A window constrained nonlinear inversion method for interpretation of aeromagnetic data

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SUMMARY

We introduce a nonlinear constrained inversion technique for 2D interpretation of aeromagnetic data along flight lines using a simple dike model. We first estimate the strike direction of a quasi 2D structure based on the eigenvector corresponding to the minimum eigenvalue of the pseudogravity gradient tensor (PGGT) derived from gridded magnetic field anomalies, assuming that the magnetization direction is known. Then the measured magnetic field can be transformed into the strike coordinate system and all magnetic dike parameters horizontal position, depth to the top, dip angle, width and susceptibility contrast can be estimated by nonlinear least squares inversion of the magnetic field data along the flight lines.

We use the Levenberg-Marquardt algorithm together with the trust-region-reflective method which enables users to define inequality constraints on model parameters such that the estimated parameters always lie in a trust region. Assuming that the maximum of the calculated g_{zz} (vertical gradient of the pseudogravity field) is approximately located above the causative body, data points enclosed by a window, along the profile, centered at the maximum of g_{zz} are used in the inversion scheme for estimating the dike parameters. The size of the window is increased until it exceeds a predefined limit. Then the solution corresponding to the minimum data fit error is chosen as the most reliable one.

Application of our method is demonstrated on a new aeromagnetic data set from the Särna area, West Central Sweden. Constraints from laboratory measurements on rock samples from the area are used in the inversion scheme.

Key words: Magnetics; Interpretation; Inversion; Parameter estimation; Potential field.

INTRODUCTION

A large variety of automatic methods have been developed for estimating magnetic source parameters such as depth, width, dip, susceptibility contrast and horizontal position of causative bodies. The most widely used of these methods are Werner deconvolution, Euler deconvolution, Naudy, and analytic signal techniques.

Beiki and Pedersen (2010) showed that for a gravity field caused by a quasi-2D structure, the eigenvector corresponding to the minimum eigenvalue of the gravity gradient tensor (GGT) is directed along the strike direction. Beiki *et al.* (2011) applied the same technique to the pseudogravity gradient tensor (PGGT) derived from the measured magnetic field assuming that the magnetization direction is known.

In this paper we use high resolution flight line magnetic field data to identify those anomalies satisfying quasi-2D conditions. First we interpolate the calculated PGGT from the grid at the data positions on the flight lines. In quasi-2D conditions, strike directions are assumed to be stable over distances several times greater than the flight line spacing. Then we identify maxima of the vertical gradient of the vertical component of the pseudogravity field (g_{zz}) assuming that this maximum occurs approximately above the source and at these points the signal to noise ratio is generally high. We now return to the high resolution line data and we transform the measured data into the strike coordinate system. Finally, we invert the corrected data for dike parameters within windows of variable length centered at the maxima of g_{zz} and select among all models those with the best data fit.

New data from the bedrock mapping program of the Geological Survey of Sweden (SGU) are interpreted using our technique and the results show a remarkable stability and agreement with the available geological information in the area.

METHOD

Nonlinear least squares method

The magnetic anomaly $F(x)$ caused by a dike of infinite strike and depth extent, magnetized in the direction of the Earth's main field, along a profile x perpendicular to the strike of the dike can be written as:

$$F(x) = C\left\{\cos\alpha\left[\tan^{-1}\left(\frac{x+b}{h}\right) - \tan^{-1}\left(\frac{x-b}{h}\right)\right] + \cdots \right.$$
$$\left. \frac{1}{2}\sin\alpha\,\ln\left[\frac{(x+b)^2+h^2}{(x-b)^2+h^2}\right]\right\} \qquad (1)$$

where $C=2kT\sin\beta$, $\alpha=2I-\beta-90^{\circ}$, k is the susceptibility, $I=\tan^{-1}(\tan i / \cos\gamma)$, T and i are the intensity and inclination of the magnetic field, respectively, γ is the azimuth of the profile measured clockwise from magnetic north, β is the dip angle, b is the half width and h is depth to the top surface of the dike.

