Short Note

Some Equivalent Bodies and Ambiguity in Magnetic and Gravity Interpretation

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The problem of ambiguity in potential field interpretation has long been recognised and arises from the fact that any observed field can be caused by an infinite variety of source distributions. This has not deterred modellers from proposing simple source distributions whose anomalies match the observed data but do not necessarily indicate the true geological picture. The underlying assumption in modelling is that a given geological unit is representable by a homogeneous body of restricted geometry. Knowledge of the geology enables us to justify such an assumption. This justification may take the form of a large number of rock property measurements and the consequent recognition that different lithologies may be distinguished by their responses. In addition, the expected structure is normally far from random and thus the model may be used with some degree of confidence. Where the rock properties are highly variable to the point where individual lithologies are not distinguishable and/or where the structure is complex then the inherent ambiguity may well lead to a grossly misleading model. If the observed data do not indicate this complexity then the structure cannot be reliably determined. We propose the term "observational ambiguity" for the case where the data does not distinguish between one geological structure and another. In general, we should seek the simplest source distribution consistent with our knowledge of the geology and whose anomalies match the observed data. A more complex model is not justified and should not be proposed. We also propose the term "model ambiguity" for a set of models of similar geometry, that are equally possible. The remainder of this paper is devoted to experiments to derive such equivalent models. There are a number of classic demonstrations of ambiguity in the literature (e.g. Skeels 1947) but these are confined to discussions of the gravity field. The clearest examples are the concentric spheres and the set of lens-shaped discs both of which are equivalent sets to the point mass (e.g. Johnson, Jupp and van Klinken, 1977).

We sought first to derive an equivalent set of lens-shaped, cross-sectioned bodies for a horizontal prism having a polygonal cross-section. We chose as our source model an octagon-shaped prism arranged with corners at the top and bottom. The gravity effect of this source model was computed and treated as an observed set of gravity readings. We then sought a model whose anomaly fitted this data to some prescribed tolerance. The chosen model was a polygon and had the following constraints

1) vertical symmetry about a defined horizontal median plane;

2) horizontal symmetry about a vertical plane through the origin;

3) constant density and hence mass.

The method used to obtain the optimum model was a non-linear optimisation procedure (van Klinken and Johnson, 1977) in which certain model parameters are allowed to vary until the model anomaly fits the source anomaly. The density, depth to the median plane and horizontal coordinates of the polygon vertices were kept fixed while the vertical coordinates were allowed to vary. A series of best-fitting models for median planes between the observation plane and the source depth were derived (Figure 1). Models whose median planes were close to the source depth tended to show instability effects. For shallower median planes the solutions were good and rapidly attained.

The exercise was then repeated for the magnetic case, the models having constant intensity and direction of magnetisation. An inclined field of 60° was chosen to give the characteristic bipolar nature of most magnetic anomalies. Care was taken to use the same optimisation criteria applied in the gravity cases. The resulting magnetic equivalent bodies (Figure 2) are virtually identical to the gravity equivalent bodies. The slight differences are attributed to the different shape of the objective function surfaces used in the minimisation procedure.

We propose that all cylindrical horizontal prisms have a set of equivalent bodies of lens-shaped cross-section and that these sets of equivalent bodies are identical for gravity and magnetic fields.

We continued the exercise in obtaining sets of equivalent bodies by reducing the model to its simplest form within the constraints. The source model and the approximating model were diamond shaped. The allowed variation was restricted to the vertical coordinate of the top and bottom vertices and the horizontal coordinate of the lateral vertices. The

