10^{30}	$2{\cdot}3 \times 10^{6}$	10^{17}	$3{\cdot}2 \times 10^{-17}$
Typical galaxy	$1{\cdot}3 \times 10^{41}$	$6{\cdot}3 \times 10^{15}$	$10^{17}/10^{-28}$	$2{\cdot}8 \times 10^{-25}/2{\cdot}8 \times 10^{20}$

We list in Table 2 the levitation lengths of gravitational superconductors (superfluid and cosmological dust) in gravitomagnetic fields generated by various material structures. A short analysis of these data leads to some interesting conclusions:

(i) such an experiment cannot be performed in terrestrial laboratories;
(ii) the relativistic stars (pulsars) behave like gravitational mini-laboratories in the presence of a superfluid;
(iii) the fact that the levitation length is of the same order of magnitude as a galaxy's dimensions shows that the cosmological dust can be expelled by galactic nuclei through a cosmological gravitational Meissner effect, giving birth to the galactic arms.

4. Fractal Gravitational Fluxoid and Rotons

An equation similar to (10), but for the current density $\mathbf{j}$, i.e.

$$\Delta\mathbf{j} - \frac{1}{\Lambda^2}\mathbf{j} = 0\,, \tag{30}$$

is obtained by substituting in the curl of equation (3), where one allows negative energy densities and the gauge condition

$$\nabla\,.\,\mathbf{j} = 0\,, \tag{31}$$

the equation (4) for $\nabla \times \mathbf{B}_{\rm g}$, where we assumed $\partial_t \mathbf{g} = 0$. The gauge condition (31) results from the continuity equation $\partial_t \rho + \nabla\,.\,\mathbf{j} = 0$ with $\rho = \text{const}$. This is equivalent, by considering

$$\mathbf{j} = 2Dg\rho\mathbf{A}_{\rm g}\,, \tag{32}$$

a result obtained by integrating equation (3) for negative energy densities, to

$$\nabla\,.\,\mathbf{A}_{\rm g} = 0\,. \tag{33}$$

In other words, at the vacuum–gravitational superconductor interface, the normal components $j_{\rm n}$ and $A_{\rm gn}$ of the current density and vector potential, respectively, are null.

Let us now rewrite the current density (2) as

$$\mathbf{j} = 2D\rho(\nabla\varphi - g\mathbf{A}_{\rm g})\,. \tag{34}$$

If inside a gravitational superconductor one considers an arbitrary contour Γ, along this contour, due to the fractal gravitational Meissner effect (30),

$$\oint_{\Gamma} \mathbf{j} \,\boldsymbol{.}\, \mathrm{d}\mathbf{l} = 0\,, \tag{35}$$

which implies

$$g^{-1} \oint_{\Gamma} \nabla\varphi \,\boldsymbol{.}\, \mathrm{d}\mathbf{l} = \oint \mathbf{A}_{\mathrm{g}} \mathrm{d}\mathbf{l} = \Phi_{\mathrm{g}}\,, \tag{36}$$

with Φ_{g} the flux of the gravitomagnetic field. Since

$$\oint_{\Gamma} \nabla\varphi \,\boldsymbol{.}\, \mathrm{d}\mathbf{l} = 2n\pi\,, \tag{37}$$

relation (36) becomes

$$\Phi_{\mathrm{g}} = 2n\pi g^{-1}, \qquad n = 1, 2... \tag{38}$$

or, for $g = 1/2D$ and

$$\Phi_{\mathrm{g}0} = 2\pi g^{-1} = 4\pi D\,, \tag{39}$$

$$\Phi_{\mathrm{g}} = n\Phi_{\mathrm{g}0}\,. \tag{40}$$

Therefore the flux of the gravitomagnetic field is quantised. We call $\Phi_{\mathrm{g}0}$ the 'gravitational fluxoid'. Here too, the fractal scale is involved though the coupling constant g, and thus for $D = \hbar/2m$, equation (39) defines the gravitational microfluxon, $\Phi_{\mathrm{g}0} = h/2m$, and for $D = Gm/2\alpha_{\mathrm{g}}c$, the gravitational macrofluxon, $\Phi_{\mathrm{g}0} = 2\pi Gm/\alpha_{\mathrm{g}}c$. Relation (40) stipulates that in the presence of the gravitomagnetic field, vortices should appear in a gravitomagnetic superconductor. If the area of such a vortex in the cross section of the gravitomagnetic field is $A = \pi R^2$, where R is the vortex radius, the number of vortices per unit area is