The nonlinear equation (1) can be solved for estimating dike parameters within a window enclosing data points measured approximately above the dike. The parameter estimation problem is overdetermined if the window is sufficiently large containing number of data points greater than model parameters.

We used the Levenberg-Marquardt (LM) method together with the trust-region-reflective algorithm for solving equation (1). For brevity the formulation of the algorithm is omitted here and readers are referred to Aster *et al.* (2005) for more details of the LM algorithm.

Assuming that the data and model parameters are related through a nonlinear system of equations **g(m)=d**, the objective function to be minimized is

$$s = \sqrt{\frac{\sum_{i=1}^{N} r_i^2}{N - M}} \tag{2}$$

where residuals **r**=**g** ($\mathbf{m}^{est}$)- **d**, i is the index of data points, N and M are the number of data points and model parameters, respectively, $\mathbf{m}^{est}$ is the estimated model parameters and the data vector **d** is the measured magnetic field.

Interference effects

Using a large window more data points are incorporated in the algorithm and data fit error decreases. Also, in the presence of random noise, a large window is preferred to reduce the effect of local high frequency noise. However, this is often counter-balanced by increasing interference effects from neighbouring bodies. In practice, use of a very large window is risky as we may include some data points influenced by neighbouring sources. To find the optimum window size, we form a window centered at the maximum of the anomaly and we use data points located within this window for estimating the model parameters. Then we increase the size of the window in both directions until it exceeds a predefined limit. Finally we pick the estimate with minimum data fit error as the best solution from a set of estimates. We can also incorporate a linear function (first order polynomial) to the model (equation 1) to represent the interference effects.

We applied our method to some synthetic examples in the presence of interfering sources and Gaussian noise. Figure 1 shows the data fit errors plotted versus window lengths centered at the maximum of the magnetic field anomaly caused by a dike located between two other sources. Gaussian noise with zero mean and a standard deviation of 15 nT is added to the calculated magnetic field. It is also assumed that the magnetic field is perturbed by a background field of first order. As expected, by increasing the length of the window, the data fit error decreases and then starts to increase. For small windows the estimated parameters are more influenced by random noise and when the windows are large they are more affected by the interfering sources.

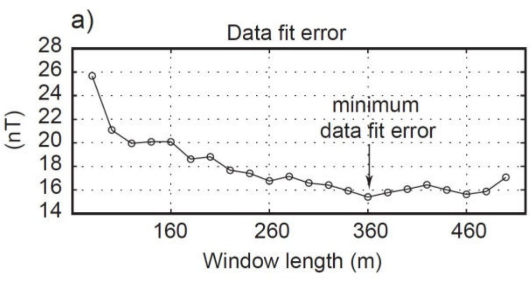

Figure 1. Data fit errors calculated for windows centered at the maximum of the magnetic field anomaly caused by a dike in the presence of random noise and interfering sources

Geological constraints

As mentioned earlier, we have used the trust-region-reflective algorithm together with LM algorithm to enable users to define inequality constraints on model parameters, such that estimated model parameters are always in a trust region. A priori geological information and an interpreter's own ideas can be given to the algorithm to obtain stable solutions and avoid unrealistic solutions. For example, negative depths can be avoided by defining a constraint of depths greater than zero (ground surface) or a susceptibility contrast varying between a lower and upper bound based on laboratory measurements of rock samples.

Beiki and Pedersen (2011) studied the effect of using inappropriate models to represent a given anomaly for the gravity case. They showed that the representation of the anomaly from a contact model using a dike model including a linear "regional" (low order polynomial) to some extent can fit the data within small windows making discrimination difficult. By excluding this "regional" (a low order polynomial) as part of the model, such discrimination is much easier. In this paper, we have included a first order polynomial in the model since the available geological information for the real data example (from the Särna area) indicates that the linear magnetic anomalies in this study area correspond to dolerite dikes.

