

# A robust scheme to output angle-domain common image gathers for shot-profile migration

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## SUMMARY

Angle-domain common image gathers (CIGs) play an important role in migration-based velocity updating. One popular criterion of migration velocity analysis (MVA) is the flatness of common-angle image gathers. We first revisit the popular schemes of generating angle-domain CIGs by wavefield continuation migration. Then, we present an alternative approach to produce commonangle image gathers. Our method, one of multiple-weight methods, is also for shot-profile migration based on oneway wave equation. To avoid the stability problem of multiple-weight methods, our scheme is implemented in complex number domain for every frequency after imaging. The ability of anti-alias resulted from sparse shot geometry is investigated. Numerical examples show that our strategy is efficient, stable, and easy to implement.

**Key words:** angle-domain common image gathers, oneway wave equation, shot-profile migration, sparse shot geometries

## INTRODUCTION

With the advances of computer technologies, wave-equationbased prestack depth migration (PSDM) is more affordable and has become a routine processing procedure for seismic data from complex regions. However, PSDM is dependent on the accuracy of interval velocity model, which can be derived through the migration velocity analysis based on angle-domain common image gathers (ADCIGs). Common angle image gathers can be obtained by either using Kirchhoff migration (Xu et al., 2001; Brandsberg-Dahl et al., 1999) or using waveequation migration. Stolk and Symes (2004) asserted that Kirchhoff migration in common angle domain bears kinematic artifacts due to the asymptotic assumption. Thus, we focus on ADCIGs produced from wave-equation migration, which can easily accommodate complex velocity structures.

There are two possible ways to generate ADCIGs from waveequation migration: 1) as a function of offset ray parameter ( $p_h$ ) (de Bruin et al., 1990; Prucha er al., 1999; Mosher and Foster, 2000); 2) as a function of reflection angle (Rickett and Sava, 2002; Xie and Wu, 2002; Sava and Fomel, 2003; Biondi and Symes, 2004; Zhang et al., 2005, 2007). In addition, ADCIGs can be obtained before imaging or after imaging (Sava and Fomel, 2003).

Although it is difficult to extract ADCIGs, shot-profile-based migration is preferable for wide-azimuth geometries. Rickett and Sava (2002) presented an indirect method to compute ADCIGs during shot-profile migration in two steps: first generating offset-domain CIGs, and then converting them into ADCIGs via the formulae of radial-trace transformation (Sava and Fomel, 2003). The method proposed by Rickett and Sava (2002) may suffer shot-aliasing while the shots are undersampled. Zhang et al. (2005, 2007) demonstrated a method of producing true-amplitude ADCIGs from shot-profile migration directly and systematically. However, they did not evaluate the ability of their method to tolerate shot-aliasing while the shot is under-sampled.

This paper aims to provide an alternative method to compute ADCIGs for shot-profile migration. The idea of the method is based on multi-weighted migration (Tygel et al., 1993), which is similar to that of Zhang et al. (2005, 2007). We propose a direct method to compute ADCIGs for shot-profile migration and evaluate its abilities for sparse shot geometry. First we briefly review some practical schemes of computing commonangle image gathers. Then we describe an efficient and stable algorithm to compute ADCIGs by starting from angle-dependent one-way wave propagators (Sun et al., 2010). Finally the effects of sparse shot sampling are numerically investigated and the results show that the method is stable and is suitable for sparse shot sampling geometries.

# REVIEWS

In this section, we briefly review some practical algorithms of producing ADCIGs. All of these algorithms are based on wave-equation migration after imaging, since they are more efficient and convenient for implementation than those before imaging (Sava and Fomel, 2003). Here, we classify the methods after imaging into two categories according to propagator's implementation: shot-receiver-based method and shot-profile-based method.

### Shot-receiver-based method

Sava and Fomel (2003) presented an accurate and robust method to produce ADCIGs. Their algorithm converts offsetdomain common image gathers (ODCIGs) to common-angle gathers via radial-trace or  $\tau$ -p transform in the Fourier domain. The relation in the frequency-wavenumber domain is

$$\tan\theta = -\frac{k_{x_h}}{k_z},\tag{1}$$

where  $\theta$  is the reflection angle,  $k_{x_h}$  and  $k_z$  are the horizontal half-offset and depth frequency wavenumbers, respectively.

#### Shot-profile-based method

For a single shot gather migration, the correlation-type imaging condition can be expressed as

$$I(\mathbf{x}, z) = real\left[\sum_{\omega} u_g(\mathbf{x}, z, \omega) u_s^*(\mathbf{x}, z, \omega)\right],$$
 (2)

where  $u_g$  and  $u_s$  are upward (receiver) and downward (shot) wavefields, respectively. The asterisk denotes conjugation.

