Using a multi-state model to enhance understanding of geriatric patient care

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Abstract

Objectives: To use multi-state Markov chain modelling to analyse data on geriatric patient care, and to make comparisons between male and female patients.

Methods: Estimation, from observed data, of covariate (age of patient and date of admission to hospital or community care) dependent parameters of statistical models for time in care and subsequent events.

Results: Differential effects of these covariates shown on the parameters of the models for female and male patients, where these parameters can be interpreted as affecting different features of the distributions of time in care.

Conclusions: Multi-state modelling is an appropriate means of analysing data on geriatric patient care and can reveal underlying patterns of differential effects, some of which may not be apparent from more routine data processing.

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What is known about the topic?
Previous studies have effectively used Markov chain modelling to analyse health services utilisation data.

What does this paper add?
Assessment/diagnosis, treatment, rehabilitation and long stay were identified as phases of hospital care, with differences noted between males and females in their progress through these phases. In the community, three phases were identified: dependent on some form of continued care, convalescent and recovered. Females were more likely to be re-admitted to hospital from an earlier phase than males.

What are the implications for practitioners?
Similar techniques can be used to examine other patient care data.

HOSPITAL AND COMMUNITY CARE of geriatric patients can be thought of as a process of progression through different stages of assessment, treatment, convalescence, recovery, relapse, etc. which is amenable to modelling by continuous time Markov processes with discrete states. Such modelling has been described in a previous paper,[1] and applied to some data on male geriatric patients from Millard.[2] In this paper, similar modelling and analysis is done using data from the same source on female geriatric patients. Since women make up the majority of geriatric patients, differences between them and men in their hospital and community care requirements are likely to be of interest. The main purpose of this paper is to make comparisons between multi-state Markov models fitted to data from both male and female geriatric patients. Since substantially more female data are available, more significant effects may emerge than from the male data.

These data related to 2090 male and 4899 female geriatric patients admitted to St George's Hospital, London, over the period 1969–85. Duration of hospital treatment and time spent back in the community after discharge from hospital before possible readmission were available for these patients, along with the two covariates, age at admission to hospital and date of admission. These duration time data typically have distributions that are strongly right skewed,

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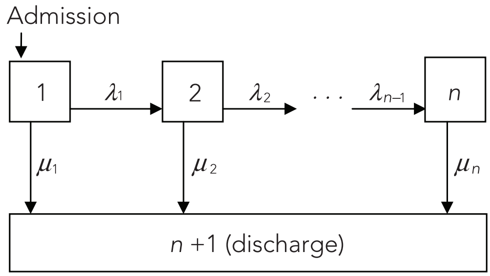
1 Schematic representation of the model for patients' hospitalisation



and for such distributions the *mean* may not be particularly informative. Indeed, Millard[2] argues that the mean duration of stay in hospital is a poor measure of resource use in geriatric medicine, because beds contain a mix of short and long-term patients. The Markov modelling used here leads to very general distributions for duration times, enabling differential inferences to be made for short and long-term durations.

Methodology

Full details of the statistical modelling and parameter estimation are described in the previous paper,[1] with only a brief description of the model given here. A Markov chain on states 1, 2, . . . , $n+1$, represented schematically in Box 1, is used as a model for patients' hospitalisation from admission (state 1) until discharge (state $n+1$). Here the λ_is describe sequential transitions between the *transient* states 1, 2, . . . , n (or phases) and the μ_is describe transitions from these states to the absorbing state $n+1$.

The resulting probability distribution of time from admission to discharge is known as phase-type after Neuts,[3] Chapter 1, and is a generalisation of the exponential ($n=1$) and gamma or Erlang ($\lambda_1 = \lambda_2 = \ldots = \lambda_{n-1} = \mu_n$ and $\mu_1 = \mu_2 = \ldots = \mu_{n-1} = 0$) distributions. It has coefficient of variation $\geqslant 1/\sqrt{n}$, and can show long right tails with a mode near 0 — typical of hospital length of stay data. Given such data, the λ_i and μ_i parameters can be estimated by maximum likelihood.[1] Taking a holistic view of a model with *discrete* phases, parameters describing the later phases (μ_{n-1}, λ_{n-1} and μ_n) will have greater influence on the upper tail of the distribution, while those from the early phases (μ_1,λ_1 and μ_2,λ_2) will tend to affect the lower part of the distribution. Individually, phase i can be described in terms of the mean sojourn time $1/(\mu_i+\lambda_i)$, and probability $\lambda_i/(\mu_i+\lambda_i)$ of a subsequent transition to the next phase (or probability $\mu_i/(\mu_i+\lambda_i)$ of absorption into state $n+1$), where λ_n is here taken to be 0 since the nth phase is the last in the sequence.

