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Analytic Approximation to the Bounce-average Drift Angle for Gyrosynchrotron-emitting Electrons in the Magnetosphere of V471 Tauri

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Abstract: Numerical modelling (Nicholls & Storey 1999) suggests that the eclipse of a wedge of enhanced number density of mildly relativistic electrons is responsible for the variations in quiescent radio emission of the binary system V471 Tauri. In the model, the wedge of enhanced density is created by electrons accelerated in the interaction region of the magnetospheres of the two stars, which subsequently drift in azimuth while emitting gyrosynchrotron emission. We present here an analytic approximation to the opening angle of the wedge of enhanced density and show that it is consistent with the opening angle derived from numerical modelling for reasonable values of the input parameters.

Keywords: stars: individual (V471 Tauri) — stars: magnetic fields — binaries: close — radiation mechanisms: nonthermal

1 Introduction

The eclipsing binary system, V471 Tauri, is an interesting system for several reasons, not least because it is the only known pre-cataclysmic system to exhibit non-thermal radio emission. V471 Tau comprises a white dwarf and a K2 dwarf only slightly distorted by the Roche potential. Values of the system parameters are given in Table 1. As well as emission due to flares (Crain et al. 1986), the quiescent radio emission has been shown to vary in intensity, in phase with the optical light curve (Patterson, Caillaut & Skillman 1993; Lim, White & Cully 1996).

In another paper (Nicholls & Storey 1999) we calculated the gyrosynchrotron intensity and circular polarisation from a three-dimensional model of the system's magnetic field, comprising a dipole field region around the K2 dwarf, and a magnetised stellar wind beyond the closed field lines (based on a model for RSCVn binary systems—see Storey 1996), with a region of enhancement of mildly-relativistic electron number density between the two stars. We assumed that the enhancement in density of mildly relativistic electrons between the two stars was due to the acceleration of electrons in the region where the magnetic fields of the two stars interact. The accelerated electrons are trapped in the region of the dipolar magnetic field of the K2 star and experience curvature/gradient drift, resulting in a drift in azimuthal angle from the white dwarf phase. We explored several models for the variation of electron number density in the region of enhanced number density, including different models for the decrease in number density with azimuthal angle ϕ, caused by gyrosynchrotron radiation losses. We showed that a wedge of enhanced mildly relativistic electron density that precedes the white dwarf, and in which the number density falls as a power law, provides the best fit (and a good fit) to the observed variation in intensity.

The numerical modelling indicated that the enhancement in number density of relativistic electrons fills a significant fraction of the magnetosphere around the K2 star, with the angular extent of the enhancement lying between about 90° and 200°. However, in the numerical model we assume that such an enhancement forms, and the angle through which the mildly-relativistic electrons drift is determined by the best fit to the data. In this paper we derive an analytic approximation to the angle through which gyrosynchrotron-emitting electrons will drift to before thermalising. We show that, given the assumptions made during the derivation, electrons neither drift so far that any azimuthal structure is smeared out, nor radiate their energy before a wedge of enhanced density has time to form, but that the gyrosynchrotron-emitting electrons are expected to form a wedge of enhanced density with an opening angle that is consistent with the best-fit numerical models.

To accurately calculate the angle through which the mildly relativistic electrons drift before thermalising, it is necessary to consider the non-uniformity of

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Table 1. System parameters for V471 Tau

Parameter	Value	Parameter	Value
Orbital period	$12{\cdot}51$ hours	Inclination angle	80°
Distance from Earth	45 pc	Presumed age	$0{\cdot}6$ Gyr
R_{K2}	$0{\cdot}8\ R_\odot$	R_{WD}	$0{\cdot}01\ R_\odot$
Combined mass	$1{\cdot}4\ M_\odot$	Rotation period of WD	$9{\cdot}25$ min
Distance between stars d_{WD}	$3{\cdot}1\,R_\odot = 3{\cdot}9\,R_{K2}$		

the magnetic field in calculating both the radiation timescale and the variation in drift velocity as the electrons lose energy. An order of magnitude estimate ignores these significant variations, and hence overestimates the angle through which the electrons drift.

The steps we take are

- to derive the bounce-average angular drift speed of the electrons, which depends on the Lorentz factor γ
- to derive an expression for the change of γ with time and hence the radiation timescale for the electrons
- to consider the effect of the dipole field on both $\gamma(t)$ and the radiation timescale
- to integrate the bounce-averaged drift speed over time
- and to average over pitch angle,

which leads to our final expression for the drift angle. Throughout the paper we use the frame of reference corotating with, and centred on, the K dwarf. The drifts due to the curvature and gradient of the magnetic field that we consider are drifts relative to the corotating plasma (Schulz & Lanzerotti 1974, pp. 4–6).