Shot-profile migration is preferable in practical application since it is quite efficient for sparse-shot wide-azimuth geometries. Rickett and Sava (2002) described an approach for extracting ADCIGs during shot-profile migration by introducing the concept of "subsurface offset". They extracted an image that contains multiple offsets for a single shot migration through the equation

$$I(\mathbf{x},\mathbf{h},z) = real\left[\sum_{\omega} u_g(\mathbf{x}+\mathbf{h},z,\omega)u_s^*(\mathbf{x}-\mathbf{h},z,\omega)\right].$$
 (3)

After obtaining the ODCIGs, we can convert it to ADCIGs via slant stack shown in equation (1).

Another scheme is presented by Zhang et al. (2005, 2007) from shot-profile migration. Their method is based on true amplitude one-way wave equations (Zhang et al., 2003). Here, we review their work based on the conventional one-way wave equation (Claerbout, 1985) for migration velocity estimation. Their method is based on multiple-weight migration techniques, which has been applied to Kirchhoff migration (Tygel et al., 1993). Zhang et al. (2005, 2007) introduced a weight  $\cos\theta$  for imaging given by

$$I_{1}(\mathbf{x},z) = real\left[\sum_{\omega} \cos\theta u_{g}(\mathbf{x},z,\omega)u_{s}^{*}(\mathbf{x},z,\omega)\right].$$
(4)

Combined with equation(2), the ratio  $\cos\theta$  can give the subsurface-angle information at each image point by

$$\cos\theta = \frac{I_1(\mathbf{x}, z)}{I(\mathbf{x}, z)},\tag{5}$$

where  $\theta$  is the reflection angle. Then, we can convert shotdomain common image gathers (SDCIGs) to common-angle image gathers, according to the angle information at each imaging location (equation (5)).

#### METHOD

In this section, we will present our algorithm to obtain angledomain CIGs, starting from angle-dependent one-way wave propagators proposed by Sun et al. (2010). We introduce a weight in downward wavefield continuation, which can be expressed in the frequency-wavenumber domain as (Sun et al., 2010)

$$\tilde{u}_s(K_T, z; \omega) = \frac{k_0}{k_z(z)} u_s(K_T, z; \omega) , \qquad (6)$$

where  $\tilde{u}_s$  is the angle-dependent source wavefield in the frequency-wavenumber domain,  $u_s$  is the conventional source wavefield, and  $K_T = (k_x, k_y)$ .  $k_0$  is the reference wavenumber,

and  $k_z$  the vertical wavenumber at depth z and satisfies the following relationship

$$k_z = k_0 \cos\theta \,, \tag{7}$$

where  $\theta$  is propagation angle. Rewrite equation (2) into complex form for a single frequency

$$I_{cmplx}(\mathbf{x}, z, \omega) = u_g(\mathbf{x}, z, \omega)u_s^*(\mathbf{x}, z, \omega) .$$
(8)

Substituting equation (6) and (7) into equation (8), we have 
$$\frac{1}{2}$$

$$I'_{cmplx}(\mathbf{x}, z, \omega) = \frac{1}{\cos\theta} u_g(\mathbf{x}, z, \omega) u_s^*(\mathbf{x}, z, \omega) .$$
(9)

The ratio between the two images provides the subsurface angle information at every image point as

$$\theta(\mathbf{x}, z) = \cos^{-1} real \left[ \frac{\sum_{\omega} \left[ I_{cmplx}(\mathbf{x}, z, \omega) \right]}{\sum_{\omega} \left[ I'_{cmplx}(\mathbf{x}, z, \omega) \right]} \right].$$
(10)

Please note that Equation (12) is singular or unstable when the denominator approaches zero. In order to make the computation of the above equation stable, we rewrite it into the following form

$$\theta(\mathbf{x}, z) = \cos^{-1} real \left[ \frac{\mathbf{I}(\mathbf{x}, z) \mathbf{I}^{\prime*}(\mathbf{x}, z)}{\mathbf{I}^{\prime}(\mathbf{x}, z) \mathbf{I}^{\prime*}(\mathbf{x}, z) + i\eta} \right],$$
(11)

where the asterisk stands for conjunction;  $i = \sqrt{-1}$  is imaginary unit;  $\eta$  is a small positive real number; and  $I(\mathbf{x}, z)$  and  $I'(\mathbf{x}, z)$  are

$$\mathbf{I}(\mathbf{x}, z) = \sum_{\omega} \left[ I_{cmplx}(\mathbf{x}, z, \omega) \right]$$
(12)

and

$$\mathbf{I}'(\mathbf{x}, z) = \sum_{\omega} \left[ I'_{cmplx}(\mathbf{x}, z, \omega) \right], \qquad (13)$$

respectively. In this paper, we use equation (10) to avoid the instability. After obtaining the angle information at every position, we can follow the implementation proposed by Zhang et al. (2005, 2007) to stack shot-indexed CIGs at different angle bins to produce angle-domain CIGs.