Other data were available about what happened to the patients after their time in hospital (or community) care ended — for example, in the case of hospital treatment the patients were either *transferred* to another hospital, *discharged* (back into the community) or *died*. In the context of the above modelling this time would end with absorption from one of the phases $i = 1, 2, \ldots, n$, so that probabilities θ_{ij} can be defined as *event j* occurring after time ending with absorption from phase i. With the above holistic interpretation of the model, times ending from an early phase would tend to be shorter than those ending from a later phase, so that any changes in the θ_{ij} probabilities with increasing i can be interpreted similarly. Estimation of these θ_{ij} probabilities from the available data can also be done by maximum likelihood.[1]

The two *covariates*, date of admission to hospital and age at admission, can be incorporated into the above parameters λ_i and μ_i by having these parameters dependent on the covariates.[1] Differential effects of the covariates across the phases ($i = 1, 2, \ldots, n$) will give information about how these covariates are affecting short and long-term durations of time in care. Additional covariate effects on the θ_{ij} event probabilities can be estimated from similar dependence on the covariates.[1]

Results

Choosing an appropriate number of phases (n) to adequately describe the data involves a number of criteria. Increasing n from a single-phase (exponential distribution) fit will always increase the

2 Estimates for males in hospital

Phase i	Mean sojourn time $1/(\mu_i+\lambda_i)$ (days)	Probability $\lambda_i/(\mu_i+\lambda_i)$ of transition to next phase	Probability θ_{i1} of transfer	Probability θ_{i2} of discharge	Probability θ_{i3} of death
a) for age 80 years, and dates (i) 1969 and (ii) 1985					
1	(i) 12.7	(i) 0.756	(i) 0.008	(i) 0.153	(i) 0.839
	(ii) 8.6	(ii) 0.747	(ii) 0.008	(ii) 0.496	(ii) 0.496
2	(i) 12.7	(i) 0.228	(i) 0.166	(i) 0.727	(i) 0.107
	(ii) 8.6	(ii) 0.369	(ii) 0.001	(ii) 0.870	(ii) 0.129
3	(i) 56.7	(i) 0.034	(i) 0.390	(i) 0.350	(i) 0.260
	(ii) 56.7	(ii) 0.034	(ii) 0.132	(ii) 0.498	(ii) 0.370
4	(i) 465	–	(i) 0.177	(i) 0.003	(i) 0.820
	(ii) 465	–	(ii) 0.177	(ii) 0.581	(ii) 0.242
b) for ages (i) 70 and (ii) 90 years, and date 1977					
1	(i) 10.4	(i) 0.751	(i) 0.008	(i) 0.492	(i) 0.500
	(ii) 10.4	(ii) 0.751	(ii) 0.008	(ii) 0.155	(ii) 0.837
2	(i) 10.4	(i) 0.302	(i) 0.010	(i) 0.862	(i) 0.128
	(ii) 10.4	(ii) 0.302	(ii) 0.010	(ii) 0.862	(ii) 0.128
3	(i) 68.6	(i) 0.110	(i) 0.238	(i) 0.437	(i) 0.325
	(ii) 44.3	(ii) 0.010	(ii) 0.238	(ii) 0.437	(ii) 0.325
4	(i) 465	–	(i) 0.177	(i) 0.543	(i) 0.280
	(ii) 465	–	(ii) 0.177	(ii) 0.004	(ii) 0.819

maximised log-likelihood, but after a certain value of n subsequent increases will have little effect: *information criteria* (Davison,[4] Chapter 4) will be a useful guide here. Some assessment of the goodness of fit of the estimated distribution to the data, such as *quantile-quantile* plots, will provide further information about the fitted model — typically, extra phases may be necessary to accommodate a long upper tail of the distribution of the observed data. Numerically, if the quantities $(\mu_1+\lambda_1), (\mu_2+\lambda_2), \ldots, (\mu_{n-1}+\lambda_{n-1})$ and μ_n are very disparate then the resulting phase-type probability density function can be multimodal (in particular, with a spike at time 0 as occurred when a fifth phase was included in the models for the hospital data); imposing a *penalty* on such distributional shapes can help resolve this.[5] Finally, when the covariates are introduced, backwards elimination after fitting a fully parameterised model can be used to remove those effects that are deemed not significant.