In Section 2 we present a short review of concepts and quantities needed to calculate bounce-average quantities in a dipolar magnetic field. In Section 3 we discuss how the Lorentz factors of the relativistic electrons change with time due to synchrotron losses in a dipolar magnetic field, and in Section 4 we calculate the extent of the angular drift of relativistic electrons in the time taken for them to radiate all their energy and thermalise. The discussion in Section 5, uses this analytic expression to calculate the extent of the drift for V471 Tau.

2 Review of Bounce-average Quantities

In this section we present a brief review of the quantities and concepts involved in calculating a bounce-average quantity in a dipolar magnetic field, and derive a bounce-average angular drift speed for the mildly relativistic electrons. As we are concerned with gyrosynchrotron emission by electrons we will refer to electrons, but many remarks are equally valid for other charged particles. Similarly, although we use a dipolar field, many of the concepts are applicable to other magnetic field geometries.

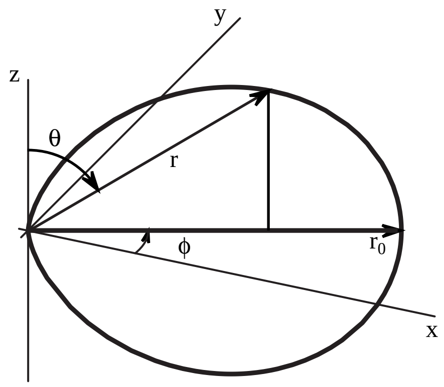

Figure 1—Diagram of a dipolar field line showing the coordinates r and θ and the equatorial radius r_0.

In spherical polar coordinates (r, θ, ϕ), (see Figure 1) the dipolar magnetic field, in SI units, is

$$B_r = \frac{\mu_0 m_m}{4\pi}\frac{2\cos\theta}{r^3}\,,$$

$$B_\theta = \frac{\mu_0 m_m}{4\pi}\frac{\sin\theta}{r^3}\,,$$

$$B_\phi = 0\,,$$

where μ_0 is the permeability of free space and m_m is the dipole magnetic moment. The differential equations of a field line are

$$\frac{dr}{B_r} = \frac{r d\theta}{B_\theta}, \qquad d\phi = 0\,, \tag{1}$$

which can be integrated to give

$$r = r_0 \sin^2\theta; \qquad \phi = \text{const}\,, \tag{2}$$

where r_0 is the equatorial radius of the field line, (see Figure 1). A field line is totally specified by its equatorial radius and its ϕ coordinate. The element of field line arc length ds follows from equation (2):

$$\begin{aligned} ds &= (dr^2 + r^2 d\theta^2)^{\frac{1}{2}} \\ &= r_0 \sin\theta(1 + 3\cos^2\theta)^{\frac{1}{2}} d\theta\,. \end{aligned} \tag{3}$$

Setting $B_0 = \mu_0 m_m / 4\pi r_\star^3$ to be the strength of the magnetic field on the surface of the star at the equator, with $r_\star$ being the radius of the star, we have

$$B(\theta) = B_0 \left(\frac{r_\star}{r}\right)^3 (1 + 3\cos^2\theta)^{\frac{1}{2}}$$
$$= B_0 \left(\frac{r_\star}{r_0}\right)^3 \frac{(1 + 3\cos^2\theta)^{\frac{1}{2}}}{\sin^6\theta} . \quad (4)$$

In a dipolar magnetic field there are three important timescales relating to the motion of charged particles — the cyclotron time t_c, the bounce time t_b and the drift time t_d — which we will consider in turn. In general the cyclotron time is very much shorter than the bounce time, which in turn is very much shorter than the drift time, $t_c \ll t_b \ll t_d$, and the derivations below depend on these relations holding. For more extensive reviews of this material see Roederer (1970) or Schulz & Lanzerotti (1974) and references therein.

The pitch angle α of an electron is the angle between the velocity vector and the magnetic field. The electron experiences a Lorentz force which causes it to gyrate about the magnetic field line. The period of one gyration is known as the cyclotron period or gyroperiod. In the frame of reference in which an observer sees the electron in a periodic orbit perpendicular to the magnetic field, known as the guiding centre system (GCS), the cyclotron period is

$$\tau_c = \frac{2\pi m_e \gamma}{eB} , \quad (5)$$

where m_e is the rest mass of the electron and γ is its Lorentz factor. The gyroradius, also known as the cyclotron radius and Larmor radius, is the radius of gyration of the electron and is given by

$$\rho_c = \frac{p_\perp}{eB} , \quad (6)$$

where $p_\perp = p \sin\alpha$ is the component of the electron's momentum p perpendicular to $\mathbf{B}$.