$$N = \pi^{-1} R^{-2}\,. \tag{41}$$

On the other hand, from equation (40) written as $B_{\mathrm{g}}\pi R^2 = 4\pi nD$, the vortices' density is

$$N = \frac{B_{\mathrm{g}}}{4\pi nD} \tag{42}$$

thus substituting (42) in (41) yields

$$R_n = n^{\frac{1}{2}} R_0\,, \tag{43}$$

where the elementary vortex has the radius

$$R_0 = \left(\frac{4D}{B_{\mathrm{g}}}\right)^{\frac{1}{2}}. \tag{44}$$

This means that the vortices' radii are quantised. For $D = \hbar/2m$ the macroscopic vortices, and for $D = Gm/2\alpha_{\rm g}c$ the microscopic vortices, are quantised. Therefore one can detect the gravitomagnetic field magnitude by measuring the vortex radius. In such a test, the gravitoelectric field may be compensated on the basis of the local equivalence principle.

Table 3. Radii of vortices, R_0

Structure type	M (kg)	R (m)	$B_{\rm g}$ ($\rm s^{-1}$)	$D = \hbar/2m$ ($\rm m^2\ s^{-1}$)	R_0 (m)
Typical neutron star (pulsar)	10^{30}	10^4	$0{\cdot}46$	8×10^{-9}/ $5{\cdot}2 \times 10^{-4}$	$2{\cdot}6 \times 10^{-4}$/ $6{\cdot}7 \times 10^{-2}$
Earth	6×10^{24}	$6{\cdot}4 \times 10^6$	5×10^{-14}	8×10^{-9}/ $5{\cdot}2 \times 10^{-4}$	$0{\cdot}8 \times 10^3$/ 2×10^5
Sun	2×10^{30}	7×10^8	$5{\cdot}7 \times 10^{-12}$	8×10^{-9}/ $5{\cdot}2 \times 10^{-4}$	$0{\cdot}7 \times 10^2$/ 2×10^4
Typical galaxy	$1{\cdot}3 \times 10^{41}$	10^{21}	$0{\cdot}94 \times 10^{-22}$	8×10^{-9}/ $5{\cdot}2 \times 10^{-4}$	$1{\cdot}8 \times 10^7$/ $4{\cdot}9 \times 10^9$

We enumerate in Table 3 the radii R_0 of vortices induced by the gravitomagnetic fields generated by various cosmic structures in gravitational superconductors (superfluid with generic particles taken as helium ions, $D \sim 8 \times 10^{-9}$ $\rm m^2\ s^{-1}$, and an electron–positron vacuum, $D \sim 5{\cdot}2 \times 10^{-4}$ $\rm m^2\ s^{-1}$). The gravitomagnetic fields are calculated using the first relation (27) with $z = 0$.

Analysing these data yields some interesting conclusions:

(i) such an experiment cannot be performed in terrestrial laboratories;
(ii) the relativistic stars (pulsars) behave like gravitational mini-laboratories for a superfluid of generic particles taken as helium ions, and an electron–positron vacuum.

The same result (40) may be obtained using the locally U(1)-gauge-invariant Lagrangian

$$L = (\overline{\nabla}\Psi)^*(\overline{\nabla}\Psi) - \mu^2\Psi^*\Psi - \tfrac{1}{4}F_{il}F_{il}\,, \tag{45}$$

where $\overline{\nabla}$ is the covariant derivative, $F_{il} = \partial_i A_{gl} - \partial_l A_{gi}$ the tensor of the gravitomagnetic field, and μ^2 the mass coefficient. The field equations corresponding to the Lagrangian (45) are then

$$\Delta\Psi - 2ig\mathbf{A}_{\rm g}\,.\,\nabla\Psi + (\mu^2 - g^2\mathbf{A}_{\rm g}\mathbf{A}_{\rm g})\Psi = 0\,, \tag{46}$$

$$\Delta\Psi^* + 2ig\mathbf{A}_{\rm g}\,.\,\nabla\Psi^* + (\mu^2 - g^2\mathbf{A}_{\rm g}\mathbf{A}_{\rm g})\Psi^* = 0\,, \tag{47}$$

$$\Delta\mathbf{A}_{\rm g} + {\rm i}g(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + 2g^2\mathbf{A}_{\rm g}|\Psi|^2 = 0\,, \tag{48}$$