Parameter estimates, for example ages and admission dates, from the four-phase fit to the male hospital data[1] are shown in Box 2.

Similarly, four phases provided an adequate fit to the female hospital data with the estimates shown in Box 3 after backwards elimination of covariate effects done using a 5% level of significance; these estimates are again for example ages and admission dates.

Some doubt has been expressed about the completeness of the information on the patients while back in their communities after discharge from hospital for the later dates in the data (Millard PH, Visiting Professor of Health Informatics, Westminster University, London and Emeritus Professor of Geriatrics, St. George's, University of London, private communication, 2006) since follow-up was not carried out for such patients. Indeed, examination of the data shows that the proportion of patients with destinations that ended their time back in the community not recorded, or possibly censored times here, increased quite dramatically after 1975 from about 7% (males) and 10% (females). Accordingly, only data relating to times spent back in the community from the years 1969–75 for patients with known destinations were used in estimating model parameters. This reduced the amount of available data for such estimation, so in carrying out the backwards elimination of covariate effects a significance level of 10% was used. The results for the males, shown in Box 4, are thus different from those given previously.[1]

The events that ended the patients' time back in the community were re-admission to hospital or death. An adequate description of the distribution of times for the males was provided by a three-phase model, with parameter estimates for example ages and dates given in Box 4.

Although five phases were required to adequately describe the distribution of the times spent by the female patients back in the community, the parameters describing the last three phases were such that $\mu_3 = \mu_4 = 0$ and $\lambda_3 = \lambda_4 = \mu_5 \neq 0$ so that these last three phases could be grouped together to produce a *composite* third phase with an Erlang distributed sojourn time with mean $3/\mu_5$ and coefficient of variation $1/\sqrt{3}$ (rather than an exponential distribution with coefficient of variation 1). This had the effect of *shortening* the upper tail of the fitted distribution, in accordance with the data. The parameter estimates, again for example ages and dates, are shown in Box 5.

Discussion

The four phases of the fitted distributions to the hospital data could be interpreted as assessment/diagnosis, treatment, rehabilitation and long stay. Likewise, the three phases of the fitted distributions to the community data could be interpreted

3 Estimates for females in hospital

Phase i	Mean sojourn time $1/(\mu_i+\lambda_i)$ (days)	Probability $\lambda_i/(\mu_i+\lambda_i)$ of transition to next phase	Probability θ_{i1} of transfer	Probability θ_{i2} of discharge	Probability θ_{i3} of death
a) for age 80 years, and dates (i) 1969 and (ii) 1985					
1	(i) 11.1	(i) 0.895	(i) 0.029	(i) 0.162	(i) 0.809
	(ii) 9.8	(ii) 0.788	(ii) 0.029	(ii) 0.417	(ii) 0.554
2	(i) 11.1	(i) 0.268	(i) 0.432	(i) 0.508	(i) 0.060
	(ii) 9.8	(ii) 0.277	(ii) 0.000	(ii) 0.949	(ii) 0.051
3	(i) 53.9	(i) 0.159	(i) 0.484	(i) 0.341	(i) 0.175
	(ii) 63.4	(ii) 0.011	(ii) 0.210	(ii) 0.522	(ii) 0.268
4	(i) 633	–	(i) 0.081	(i) 0.204	(i) 0.715
	(ii) 633	–	(ii) 0.081	(ii) 0.204	(ii) 0.715
b) for ages (i) 70 and (ii) 90 years, and date 1977					
1	(i) 10.0	(i) 0.801	(i) 0.029	(i) 0.496	(i) 0.475
	(ii) 11.0	(ii) 0.888	(ii) 0.029	(ii) 0.122	(ii) 0.849
2	(i) 10.0	(i) 0.307	(i) 0.001	(i) 0.959	(i) 0.040
	(ii) 11.0	(ii) 0.231	(ii) 0.001	(ii) 0.867	(ii) 0.132
3	(i) 70.4	(i) 0.051	(i) 0.280	(i) 0.476	(i) 0.244
	(ii) 53.3	(ii) 0.039	(ii) 0.391	(ii) 0.403	(ii) 0.206
4	(i) 633	–	(i) 0.081	(i) 0.204	(i) 0.715
	(ii) 633	–	(ii) 0.081	(ii) 0.204	(ii) 0.715