In a field such as a static dipolar field, an electron moving along a field line experiences a change in the strength of the field, which in the GCS appears as a change in field strength with time. If the changes in magnitude of $\mathbf{B}$ are very much slower than the cyclotron period, it can be shown that

$$\frac{p_\perp^2}{2m_e B} = \text{const} . \quad (7)$$

This is known as the first adiabatic invariant. In a static dipolar field the field lines are equipotentials and so as long as a particle follows a given field line, its kinetic energy will remain constant. In this case the above equation reduces to

$$\frac{\sin^2\alpha(s)}{B(s)} = \frac{\sin^2\alpha_i}{B_i} = \text{const} , \quad (8)$$

where s is the field line arc length, measured from an arbitrary point, labelled i, on the field line. We use the point where the field line crosses the equator as our reference point, and denote it by the subscript 0.

As the electron moves away from the equator the strength of the field increases and hence $\sin^2\alpha(s)$ increases until it reaches 1, at which point the electron's speed parallel to the magnetic field is zero. Since the component of the gradient of $\mathbf{B}$ parallel to $\mathbf{B}$, $\nabla_{||} B$, is non-zero, there is a component of the Lorentz force acting on the particle that is also parallel to $\mathbf{B}$ which drives the electron back the way it came. The point at which the electron is reflected is known as the mirror point. For a dipolar field line the mirror points (with magnetic field strength B_m) occur symmetrically about the equator, and a particle will bounce between the mirror points. Since $\sin^2\alpha = 1$ at the mirror point, from equation (2) we can see that

$$B_m = \frac{B_0 (r_\star/r_0)^3}{\sin^2\alpha_0} = \frac{B(s)}{\sin^2\alpha(s)} . \quad (9)$$

Rearranging gives

$$\sin^2\alpha(s) = \frac{\sin^2\alpha_0\, B(s)}{B_0 (r_\star/r_0)^3}$$
$$= \frac{\sin^2\alpha_0 (1 + 3\cos^2\theta)^{\frac{1}{2}}}{\sin^6\theta} , \quad (10)$$

where we have used equation (4). Using this, and denoting the particle speed by v and the component of the speed parallel to $\mathbf{B}$ by $v_{||} = v(1 - \sin^2\alpha)^{\frac{1}{2}}$, the bounce period of the electron is given by

$$\tau_b = 2 \int_{s_m}^{s'_m} \frac{ds}{v_{||}(s)}$$
$$= \frac{2}{v} \int_{s_m}^{s'_m} ds \Big/ \left(1 - \frac{\sin^2\alpha_0 B(s)}{B_0 (r_\star/r_0)^3}\right)^{\frac{1}{2}} , \quad (11)$$

where the integral is along the field line between mirror points s'_m and s_m. A change of variables from s to θ, the symmetrical placing of the mirror points about the equator and using equation (3), yields

$$\tau_b = \frac{4r_0}{v} \int_{\theta_m}^{\pi/2} \frac{\sin\theta (1 + 3\cos^2\theta)^{\frac{1}{2}} d\theta}{[1 - \sin^2\alpha_0 (1 + 3\cos^2\theta]^{\frac{1}{2}} / \sin^6\theta)^{\frac{1}{2}}} . \quad (12)$$

The bounce period is much greater than the cyclotron period, $\tau_b \gg \tau_c$. A related quantity is the half-bounce length, $S_b = \frac{1}{2} v \tau_b$, i.e. the distance along a field line between the two mirror points.

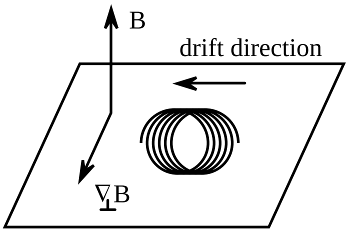

Figure 2—Diagram (exaggerated) showing the drift of an electron due to a component of the gradient of **B** perpendicular to **B**, $\nabla_\perp \mathbf{B}$. The stronger field region (bottom of figure) results in a smaller gyroradius than the weaker field region (top of figure), causing the gyroradius to vary periodically, resulting in the electron drifting across field lines.