or, in cylindrical coordinates (r,θ,z) with

$$A_{\mathrm{g}}^1 = 0, \quad A_{\mathrm{g}}^1 = A_{\mathrm{g}}(r), \quad A_{\mathrm{g}}^3 = 0, \quad \Psi = F(r)\mathrm{e}^{\mathrm{i}n\theta}\,, \tag{49}$$

$$\frac{1}{r}d_r(rd_rF) - \left(\frac{n}{r} - gA_{\mathrm{g}}\right)^2 F + \mu^2 F = 0\,, \tag{50}$$

$$d_r\left[\frac{1}{r}d_r(rA_{\mathrm{g}})\right] - 2g\left(\frac{n}{r} - gA_{\mathrm{g}}\right)F^2 = 0\,. \tag{51}$$

Since (51) admits the solution

$$A_{\mathrm{g}} = ng^{-1}r^{-1}\,, \tag{52}$$

the gravitomagnetic field flux becomes

$$\Phi_{\mathrm{g}} = \iint B_{\mathrm{g}}(x,y)\mathrm{d}x\mathrm{d}y = \oint_{\Gamma} \mathbf{A}_{\mathrm{g}}\,\boldsymbol{.}\,\mathrm{d}\mathbf{l} = ng^{-1}\int_0^{2\pi}\mathrm{d}\theta = 2n\pi g^{-1}\,, \tag{53}$$

that is, equation (38).

The ring vortex induced by a gravitomagnetic field inside a gravitational superconductor has the kinetic energy per unit length

$$\epsilon = \tfrac{1}{2}\rho\int_{R_{\mathrm{m}}}^{R_{\mathrm{M}}} \nu^2 2\pi r\mathrm{d}r \tag{54}$$

or, taking into account equations (52) and (32),

$$\epsilon = \tfrac{1}{2}\rho\int_{R_{\mathrm{m}}}^{R_{\mathrm{M}}} A_{\mathrm{g}}^2 2\pi r\mathrm{d}r = 4\pi D^2\rho n^2 \ln\frac{R_{\mathrm{M}}}{R_{\mathrm{m}}}\,, \tag{55}$$

where ρ is the density of the gravitational superconductor, R_{M} the outer radius and R_{m} the inner radius. Then the energy is

$$E = 2\pi R_{\mathrm{M}}\epsilon = 8\pi^2 D^2\rho n^2 R_{\mathrm{M}}\ln\frac{R_{\mathrm{M}}}{R_{\mathrm{m}}} \tag{56}$$

and momentum of the vortex

$$p = 4\pi^2 D\rho n R_{\mathrm{M}}^2 \tag{57}$$

and we assume, for $n = 1$, an energy quantum

$$E_0 = 8\pi^2 D^2\rho R_{\mathrm{M}}\ln\frac{R_{\mathrm{M}}}{R_{\mathrm{m}}} \tag{58}$$

and momentum quantum

$$p_0 = 4\pi^2 D\rho R_{\mathrm{M}}^2\,. \tag{59}$$

Now, eliminating the diffusion coefficient D between relations (58) and (59), one gets the parabolic dependence

$$E_0 = \frac{p_0^2}{2m^*}\,, \tag{60}$$

where

$$m^* = \pi^2 \rho R_{\mathrm{M}}^3 \left(\ln \frac{R_{\mathrm{M}}}{R_{\mathrm{m}}} \right)^{-1} \tag{61}$$

represents the effective mass of the vortex. From the dependence (60), in analogy with Landau's theory of superfluidity, it results that a gravitomagnetic field induces in a gravitational superconductor the elementary energy (58) and momentum (59) excitations, named 'fractal gravitational rotons'. If $D = \hbar/2m$ then the fractal gravitational roton reduces to the conventional roton from Landau's theory (Balla and Deutsch 1970).

5. Planetary Systems as Superconducting Structures

Considering the planetary systems to be superconducting systems and eliminating A_{g} in equations (32) and (52) yields

$$R_n V_n = 2nD \tag{62}$$

or, with $D = Gm/2\alpha_{\mathrm{g}} c$ and $1/\alpha_{\mathrm{g}} = 2072 \pm 7$ (Nottale 1996*b*),

$$R_n V_n = n \frac{Gm}{\alpha_{\mathrm{g}} c}\,, \tag{63}$$

i.e. the kinetic orbital moments of the planets are quantised. We show in Table 4, in contrast, the kinetic moments ($R_n\, V_n$) of the planets calculated from (63) and the experimental data (the average value of $1/\alpha_{\mathrm{g}}$ was used).