4 Estimates for males back in the community

Phase i	Mean sojourn time $1/(\mu_i+\lambda_i)$ (days)	Probability $\lambda_i/(\mu_i+\lambda_i)$ of transition to next phase	Probability θ_{i1} of re-admission to hospital	Probability θ_{i2} of death
a) for age 80 years, and dates (i) 1969 and (ii) 1975				
1	(i) 30.6	(i) 0.719	(i) 0.674	(i) 0.326
	(ii) 30.6	(ii) 0.719	(ii) 0.674	(ii) 0.326
2	(i) 303	(i) 0.133	(i) 0.624	(i) 0.376
	(ii) 123	(ii) 0.648	(ii) 0.624	(ii) 0.376
3	(i) 870	–	(i) 0.529	(i) 0.471
	(ii) 870	–	(ii) 0.529	(ii) 0.471
b) for ages (i) 70 and (ii) 90 years, and date 1972				
1	(i) 34.2	(i) 0.803	(i) 0.674	(i) 0.326
	(ii) 26.3	(ii) 0.617	(ii) 0.674	(ii) 0.326
2	(i) 135	(i) 0.614	(i) 0.727	(i) 0.273
	(ii) 297	(ii) 0.151	(ii) 0.509	(ii) 0.491
3	(i) 870	–	(i) 0.529	(i) 0.471
	(ii) 870	–	(ii) 0.529	(ii) 0.471

5 Estimates for females back in the community

Phase i	Mean sojourn time $1/(\mu_i+\lambda_i)$ (days)	Probability $\lambda_i/(\mu_i+\lambda_i)$ of transition to next phase	Probability θ_{i1} of re-admission to hospital	Probability θ_{i2} of death
a) for age 80 years, and dates (i) 1969 and (ii) 1975				
1	(i) 57.9	(i) 0.734	(i) 0.808	(i) 0.192
	(ii) 50.1	(ii) 0.635	(ii) 0.808	(ii) 0.192
2	(i) 475	(i) 0.267	(i) 0.624	(i) 0.376
	(ii) 475	(ii) 0.267	(ii) 0.624	(ii) 0.376
3	(i) 1290*	–	(i) 0.384	(i) 0.616
	(ii) 1290*	–	(ii) 0.384	(ii) 0.616
b) for ages (i) 70 and (ii) 90 years, and date 1972				
1	(i) 92.0	(i) 0.565	(i) 0.808	(i) 0.192
	(ii) 30.1	(ii) 0.787	(ii) 0.808	(ii) 0.192
2	(i) 641	(i) 0.361	(i) 0.624	(i) 0.376
	(ii) 339	(ii) 0.191	(ii) 0.624	(ii) 0.376
3	(i) 1290*	–	(i) 0.384	(i) 0.616
	(ii) 1290*	–	(ii) 0.384	(ii) 0.616

* These are $3/\mu_5$

as dependent (on some form of continued care), convalescent and recovered.

Males in hospital spent less time in the assessment/diagnosis and treatment phases at later dates, and were less likely to progress to the second of these phases but more likely to progress from the treatment to the rehabilitation phase; they were also less likely to progress to the long-stay phase at older ages. They were more likely to be discharged at later dates, whatever the phase from which their time in hospital ended, but were more likely to die at older ages if their time in hospital ended from the initial assessment/diagnosis phase or the final long-stay phase.