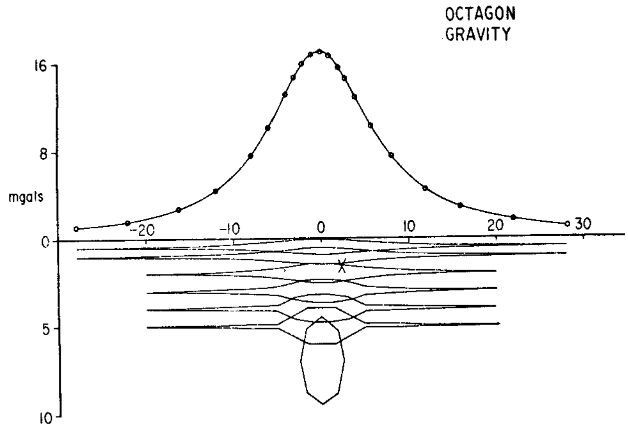

FIGURE 1

Equivalent gravity bodies to an octagonal horizontal prism

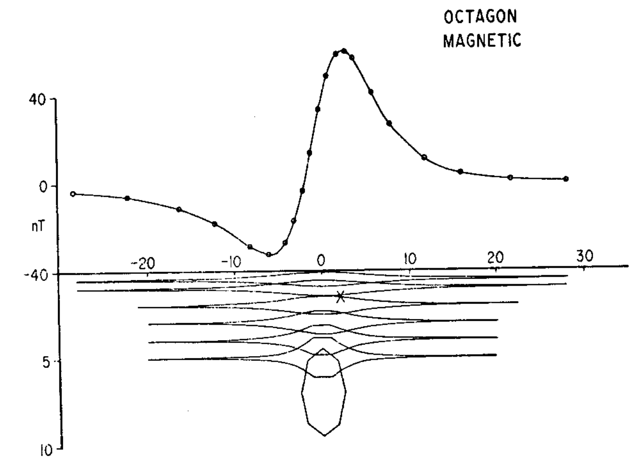

FIGURE 2

Equivalent magnetic bodies to an octagonal horizontal prism

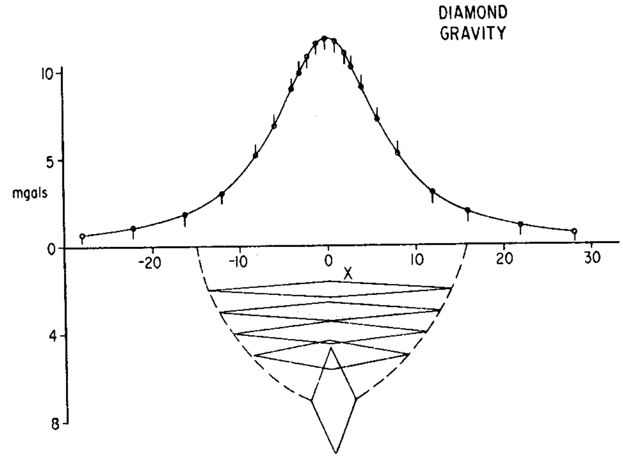

FIGURE 3

Equivalent gravity bodies to a diamond horizontal prism

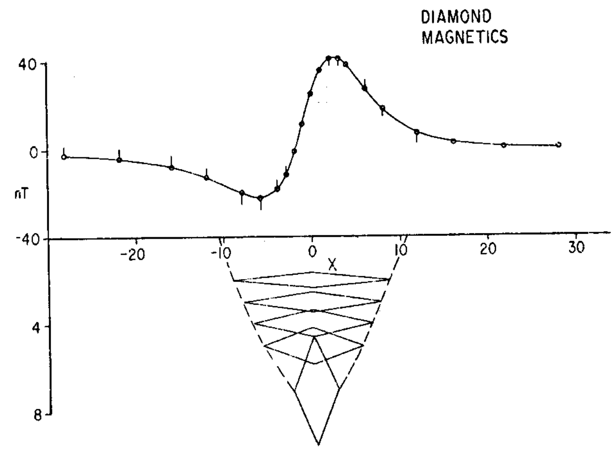

FIGURE 4

Equivalent magnetic bodies to a diamond horizontal prism

constraints of symmetry and rock properties were maintained. A set of equivalent bodies was thus calculated for gravity and magnetic fields for a set of median planes between the observation plane and the source plane (Figures 3 and 4). The approximations to the source anomalies were relatively poor and became worse for median planes farthest removed from the source plane. The lack of exact equivalent between the bodies is a demonstration of Al-Chalabi's (1971) statement that simple polygonal bodies are unique. A comparison between the equivalent bodies for magnetic and gravity fields shows that the variation in lateral extent of the magnetic bodies is not as great as for the gravity bodies. The difference in behaviour of these sets of bodies may indicate that magnetic fields are better at determining horizontal extents of structures whereas gravity may be better at determining the mass or depth of the structure.

In order to assess the importance of ambiguity in any model interpretation we recommend the application of a form of sensitivity analysis, in which the model parameters are perturbed by small amounts. The resulting effects on the calculated anomalies can thus be used to determine which of the model parameters are relatively important and which observations are most sensitive to variations in the model.

Acknowledgements

Mr Aksheya Kumar carried out some of the preliminary computations in these experiments using a program he developed in his doctoral research. The authors are grateful for support from a Macquarie University Research Grant.

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