#### NUMERICAL EXAMPLES

In this section, we use a numerical example to verify the accuracy and feasibility of our scheme. The model is a multilayer strata modified from Thorbecke and Berkhout (2006). The layered model, defined on a 481 by 301grids, is shown in Figure 1. The horizontal space sampling interval is 12.5m and the depth sampling interval is 10m. The dataset is synthesized using acoustic finite difference (FD) method by the package CWP/SU: Seismic Un\*x. The source function is a Ricker wavelet with a peak frequency of 30Hz. The shot interval is 12.5m with 481 receivers per shot. Receivers are set on the surface of model with 12.5m space interval.



Figure 1. A multi-layered model modified from Thorbecke and Berkhout (2006). The dashed line denotes the CIG location.

Figure 2 shows the migration profile computed by the splitstep Fourier method (Stoffa et al., 1990) and SDCIGs at location X=4000m. The image suffers numerical migration noise resulted from artificial reflections. Correspondingly, there are obvious phenomena in SDCIGs (Figure 2b). Figure 3 illustrates the stacked profile of common-angle gathers with angle ranging from 0° to 30°, and ADCIGs at location X=4000m with an angular bin size of  $2^{\circ}$ . It can be seen that the ADCIG migration result avoids the artificial reflections in Figure 3 by stacking common angle gathers in a specified ranges.



Figure 2. a) Migrated image of the multi-layered data set computed by conventional one-way method and b) corresponding shot-indexed CIGs at location X=4000m (the dashed line in Figure 1).



Figure 3. a) Migrated image of the multi-layered data set obtained by stacking ADCIGs and b) corresponding ADCIGs at location X=4000m (the dashed line in Figure 1).

Figure 4 shows SDCIGs and ADCIGs at every 80 shots (1000m), respectively. The angular bin size is 2° for ADCIGs. As can be seen, shot-indexed CIGs are suffered from much more numerical migration noise than angle-domain CIGs. This leads to much noise in conventional migrated image stacking shot by shot (Figure 2a), while little noise in angle-domain migrated image stacking common angle gathers (Figure 3a). Since the shot-indexed CIGs are contaminated by artificial noises, there are noises in ADCIGs where locates at high propagation angles. To obtain good images, we can stack common angle gathers using specified angle ranges. In another words, it can be considered transforming the migrated shot gather data into the ADCIGs functions as a noise filtering process.

Figure 5 illustrates the common angle gathers at location X=4000m computed with incorrect velocity, which notes the effect of velocity on ADCIGs. The three panels show CIGs produced by migrating the synthetic data set with 6% lower (Figure 5a) and 6% higher (Figure 5c) than with correct velocity (Figure 5b). As expected, events are over-corrected (curve up) when the migration velocity is lower than correct velocity, and events are under-corrected (curve down) when the migration velocity is higher than correct velocity.



Figure 4. CIGs corresponding to multi-layered model shown in Figure 1 indexed by shot a) and indexed by angle b).



Figure 5. Angle-domain CIGs, at location X=4000m, computed with a) 6% lower than, b) equal to and c) 6% higher than correct interval velocity.

## EFFECT OF SPARSE SHOT SAMPLING

Generally, sparse shot geometries are suitable for shot-profile migration (Rickett and Sava, 2002). They gives two migrated images with fully sampled and sparsely sampled (every 20 shots). The results show that the method suffers the serious problem of shot aliasing both in migration profile and ADCIGs. To investigate the ability of anti-alias of our method, we show ADCIGs produced with different shot intervals in Figure 6. The shot intervals are 50m, 100m, 200m, 300m, 400m, and 500m, respectively. Although it shows poorer illumination as the shot intervals become larger, shot aliasing can be found in migrated images (see Figure 7). One dominate reason of poor illumination is the fold decreasing as the shot interval increases.

The gathers computed by multi-weighted method are still regular and interpretable as the shot samplings become sparse (see Figure 6), although the problem of shot aliasing also exists. However, the common image-point (CIP) gathers computed by Rickett and Sava (2003) cannot be identified clearly (original Figure 7) while they migrated zero-offset images at shot interval of 500m.



Figure 6. Angle-domain CIGs, at location X=4000m, produced with different shot interval: a) 50m, b) 100m, c) 200m, d) 300m, e) 400m, and f) 500m.

## CONCLUSIONS

We presented an efficient and stable algorithm to compute common angle image gathers for one-way shot-profile migration. The angle information is obtained from the ratio between angle-dependent wavefields and conventional wavefileds computed by classical one-way wave equation. Our method can be easily extended to 3D case with no algorithm structure changed. The common angle gathers computed by multi-weighted method can handle shot aliasing resulted from sparse shot geometry well. However, the problem of shot aliasing still exists while the shot intervals are large. Numerical results demonstrate that the described scheme is efficient and stable, and can deal with the problem of shotaliasing as well.

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