

# Self-gradient effects for airborne gravity gradiometry

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## SUMMARY

The method by which an airborne gravity gradiometer measures gravity gradients and is stabilised within the aircraft affects the magnitude of the observed gravity gradient due to the nearby masses in the aircraft. To first order, the gravity gradient due to masses within the aircraft can be modelled using point masses. When the centre of mass of the instrument is stationary with respect to the aircraft, self-gradient is caused by rotation of masses about the centre of mass of the instrument resulting in a modest contribution to changes in observed gravity gradient. Movement of the centre of mass of the instrument with respect to the aircraft produces a larger self-gradient signal. In either case, the self-gradient signal correlates well with aircraft motion and can be easily removed from the observations by post-processing without the need for a complex model of the mass distribution within the aircraft.

Key words: airborne gravity gradiometry, gravity gradient.

## **INTRODUCTION**

Several high precision airborne gravity gradiometers are currently being developed (eg. Anstie *et al.*, 2010, Carroll *et al.*, 2010, Lumley *et al.*, 2004) with the aim to be able to recover the gravity gradient with a noise level better than 1 Eötvös (Eö) in a bandwidth of 1 Hz. There are significant challenges to achieving this goal for both the instrument and data processing.

Instrument challenges include achieving sufficient precision and reliability of the instrument and stabilisation of the instrument within the aircraft. Data processing challenges include correction for the terrain, tensor conversion and height correction. Another data processing challenge is the removal of the gravity gradient due to masses within the aircraft (e.g. Lee 2001). The result of an airborne gravity gradiometry survey should be a signal that is independent of aircraft motion. Repeated flights over the same line should produce the same gravity gradient signal to within measurement noise.

Airborne gravity gradiometers require some form of stabilisation within the aircraft so that the instrument receives minimal angular motion. Angular motion contributes to the signal measured by the instrument (and therefore needs to be kept to a minimum to maximise the signal to noise ratio. Linear accelerations experienced by the instrument exert torques due to imbalances in the stabilisation system. Therefore linear accelerations also result in angular motion of the instrument and can degrade the signal to noise ratio.

A stabilisation system consists of a set of concentric gimbals that prevent motion of the instrument in the roll pitch and yaw axes. When the aircraft rotates about any of these axes, the instrument remains at a constant orientation in space. Therefore the aircraft rotates with respect to the instrument. This change in mass distribution at such close proximity to the instrument must be considered, and if sufficiently large, corrected for.

This article discusses the magnitude of rotation of the aircraft and stabilisation system about the airborne gravity gradiometer and the resulting change in gravity gradient experienced by the instrument. Because gravity gradient is inversely proportional to the cube of the distance, it is important to distinguish between stabilisation systems that rotate within the aircraft from those that also allow vertical and lateral motion. The gravity gradient is calculated by approximating the main components of the aircraft and stabilisation system as point masses. The total variation in gravity gradient due to aircraft motion is shown to be less than 3 Eö. If the motion of the aircraft is recorded simultaneously with the gravity gradient, then the self-gradient, which is correlated with the aircraft motion, can be effectively removed leaving a signal that is independent of the aircraft motion.

## METHOD AND RESULTS

The gravity gradient due to a point mass M located at a point  $(x_1, x_2, x_3)$  with respect to the instrument is

$$G_{ij}(x_1, x_2, x_3) = \gamma \frac{M}{r^5} \begin{cases} 3 x_i^2 - r^2 & i = j \\ 3 x_i x_j & i \neq j \end{cases}$$

where  $r^2 = x_1^2 + x_2^2 + x_3^2$  and  $\gamma$  is the gravitational constant. The magnitude of any component of the gravity tensor is less than  $2\gamma M/r^3$ . The magnitude of the derivative of the gravity gradient tensor with respect to distance for any component of the tensor is less than  $9\gamma M/(2r^4)$  and with respect to rotation is less than  $3\gamma M/r^3$ . We can therefore give an upper bound on the change in gravity gradient at the instrument due to rotation or translation independently of which component is being measured by multiplying the magnitude of the appropriate derivative by the rotation angle or translation distance.