Strike correction

In practice profiles are not generally orthogonal to the strike direction of the 2D structure. This causes errors in the estimation of source parameters depending upon the deviation from normal of the angle between the strike and profile directions. Beiki *et al.* (2011) showed that the strike direction of magnetic quasi-2D bodies can be estimated from the eigenvector corresponding to the smallest eigenvalue of the PGGT derived from the magnetic field data. Assuming that the maximum of calculated g_{zz} component (equivalent to the RTP field) is approximately located above the causative body, we form the windows along the line centered at the maxima of g_{zz}. Once the strike angle ψ (counted positive clockwise) is estimated, the measured magnetic field at the data points located within the windows are transformed into the strike coordinate system. The x-coordinate of the measurement points in the new coordinate system is

$$x' = x \cos \psi \ . \tag{3}$$

Once the magnetic field data in a given window is corrected for the strike angle, the presented constrained nonlinear inversion method is used for estimating the source parameters.

REAL DATA EXAMPLE

We applied our constrained inversion technique to an aeromagnetic data set from the Särna area, west central Sweden. The magnetic anomalies are dominated by several dike swarms. In 2004, the study area was covered with aeromagnetic measurements conducted by SGU. The nominal height of the aircraft was 60 m above ground with a sampling interval of about 17 m along flight lines. The flight lines were flown in an E-W direction with spacing of 400 m. The measured total magnetic field was reduced by subtracting the IGRF 2005 field. Figure 2a shows the gridded magnetic field anomaly of the study area with cell size of 100 m.

For the map shown here, to reduce aliasing effects between the fight lines, a low-pass filter with cut-off wavelength of 800 m was applied to the line data. Then the data was re-sampled to 100 m along and perpendicular to the flight line direction.

Assuming the magnetization vector to be parallel to the main field (inclination *Inc*=74.7^o and declination *Dec*=3.8^o), the PGGT components are calculated as described in Beiki *et al.* (2011). Once the PGGT components are calculated for the re-sampled data, they are interpolated to the actual measurement points along the flight lines.

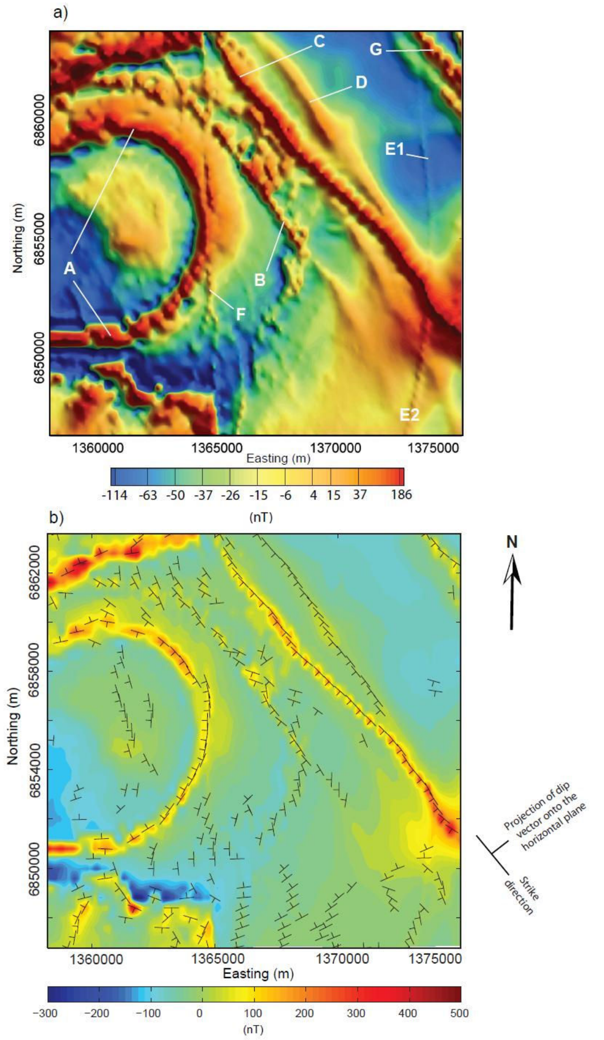

Figure 2. a) TMI Magnetic field and b) estimated strike directions together with dip angles acquired from the window constrained inversion technique at the maxima of g_{zz} with a dimensionality indicator smaller than 0.5 superimposed on the calculated g_{zz} map