The third timescale to consider is that of the azimuthal drift of the electrons, which arises because the dipolar field is not uniform in space. The gradient of the field means that during one gyration the electron does not experience the same field at all points on its path, which results in the radius of gyration changing in a periodic fashion. Hence the electron does not follow a strictly circular orbit, resulting in a drift across the field lines (see Figure 2). The curvature of the field also gives rise to a drift related to a centrifugal force, $\mathbf{F_c} = (mv_{||}/R_c)\mathbf{n}$, where R_c is the radius of curvature of the field line and $\mathbf{n}$ is the unit vector normal to **B** along the radius of curvature. Both the curvature and the gradient drifts are in the same direction and always appear together, and if the curvature and gradient of the field are small enough that there is very little change in the magnetic field strength over a gyroperiod, then the drift velocity, $\mathbf{V}_{CG}$, of an electron, caused by the curvature and gradient of **B**, is given by

$$\mathbf{V}_{CG} = \frac{\gamma m_e v^2}{2eBR_c}(2 - \sin^2\alpha)\,\mathbf{e} \times \mathbf{n}\,, \tag{13}$$

where e is the charge on the electron, and $\mathbf{e}$ is the unit vector tangent to **B**.

Equation (13) is the instantaneous drift speed of the electron, and as B, R_c and $\sin^2\alpha$ all change with θ along a field line, then V_{CG} changes along a field line as well. A more useful quantity can be defined as follows. For an electron passing through a point P there is a related point on the equator, 0, found by tracing down the field line passing through P to the equator (see Figure 3). While the electron at P is being displaced by $V_{CG}\delta t$, the associated point 0 is being displaced by $V_{0s}\delta t$. From Figure 3 we can see that the associated speed V_{0s} is related to the instantaneous speed $V_{CG}(\theta)$ through $r_0\phi$:

$$V_{0s}\delta t = r_0\phi\,, \tag{14}$$

$$V_{CG}(\theta)\delta t = r_0 \sin^3\theta\phi\,, \tag{15}$$

so that

$$r_0\phi = V_{0s}\delta t = \frac{V_{CG}(\theta)}{\sin^3\theta}\delta t\,. \tag{16}$$

Hence we can define the bounce-average drift speed, which is the drift speed of the electron's guiding centre at the equator, averaged over all the associated speeds V_{0s}, during one bounce, as

$$\langle V_0\rangle = \frac{2}{\tau_b}\int_{s_m}^{s'_m} V_{0s}\frac{ds}{v_{||}(s)}\,. \tag{17}$$

Hence the bounce-average *angular* drift speed is

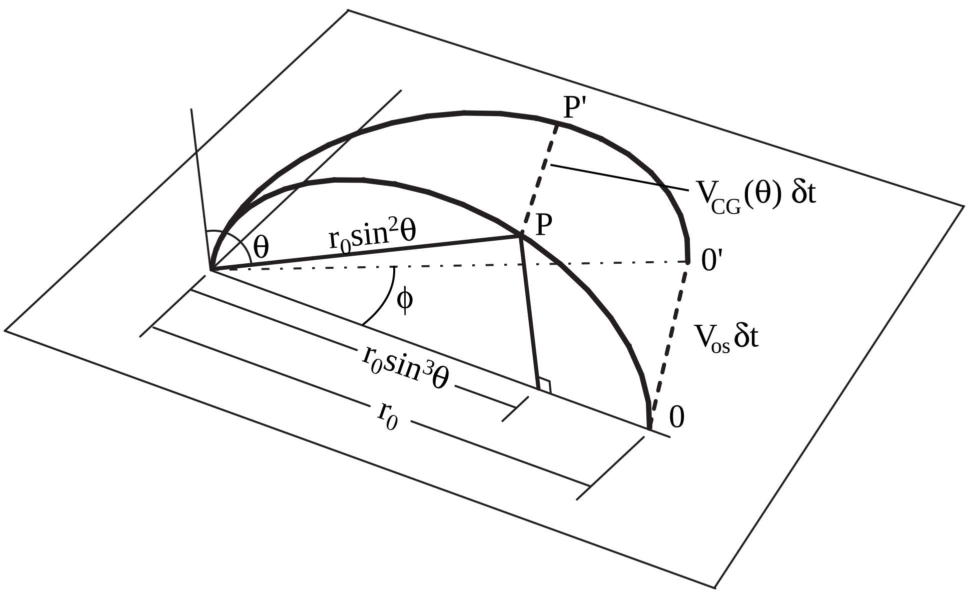

Figure 3—Diagram showing how the associated speed V_{0s} is related to the instantaneous drift speed V_{CG}.