Figure 1 shows a gravity gradiometer inside an aircraft – the approximate dimensions are taken from a Cessna 205 Caravan which is an industry standard for airborne gravity gradiometry due to its stability. The most extreme rotations likely to be experienced consistently by the aircraft is about 6° away from the desired survey orientation. The upper bound for the change in gravity gradient is then approximately  $0.02 M/r^3$  Eö where the mass is in kg and distance in metres. Figure 1 shows the magnitude of the maximum change in gravity gradient due to the largest masses on the aircraft. The largest masses likely to affect the gravity gradient measurement are the engine, fuel and the pilot and operators (lighter but closer to the instrument). The figure shows that the fuel has the largest effect of slightly more than 1 Eö and the total effect is less than 2 Eö. This is comparable with the required precision for instruments under development (e.g. Anstie et al. 2010). Note that the calculations give the maximum effect assuming the rotation is in the direction in which the instrument is most sensitive.



Figure 1. The maximum change in gravity gradient observed by the instrument due to rotation by  $6^{\circ}$  about any axis. Values are given for each of the significant masses in the aircraft.

As mentioned above, some gravity gradiometers may require a linear stabilisation system to reduce linear accelerations experienced by the instrument. The limited space within the aircraft allows only relatively small movements in the horizontal plane and allows no room for vertical motion. The maximum change in observed gravity gradient due to motion of a mass with respect to the instrument by a distance dr is  $0.3 M dr/r^4$  Eö.

Figure 2 shows a gravity gradiometer inside a Cessna 205 Caravan. The largest linear motions are assumed to be 0.5 metres along the aircraft and 0.25 m across. Figure 2 shows the magnitude of the maximum change in gravity gradient due to the largest masses on the aircraft. Linear motions produce larger maximum changes in the observed gravity gradient than for the rotations shown in Figure 1. As for the case of angular motion, the maximum change is given assuming the motion is along the most sensitive axis of the instrument. Again, the change in gravity gradient due to the fuel is largest and the magnitude of the signal is comparable to the required precision of airborne gravity gradiometers currently under development.



Figure 2. The maximum change in gravity gradient observed by the instrument due to linear motion of the instrument along and across the aircraft. Values are given for each of the significant masses in the aircraft.

The calculations above show that the gradient due to masses within the aircraft are expected to be observable by modern AGG instruments. Given that the magnitude of the selfgradient is relatively moderate, it should be easy to correct for this part of the signal. This could be done by creating a more detailed model of the aircraft and stabilisation platform, taking into consideration the components of the gradient tensor measured by the AGG, recording the angles and distances by which the stabilisation platform moves and forward modelling the response of the instrument (van Leeuwen et al., 2005). However, the motions of the stabilisation platform are uncorrelated with external factors such as the topography and density anomalies on the ground. It is therefore possible to remove the component of the signal which correlates with the platform rotations and displacements by post-processing leaving a signal with no self-gradient component. The postprocessing procedure can be tested independently by conducting tests where only the self-gradient is changing (e.g. rotating the aircraft on the ground while keeping the instrument stationary or performing roll, pitch and yaw tests at high altitude where the gravity gradient is relatively uniform).

## CONCLUSIONS

The maximum change in the gravity gradient caused by linear and angular motion of an AGG with respect to the aircraft using analytical expressions for the gravity gradient tensor has been presented. The form of the expression identifies that large masses in close proximity to the instrument produce the largest effects and that relatively small linear motion of masses with respect to the instrument can produce larger signals than rotation of the instrument with respect to the aircraft. The overall magnitude of the self-gradient effect is not sufficiently large to warrant a detailed model of the aircraft and platform system. Post-processing to determine the part of the measured gravity gradient signal that correlates with the rotation and translation of the instrument within the aircraft will be sufficiently precise to reduce the self-gradient signal below the noise floor of modern airborne gravity gradiometers.

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