Table 4. Quantisation of planetary kinetic moments

Planet	n	$R_0\, V_0 \times 10^{15}$ ($\mathrm{m^2\ s^{-1}}$)	$R_n\, V_n \times 10^{15}$ ($\mathrm{m^2 s^{-1}}$)	$R_n\, V_{n\mathrm{exp}} \times 10^{15}$ ($\mathrm{m^2\ s^{-1}}$)
Mercury	3	0·9165	2·74	2·76
Venus	4	0·9165	3·66	3·78
Earth	5	0·9165	4·58	4·45
Mars	6	0·9165	5·49	5·49
Ceres	8	0·9165	7·33	—
Jupiter	11	0·9165	10·08	10·15
Saturn	15	0·9165	13·74	13·75
Uranus	21	0·9165	19·24	19·55
Neptune	26	0·9165	23·82	24·55
Pluto	30	0·9165	27·49	27·91

Since $Gmm'/R_n^2 = m'\ V_n^2/R_n$, which implies

$$R_n V_n^2 = Gm\,, \tag{64}$$

substituting (63) in (64) leads to

$$V_n = \frac{V_0}{n}, \qquad V_0 = \alpha_{\mathrm{g}} c\,, \tag{65}$$

i.e. the revolution speeds of the planets are quantised. For V_0 we used the value $V_0 = 144{\cdot}7 \pm 0{\cdot}5$ km s^{-1} (Nottale 1996*b*). Table 5 gives, in contrast, the revolution speeds V_n computed from relation (65) and the experimental ones $V_{n\mathrm{exp}}$ (we choose for V_0 the mean value).

Table 5. Quantisation of planetary revolution speeds

Planet	n	V_0 (km s^{-1})	V_n (km s^{-1})	$V_{n\mathrm{exp}}$ (km s^{-1})
Mercury	3	144·7	48·2	47·9
Venus	4	144·7	36·17	35·0
Earth	5	144·7	28·94	29·8
Mars	6	144·7	24·11	24·1
Ceres	8	144·7	18·0	—
Jupiter	11	144·7	13·15	13·0
Saturn	15	144·7	9·64	9·0
Uranus	21	144·7	6·89	6·8
Neptune	26	144·7	5·56	5·4
Pluto	30	144·7	4·82	4·7

Eliminating V_n in relations (64) and (65) gives

$$R_n = n^2 R_0, \qquad R_0 = \frac{Gm}{\alpha_{\mathrm{g}}^2 c^2}\,, \tag{66}$$

i.e. the radii of the planetary orbits are quantised. In Table 6, in contrast, the radii R_n of the planetary orbits are given calculated from (66) and the major semiaxes $R_{n\mathrm{exp}}$.

Table 6. Quantisation of planetary orbit radii

Planet	n	R_0 (a.u.)	R_n (a.u.)	$R_{n\mathrm{exp}}$ (a.u.)
Mercury	3	0·0423	0·380	0·387
Venus	4	0·0423	0·676	0·723
Earth	5	0·0423	1·057	1·0
Mars	6	0·0423	1·522	1·523
Ceres	8	0·0423	2·707	2·766
Jupiter	11	0·0423	5·118	5·202
Saturn	15	0·0423	9·517	9·536
Uranus	21	0·0423	18·654	19·210
Neptune	26	0·0423	28·594	30·138
Pluto	30	0·0423	38·07	39·390

The planetary revolution periods

$$T_n = 2\pi \frac{R_n}{v_n} = n^3 T_0, \qquad T_0 = 2\pi \frac{Gm}{(\alpha_{\mathrm{g}} c)^3} \tag{67}$$

are also quantised. In Table 7 the revolution periods T_n of the planets are given calculated from (67) and the experimental values.

Table 7. Quantisation of planetary revolution periods

Planet	n	T_0 (yr)	T_n (yr)	$T_{n\exp}$ (yr)
Mercury	3	0·00870	0·234	0·241
Venus	4	0·00870	0·556	0·615
Earth	5	0·00870	1·087	1
Mars	6	0·00870	1·879	1·881
Ceres	8	0·00870	4·454	4·67
Jupiter	11	0·00870	11·579	11·962
Saturn	15	0·00870	29·362	29·458
Uranus	21	0·00870	81·570	84·013
Neptune	26	0·00870	152·911	164·794
Pluto	30	0·00870	234·900	247·686

Such a quantisation of motion is not singular to the solar system; it can be observed in the rings of Saturn as well. In Table 8 we present the radii R_n of the ring-shaped orbits calculated using equation (66) and the experimental data, $R_{n\exp}$. These results are close to the ones given in Landolt-Börnstein (1993).