$$\langle\dot{\phi}\rangle = \langle V_0/r_0\rangle = \frac{2}{r_0\tau_b}\int_{s_m}^{s'_m} V_{0s}\frac{ds}{v_\parallel(s)}$$
$$= \frac{4}{r_0 v\tau_b}\int_{\theta_m}^{\pi/2}\left(\frac{V_{CG}(\theta)}{\sin^3\theta}\right)$$
$$\times\frac{r_0\sin\theta(1+3\cos^2\theta)^{\frac{1}{2}}d\theta}{[1-\sin^2\alpha_0(1+3\cos^2\theta)^{\frac{1}{2}}/\sin^6\theta]^{\frac{1}{2}}}\,, \quad (18)$$

where we have used the same change of variables as for equation (12). Upon substitution of $V_{CG}(\theta)$ using equation (13), using equation (4) for $B_0(\theta)$, and

$$R_c = \left(\frac{r_0}{3}\right)\frac{\sin\theta(1+3\cos^2\theta)^{\frac{3}{2}}}{(1+\cos^2\theta)}\,, \quad (19)$$

we obtain

$$\langle\dot{\phi}\rangle = \frac{3m_e c^2\gamma\beta^2}{2eB_0 r_\star^2}\left(\frac{r_0}{r_\star}\right)g(\alpha_0)\,, \quad (20)$$

where $\beta = v/c$ and c is the speed of light. Here $g(\alpha_0)$ is the ratio of two integrals over θ. Both integrals have an integrable singularity at θ_m and due to the complexity of their integrands can only be evaluated numerically. A reasonable fit to this ratio, i.e. better than 1% for $6^\circ \leq \alpha_0 \leq 180^\circ$ (and even at 3° having only a $1{\cdot}5\%$ error), is

$$g(\alpha_0) = 0{\cdot}7 + 0{\cdot}3\sin\alpha_0\,. \quad (21)$$

3 The Lorentz Factor

In the previous section we reviewed the basic concepts necessary for calculating bounce-average quantities, finishing with the derivation of the bounce-average angular drift speed, which is dependent on several quantities, not least in this context being the Lorentz factor γ. In order to calculate how far an electron drifts in a given time, we need to know how γ changes with time due to synchrotron losses, which is derived in this section. We also demonstrate the validity of our assumption that the timescale of synchrotron losses is very much greater than the bounce time.

For the application under consideration here, the opening angle of the wedge of enhanced number density of mildly relativistic electrons in our model of V471 Tau, the important timescale is the time taken for a group of mildly relativistic electrons to lose its energy and thermalise. In the case of V471 Tau only synchrotron losses are taken into consideration. In this case Coulomb collisions are unimportant until the number density of thermal electrons is so high that Razin supression reduces the gyrosynchrotron emission to unobservable levels. The other potentially important loss mechanism is precipitation onto the K2 dwarf surface of those electrons with equatorial pitch angle sufficiently small that their mirror points are inside the star. It can be shown (Nicholls & Storey 1999; Kundu et al. 1987) that scattering is efficient in replenishing those pitch angles that lead to precipitation, and so it is reasonable to assume an isotropic distribution throughout the magnetosphere. However, scattering is not expected to take place uniformly throughout the magnetosphere and the loss due to precipitation is reduced by a factor related to the volume of the magnetosphere in which scattering occurs. In this paper loss through precipitation is assumed to be negligible, so the opening angle calculated below will be an upper limit.

Kardashev (1962) showed that for synchrotron radiation the energy of the particle decreases as

$$\frac{dE}{dt} \propto -B^2\sin^2\!\alpha E^2\,, \quad (22)$$

or in terms of the Lorentz factor (Petrosian 1985)

$$\frac{d\gamma}{dt} = -\frac{2e^4}{3(4\pi\epsilon_0)c^3m_e^3}B^2\sin^2\!\alpha\gamma^2\beta^2$$
$$= -aB^2\sin^2\!\alpha(\gamma^2-1)\,, \quad (23)$$

where $a = 2e^4/3(4\pi\epsilon_0)c^3m_e^3$. Integrating equation (23) yields

$$\gamma(t) = \frac{(\gamma_0+1)+(\gamma_0-1)\exp(-2aB^2\sin^2\!\alpha t)}{(\gamma_0+1)-(\gamma_0-1)\exp(-2aB^2\sin^2\!\alpha t)}\,, \quad (24)$$

where γ_0 is the initial Lorentz factor of the electron.