Beiki and Pedersen (2010) used the dimensionality indicator, *I* (Pedersen and Rasmussen, 1990) to discriminate between quasi-2D and 3D geological bodies. The dimensionality indicator *I* varies between zero (pure 2D) and unity (pure 3D). Here, the dimensionality indicator *I* is calculated based on the PGGT components. We have defined a threshold of 0.5 to indicate the transition from 2D to 3D bodies. The data points with I<0.5 along the profiles and enclosed by a window centered at the maxima of g_{zz} are used in the analysis. We use a minimum window length of 100 m (7 data points) and the window length is increased by one data point in both directions until it exceeds the maximum window size of 350 m (21 data points). The solution corresponding to the minimum data fit error for the infinite dike model is chosen as the most reliable model estimate. Figure 2b illustrates the estimated strikes using an eigenvector analysis of the PGGT together with dip angles estimated using the inversion of the magnetic field data at the maxima of g_{zz} with *I* smaller than 0.5. A rose diagram of the estimated strikes is shown in Figure 3 illustrating that the strikes in the study area are strongly dominated by the N50W direction. The estimated dip directions of the dolerite dikes (Figure 2b) are very stable along the anomalies B, C, D and G and mainly dip westward. For the ring shape dolerite dike (anomaly A) they dip inward.

We have used the following constraints (bounds) in our inversion procedure:

1- The minimum and maximum depths to the top are 0 and 600 m, respectively.
2- The width of infinite dike model varies between 10 m to 70 m.
3- The dip angle varies between 30^o to 150^o.
4- The susceptibility contrast is set to 0.028 SI.

These constraints are chosen based on the available geological information of the study area. In addition, the solutions with data fit errors greater than 10 nT are rejected. The estimated parameters; depth to the top, width and dip angle are displayed in Figure 4. The estimated depths for the dolerite dikes are mainly shallow. This agrees with results acquired from Euler deconvolution (Beiki *et al.* 2011). Figure 4b shows that the estimated widths of dike like bodies mainly vary from 30 m to 50 m. The estimated dip angles in Figure 4c are predominantly about 60^o.

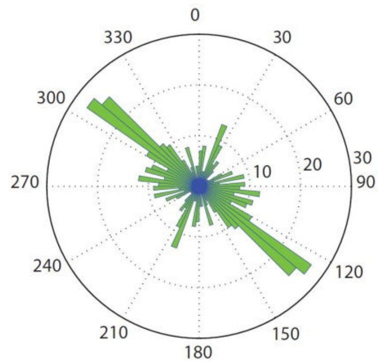

Figure 3. Rose diagram of the estimated strikes for maxima of g_{zz} with dimensionality indicator smaller than 0.5

Figures 5a and 5b show histograms of data fit errors and estimated depths for the final solutions, respectively. The depth estimates fall into two groups. A shallow one peaking around 30 m, indicating that the dikes outcrop below the glacial till cover and a deeper one around 250 m.

As the width exceeds a fraction of the depth, the data contains sufficient information to resolve both thickness and susceptibility contrast. In this particular case where the flight height was 60 m above the ground we could to some extent resolve both width and susceptibility contrast of the shallow outcropping dikes (the dolerite dikes A, C and G). However, the deeper dolerite dikes (B and D) can be well approximated by a thin sheet model because of their small width-to-depth ratio. Laboratory measurements on outcrop samples of dolerites show an average susceptibility of about 0.028 SI (private communication, L. Kübler, SGU). We therefore fixed the susceptibility contrast to 0.028 SI in our inversion procedure for the whole data set.