Females in hospital spent less time in the assessment/diagnosis and treatment phases and were less likely to progress to the second of these phases at later dates, but they spent more time in the rehabilitation phase and were less likely to progress to the long-stay phase. They spent more time in the assessment/diagnosis and treatment phases and were more likely to progress to the second of these phases at older ages, but they were less likely to progress from the treatment to the rehabilitation phase; they also spent less time in this rehabilitation phase and were less likely to progress to the long-stay phase at older ages. They were more likely to be discharged from the assessment/diagnosis phase at later dates and at younger ages, less likely to be transferred to another hospital from the treatment phase at later dates and more likely to be discharged at later dates and at younger ages; they were less likely to be transferred to another hospital and more likely to be discharged from the rehabilitation phase at later dates and at younger ages.

Males back in the community spent less time in the initial dependent phase and were less likely to progress to the convalescent phase at older ages; they spent less time in the convalescent phase and were more likely to progress to the recovered phase at later dates and at younger ages. They were less likely to be re-admitted to hospital from the convalescent phase at older ages, and less likely to be re-admitted to hospital from the final recovered phase than from the initial dependent phase.

Females back in the community spent less time in the initial dependent phase at later dates and at older ages, and were less likely to progress to the convalescent phase at later dates and at younger ages; they spent less time in the convalescent phase and were less likely to progress to the recovered phase at older ages. They were more likely to be re-admitted to hospital from an earlier phase than a later one.

There were some similarities between the distributions of males' and females' time in hospital for the first two phases of assessment/diagnosis and treatment, with less time spent in these phases and less likelihood of progressing to the second of them at later dates, which is suggestive of a tendency for less time to be spent in hospital generally at later dates. However, there were differences in later phases in that females tended to spend more time in the third, rehabilitation, phase but were less likely to proceed to the fourth, long-stay, phase at later dates, whereas the males only showed an increased likelihood of progressing from the treatment to the rehabilitation phase, suggesting that it was mainly the shorter stays that were being reduced for the males while there were some more general reductions for the females. Both males and females were less likely to progress to the long-stay phase of hospitalisation at older ages, which would seem quite sensible. Males tended to spend less time in the long-stay phase than did the females, and older males were more likely to die in this phase, whereas the older females showed no such tendency.

For time back in the community, both older males and older females spent less time in the initial dependent phase of the distribution. But males spent more time in the next convalescent phase at older ages, while the females continued to spend less time in this phase. The pattern of progression was different between these first two phases, with males being less likely to progress at older ages while the females were more likely to progress. But progression between the convalescent phase and the final recovered phase was similar with respect to age-related changes, with both showing a reduced likelihood of this at older ages. Females spent less time in the dependent phase at later dates and were less likely to progress to the convalescent phase, while the males spent less time in this convalescent phase and were more likely to progress to the recovered phase at later dates. Females tended to spend more time in all three phases than did the males, and they were more likely than the males to be re-admitted to hospital from the first phase whereas the opposite was the case in the last phase.

A tendency of older female patients to spend slightly longer times in the early phases of assessment/diagnosis and treatment in hospital is consistent with older patients being slower to respond to treatment, but such effects were not statistically significant from the smaller number of male patients. Older patients generally spent less time back in the community after discharge from hospital as they were less likely to progress to the final recovered phase, and older males were more likely to die in the convalescent phase: this is all consistent with older patients being more frail and slower to recover.

The effects of changing admission date suggest that both males and females generally tended to spend less time in hospital at later dates, which is consistent with programs to reduce "bed-blocking" over the years covered by the data. There was also a tendency for the males to spend more time back in the community after discharge at later dates, as they were more likely to progress to the final recovered phase of the distribution. But the females' tendency to spend less time in the initial dependent phase of the distribution of time back in the community at later dates, their higher (than for males) probability of being re-admitted to hospital from this phase and the lower probability of younger females progressing to the convalescent phase are suggestive of some women (particularly the younger ones and at later dates) having been discharged from hospital prematurely and at risk of subsequent relapse. On the other hand, had they remained in hospital too long there is the risk that they become long stay. Benefit, in both human and financial terms, can therefore be obtained by rehabilitating patients while in hospital and providing supportive community care after discharge. While still in hospital, they are therefore enabled to fare better when discharged; after discharge this is sustained by community care.

Conclusions

This analysis has shown that multi-state modelling is an appropriate means of analysing data on geriatric patient care and can reveal underlying patterns of differential effects, some of which may not be apparent from more routine data processing.

Competing interests

The authors declare that they have no competing interests.

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