The radiation timescale is the time taken for the electron to reach Lorentz factor γ_f, and is given by

$$t_f = \frac{1}{2aB^2\sin^2\!\alpha}\ln\left(\frac{(\gamma_0-1)(\gamma_f+1)}{(\gamma_0+1)(\gamma_f-1)}\right). \quad (25)$$

In a uniform field $B^2\sin^2\!\alpha$ is constant, and for $\sin^2\!\alpha$ close to zero, emission is suppressed as the electron is travelling almost parallel to the magnetic field line and therefore in an almost straight line. However, in a dipole field $B^2\sin^2\!\alpha$ is changing constantly. If the rate of change of $B^2\sin^2\!\alpha$ is very much greater than the radiation timescale, that is if the electron is losing energy sufficiently slowly that it undergoes many bounces before its Lorentz factor changes appreciably, then it is a reasonable approximation to average $B^2\sin^2\!\alpha$ over a bounce period and use the average in equation (28).

For the physical parameters applicable to V471 Tau, see Tables 1 and 2, the bounce period for electrons on field lines that reach the white dwarf is of the order of seconds to a few tens of seconds for Lorentz factors greater than 3 and for the full range of pitch angles. Using the model magnetic field strength at the radius of the white dwarf (Nicholls & Storey 1999), equation (25) yields a lower limit

Table 2. Model parameters for V471 Tau (from Nicholls & Storey 1999)

Parameter	Dipole magnetosphere	Wind	Enhanced region
Surface magnetic field at equator (tesla)	$3{\cdot}5\times10^{-3}$	$1{\cdot}0\times10^{-3}$	$3{\cdot}5\times10^{-3}$
Electron density at stellar surface (m^{-3})*	$4{\cdot}0\times10^{8}$	$4{\cdot}0\times10^{8}$	$6{\cdot}5\times10^{10}$
Power law dependence of electron density on radius*	$0{\cdot}0$	$-2{\cdot}0$	$0{\cdot}0$
Power law dependence of electron density on energy*	$-2{\cdot}0$	$-2{\cdot}0$	$-2{\cdot}0$
Extent in stellar radii	$3{\cdot}9$	$13{\cdot}0$	$3{\cdot}9$

* These parameters refer to the mildly-relativistic electrons.

to t_f of 8×10^8 s for an initial Lorentz factor of 3, and a final Lorentz factor of $1{\cdot}1$. For initial Lorentz factors higher than 3, t_f will be even longer. So, for V471 Tau it is a reasonable approximation to use the average of $B^2\sin^2\alpha$ over a bounce period.

The bounce-average of $B^2\sin^2\alpha$ is

$$\langle B^2\sin^2\alpha\rangle = \frac{2}{S_b}\int_{\theta_m}^{\pi/2}\left(\frac{B_0^2(r_\star/r_0)^6(1+3\cos^2\theta)}{\sin^{12}\theta}\right) \times\left(\frac{\sin^2\alpha_0(1+3\cos^2\theta)^{\frac{1}{2}}}{\sin^6\theta}\right) \times\frac{r_0\sin\theta(1+3\cos^2\theta)^{\frac{1}{2}}\,d\theta}{[1-\sin^2\alpha_0(1+3\cos^2\theta)^{\frac{1}{2}}/\sin^6\theta]^{\frac{1}{2}}}\,. \tag{26}$$

Again this yields a ratio of integrals that can only be done numerically due to the complexity of the integrands. The fit,

$$\langle B^2\sin^2\alpha\rangle = B_0^2\left(\frac{r_\star}{r_0}\right)^6\left[0{\cdot}913+\left(\frac{0{\cdot}75}{\alpha_0}\right)^{3\cdot3}\right], \tag{27}$$

to the ratio of integrals in equation (26) agrees to better than 10% for all values of α_0. Hence the time dependence of γ is

$$\gamma(t) = \frac{(\gamma_0+1)+(\gamma_0-1)\exp(-2a\langle B^2\sin^2\alpha\rangle t)}{(\gamma_0+1)-(\gamma_0-1)\exp(-2a\langle B^2\sin^2\alpha\rangle t)}, \tag{28}$$

with $\langle B^2\sin^2\alpha\rangle$ given by equation (27).