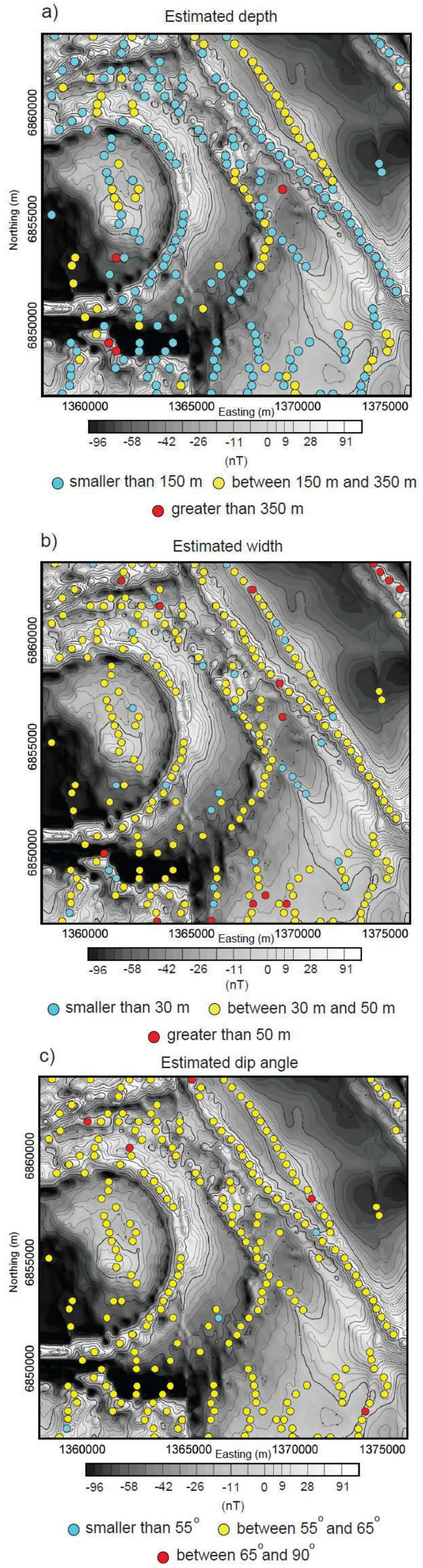

Figure 4. The estimated a) depth to the top, b) width, and c) dip angle acquired using the introduced method superimposed on a gray scale magnetic field anomaly map.

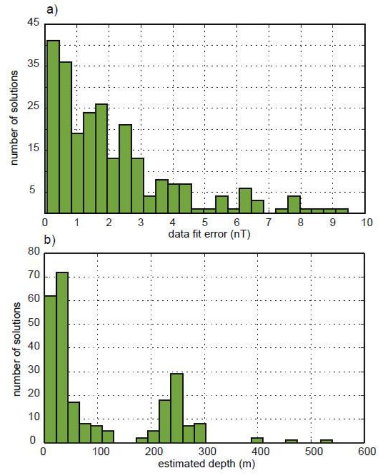

Figure 5. Histograms of a) data fit errors for final solutions and b) estimated depths to the top.

CONCLUSIONS

We have developed a constrained inversion technique for estimating magnetic dike parameters; the horizontal position, depth to the top, dip angle, width and susceptibility contrast. We use the Levenberg-Marquardt method together with the trust-region-reflective algorithm allowing for inequality constraints on the model parameters. This enables interpreters to use geological constraints in the inversion scheme such that the estimated model parameters are always in the chosen trust region.

In practice, aeromagnetic surveys are conducted along parallel lines perpendicular to the dominant geological strike in the study area. We estimate strike directions from the eigenvectors corresponding to the minimum eigenvalues of the PGGT as derived from the measured magnetic field. Once the strike direction is estimated in points of maximum g_{zz}, it is straightforward to estimate the dike parameters using the measured data in that neighbourhood.

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