Table 8. Quantisation of orbital radii of Saturn's rings

Ring name	n	$R_0 \times 10^3$ (km)	$R_n \times 10^3$ (km)	$R_{n\exp} \times 10^3$ (km)
D	6	1·817	65·4	67·0
B	7	1·817	89·0	92·0
A	8	1·817	116·2	122·2
F	9	1·817	147·1	140·4
E_1	10	1·817	181·7	180·0
E_2	16	1·817	465·1	480·0

6. Conclusions

The main conclusions of this paper are as follows:

(i) the absence of a fractal gravitational Meissner effect is thought of as an ordering of space as a crystal. For $D = \hbar/2m$ the microspace is structured, and for $D = Gm/2\alpha_g c$ the macrospace;

(ii) the gravitational superconductors exhibit fractal gravitational Meissner effects. A wormhole can be kept open by a cosmological gravitational Meissner effect, and a micro-wormhole by a microscopic gravitational Meissner effect;

(iii) a gravitational superconductor levitates in an external gravitational field. Calculating the levitation length of a superfluid in gravitomagnetic fields induced by different cosmic structures, it results that the relativistic stars (pulsars) can be considered gravitational mini-laboratories. Since the levitation length of the cosmological dust in the gravitomagnetic field of a galaxy has the same order of magnitude as the galaxy's dimensions, one can suppose the galaxy arms to occur as a result of the cosmological gravitational Meissner effect;

(iv) an external gravitomagnetic field induces vortices with quantised radii in a gravitational superconductor. Calculating the elementary radii for superfluids in gravitomagnetic fields generated by various cosmic structures, indicates that pulsars are gravitational mini-laboratories. In the same context, the elementary excitations are called gravitational rotons and are reduced, for $D = \hbar/2m$, to Landau rotons;

(v) the solar system is self-organised as a superconducting structure. Then the revolution speeds, kinetic momenta, planetary orbital radii and revolution periods are quantised;

(vi) we note that other values for $1/\alpha_{\rm g}$ and V_0 are found in the work of Agnese and Festa (1997*a*, 1997*b*).

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References

Agnese, A. G., and Festa, R. (1997*a*). *Phys. Lett.* A **227**, 165.
Agnese, A. G., and Festa, R. (1997*b*). Proc. Int. Workshop on Modern Modified Theories of Gravitational and Cosmology, Beer Sheva, pp. 237–53 (Hadronic Press).
Agop, M., Buzea, C. Gh., Griga, V., Ciubotariu, C., Buzea, C., Stan, C., and Tatomir, D. (1996). *Aust. J. Phys.* **49**, 1063.
Agop, M., Ciobanu, B., Buzea, C. Gh., Rezlescu, N., Horgo, T., and Stan, C. (1998*a*). *Nuovo Cimento* B **113** (1), 17.
Agop, M., Griga, V., Ciobanu, B., Ciubotariu, C., Buzea, C. Gh., Stan, C., and Buzea, C. (1998*b*). *Chaos, Solitons & Fractals* **7**, 1143.
Balla, D., and Deutsch, R. V. (1970). 'Fizica Temperaturilor Joase' (Ed. Academiei: Bucuresti).
Burns, G. (1992). 'High Temperature Superconductivity: An Introduction' (Academic: San Diego).
Ciubotariu, C., and Agop, M. (1996). *Gen. Relativ. Gravitat.* **28**, 1.
Kolb, E. W., and Turner, M. S. (1989). 'The Early Universe' (Addison–Wesley: New York).
Landolt-Börnstein (1993). 'The Planets and Their Satellites', Handbook VI/3a. 3.2 (Springer: Berlin).
Morris, M. S., Thorne, K. S., and Yursever, U. (1998). *Phys. Rev. Lett.* **61**, 114.
Nottale, L. (1996*a*). *Chaos, Solitons & Fractals* **7**, 877.
Nottale, L. (1996*b*). *Astron. Astrophys.* **315**, 29.
Peng, H. (1990). *Gen. Relativ. Gravitat.* **22**, 609.
Ureche, V. (1987). 'Universul Astrofizica' (Ed. Dacia: Cluj-Napoca).
Visser, M., and Hochberg, D. (1997). Generic wormhole throats, gr-qc/9710001.

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