Substituting equation (27) into the radiation timescale (25), we get

$$t_f = \frac{1}{aB_0^2(r_\star/r_0)^6[0{\cdot}913+(0{\cdot}75/\alpha_0)^{3\cdot3}]} \times\ln\left(\frac{(\gamma_0-1)(\gamma_f+1)}{(\gamma_0+1)(\gamma_f-1)}\right). \tag{29}$$

An inspection of this equation shows that as $\alpha_0\to0$, $t_f\to0$, and hence $t_f\le10^9$ s, which is a total reversal from the uniform field case stated above, which can be understood as follows. In a uniform magnetic field the magnetic field strength, and hence the pitch angle of the electron, is unchanging, so emission is most efficient for an electron with initial pitch angle of $\pi/2$, and suppressed for initial pitch angles of close to 0. However, in a dipole field the magnetic field experienced by the electron changes constantly with an associated change in pitch angle. As $\sin^2\alpha_0\to0$, $\theta_m\to0$, and so the electrons with very small equatorial pitch angles reach very high magnetic field strengths before they mirror. In other words, the electrons with smallest equatorial pitch angles spend the greatest amount of their bounce period in regions where the magnetic field is very large, and with their instantaneous pitch angles close to $\pi/2$ where their radiation is most efficient. Hence they will lose energy extremely rapidly. However, electrons with equatorial pitch angle of close to $\pi/2$ are trapped in low field regions and hence take longer to radiate their energy.

In practise the electrons with small enough pitch angle will precipitate onto the stellar surface, and the average radiation time for the remaining electrons will remain large. For electrons with $\alpha_0=\pi/2$, and hence trapped on the equator with unchanging conditions, the radiation time is the same as for electrons in a uniform magnetic field of the same strength and with pitch angle of $\pi/2$, as expected. For electrons that mirror just above the stellar surface on a field line with $r_0=d_{WD}$, the radiation time is 2×10^6 s, about 5 orders of magnitude greater than the bounce time. Hence the assumption of the bounce time being very much shorter than the radiation time, used to derive the expression for t_f and $\gamma(t)$, is valid for our parameters of V471 Tau.

4 Calculation of the Average Drift Angle $\langle\phi\rangle$

To calculate how far the electrons drift before they have lost sufficient energy to be indistinguishable from the ambient population of mildly relativistic electrons we integrate $\langle\dot\phi\rangle$ with respect to time, from $t=0$ to t_f and then average over α_0.

The integral over time is

$$\int_0^{t_f}dt\langle\dot\phi\rangle = \int_0^{t_f}\frac{3m_ec^2\gamma\beta^2}{2eB_0r_\star^2}\left(\frac{r_0}{r_\star}\right)g(\alpha_0)dt\,. \tag{30}$$

The only time-dependent quantity in equation (30) is $\gamma\beta^2=\gamma-1/\gamma$. With $\gamma(t)$ given by equation (28), the integral becomes

$$\int_0^{t_f} dt\langle\dot{\phi}\rangle = \frac{3m_e c^2}{2eB_0 r_\star^2}\left(\frac{r_0}{r_\star}\right)\frac{g(\alpha_0)}{a\langle B^2\sin^2\alpha\rangle}\ln\left(\frac{\gamma_0}{\gamma_f}\right) \quad (31)$$

All the α_0 dependence is in the $g(\alpha_0)/a\langle B^2\sin^2\alpha\rangle$ term so the average over α_0 becomes

$$\frac{1}{aB_0^2(r_\star/r_0)^6}\int_0^{\pi/2} d\alpha_0 \frac{0\cdot 7+0\cdot 3\sin\alpha_0}{1+(0\cdot 75/\alpha_0)^{3\cdot 3}} \bigg/ \int_0^{\pi/2} d\alpha_0\,.$$

$$= \frac{0\cdot 49}{aB_0^2(r_\star/r_0)^6}\,. \quad (32)$$

Bringing this all together gives the average angle through which the electrons drift in time t_f:

$$\langle\phi\rangle = 0\cdot 48\frac{9(4\pi\epsilon_0)m_e^4c^5}{4e^5B_0^3r_\star^2}\left(\frac{r_0}{r_\star}\right)^7\ln\left(\frac{\gamma_0}{\gamma_f}\right). \quad (33)$$

For a given system the pitch-angle-average drift angle depends only on the equatorial radius, and initial Lorentz factor. An electron with very large γ_0 drifts a very large distance before losing sufficient energy to thermalise regardless of its equatorial radius, for two reasons: its drift speed is initially very large and it lives for a very long time, which is why $\langle\phi\rangle\to\infty$ as $\gamma_0\to\infty$. Conversely, an electron with a small initial Lorentz factor will not drift far due to a low initial drift speed and short lifetime. For an electron of given Lorentz factor the drift speed is inversely proportional to both the magnetic field strength and the radius of curvature of the field, both of which decrease with equatorial radius. Hence an electron of given Lorentz factor will drift faster the further it is away from the star. As its lifetime also increases with decreasing field strength, an electron far from the star will drift further than one close to the star.

5 Application to V471 Tau

In evaluating $\langle\phi\rangle$ for a situation such as the opening angle of the wedge of enhanced number density of mildly relativistic electrons in the model for the radio emission from V471 Tau, we have to determine reasonable values for γ_f and γ_0, and r_0, and our determination of these is discussed in this section.

The value of γ_f is chosen so that the population of electrons accelerated in the interaction region of the magnetospheres of the white dwarf and the K2 dwarf has radiated sufficient energy that it is indistinguishable from the ambient population of gyrosynchrotron electrons. This defines the far edge of the wedge, i.e. the edge furthest from the longitude of the white dwarf. Numerical models indicate that the enhanced mildly-relativistic electron number density is much greater than the ambient mildly-relativistic electron number density (Nicholls & Storey 1999). Hence, at the far edge of the wedge most of the electrons have thermalised, so we set $\gamma_f = 1\cdot 1$. Gyrosynchrotron emission is by mildly relativistic electrons with Lorentz factors of a few to of order 10, which places an upper limit on γ_0.

If the electrons were confined to a flux tube of small cross sectional area, which extended to the white dwarf radius, then the choice of r_0 would be obvious. However, numerical modelling (Nicholls & Storey 1999) indicates that such a model is not consistent with the data, whereas models with a uniform distribution of mildly relativistic electrons throughout the wedge of enhanced electron density is much more consistent with the data, implying that radial diffusion is efficient. Hence, the mildly-relativistic electrons accelerated at $r = d_{WD}$ rapidly diffuse to fill the entire wedge from the radius of the white dwarf to the surface of the K2 star. It might be thought that the region of enhanced electron density would be narrow near the K2 surface, as electrons near the surface of the star have a slow drift speed and a short lifetime, with the region getting wider and wider as the equatorial radius of the electrons increases resulting in a faster initial drift speed and a longer drift time. However, some types of radial diffusion lead to an increase in energy of the electron [those mechanisms that do not violate the first or second adiabatic invariant, where the first is defined in equation (7) and the second is defined to be $\oint p_{||}ds$]. If this type of radial diffusion is important then the electrons that diffuse inwards will drift further in ϕ than ones that diffuse due to mechanisms that are energy conserving or energy losing. Further, the same processes that lead to radial diffusion near the white dwarf would be expected to operate in other regions of the magnetosphere of the K2 star, and so the inner regions of the wedge of enhanced density would be replenished by those electrons at the initial radius of $r_0 = d_{WD}$, and this would give rise to a region of enhanced number density that is independent of equatorial radius of the electrons, out to the radius of the white dwarf. Hence, we use $r_0 = d_{WD}$ when evaluating the drift angle. To calculate the exact shape of the enhanced-density region is not necessary for a comparison with our numerical modelling and is beyond the scope of this paper.

A more exact calculation of the pitch angle average between $\theta_{m\star}$ to $\pi/2$, where $\theta_{m\star}$ is the mirror point on the surface of the K2 dwarf for $r_0 = d_{WD}$, does not significantly change our results.

6 Results and Conclusion

We find from the analytic expression derived above that for $2 \le \gamma_0 \le 10$, we have $65° \le \langle\phi\rangle \le 240°$, and thus that a moderate-sized wedge of enhanced mildly relativistic electron number density develops for parameters applicable to V417 Tau. If the mildly-relativistic electrons drift very slowly, or radiate all their energy very swiftly, they would not drift far from the white dwarf, but pile up in a thin wedge of very small opening angle. At the other extreme, if the mildly-relativistic electrons drift very fast or radiate their energy very slowly, then they would drift many times around the star and smear out any azimuthal structure in the number density of the mildly-relativistic electrons. Our calculations show that the middle ground of a moderate-sized wedge of enhanced mildly-relativistic electron number density forms.

In our numerical work (Nicholls & Storey 1999) we use various models for the azimuthal distribution of the mildly-relativistic electrons in the region of enhanced density, to simulate the loss of energy of the electrons through synchrotron radiation. Such models include linear and power law decreases of electron density with azimuthal angle. Those models with an opening angle for the wedge of $90° \lesssim \langle\phi\rangle \lesssim 200°$, regardless of the way the electron number density decreases with azimuthal angle, best reproduce the orbital phases of the peaks and troughs of the observed data. This range of angles agrees closely with the estimates from the analytical work above.

In summary, for mildly relativistic electrons we have calculated analytically the bounce-average drift velocity and the bounce-average lifetime t_f in a dipolar magnetic field, and used these expressions to find the average angle through which such electrons drift during time t_f. We have applied this analytic expression to the model for V471 Tau and shown that the numerical results are consistent with the analytic approximation.

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