A new physical model for the pressure sensitivity of unconsolidated sands

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SUMMARY

Knowledge of the pressure dependencies of rock properties in unconsolidated sands is important for accurate time-lapse feasibility studies, pore pressure prediction, and reservoir characterization. A key problem that arises in determining such pressure dependencies is an accurate model at low effective stress. We propose a double exponential model to describe the pressure sensitivity of the bulk modulus (K) or shear modulus (G) for unconsolidated sands. The physical basis for our model relies on observed porosity-depth trends in unconsolidated sands, and the concept of critical porosity. Our new model matches laboratory measurements on unsaturated sand samples that have a range of grain size distributions and compositions. Grain size distribution data is first used to estimate critical porosity, which is then used as a zero effective pressure constraint in the data fitting process. We show that our new model more accurately predicts pressure sensitivity near zero-effective pressure compared to current methods, and is thus more accurate for situations in which core measurements at low effective stresses are not available.

Key words: Pressure sensitivity, time-lapse seismic, pore pressure prediction, unconsolidated sands, critical porosity, fluid substitution.

INTRODUCTION

Calculating the sensitivity of reservoir rock properties with production-induced changes in pressure and saturation is important to determine if time-lapse seismic monitoring will be feasible, and when inverting for reservoir properties from seismic. While rock property changes with fluid saturation can be reasonably handled with Gassmann (1951) theory, changes caused by pore pressure variations are less well understood. It should be noted that without an accurate model of the pressure sensitivity of dry frame properties, Gassmann fluid substitution predictions would also be unreliable, since the dry frame properties are required inputs to these equations.

Existing pressure sensitivity models

The pressure sensitivity of a given reservoir rock is typically determined via ultrasonic velocity measurements on core samples over a range of effective pressures. Empirical models are fit to the measurements, and the regressions are then used to forward model changes in dry rock-frame properties. The most widely used model is that of Eberhart-Phillips et al. (1989). Shapiro (2003) summarized this empirical formula as

$$V = A + KP_{eff} - Be^{(-P_{eff}D)}, \qquad (1)$$

where V is velocity (compressional or shear), P_{eff} is the effective pressure (i.e. the overburden pressure minus the pore pressure) and A, K, B, and D are fitting parameters for the given set of core measurements. At high effective pressure this model predicts that velocity has a linear relationship with pressure, because the exponential term becomes negligible. Yan and Han (2009) propose a model without this linear term since at high effective pressures, once the sample has reached its minimum porosity, velocity should not increase further. They propose the following relationship

$$V = V_{\max}\left(1 - c_P e^{\left(-\frac{P_{eff}}{b_P}\right)}\right), \qquad (2)$$

where V_{max} is the high-pressure limiting velocity, and c_P, and b_P are fitting parameters. As we will show in this study, models of this type do not accurately predict velocities at low effective pressures for unconsolidated sands.

MODEL DEVELOPMENT

Based on the limitations of the models described above, we propose a new double exponential model to describe the pressure sensitivity of unconsolidated sands. The relationship is based on observed porosity-depth trends in unconsolidated sands, and incorporates a critical porosity constraint at zero effective pressure. We calculate the critical porosity using grain size distribution data, and show that this can be used to constrain the calculated velocity-pressure relationship. The model also exhibits asymptotic behaviour at high effective pressures, honouring observed laboratory data.

Zero effective pressure constraint - critical porosity

The critical porosity, ϕ_c, is defined as the porosity at which a rock's mechanical and acoustic behaviour is separated into two distinct domains (Mavko et al., 1998); for porosities lower than ϕ_c, the grains within the rock are load bearing, while for porosities greater than $\phi_{c,}$ the grains are in suspension (Nur et al., 1995). The effective bulk modulus (K_R) of a suspension can be accurately calculated as a harmonic (or Reuss) average of the fluid and mineral constituents

$$K_R = \left(\frac{\phi_c}{K_{fl}} + \frac{1-\phi_c}{K_m} \right)^{-1}, \tag{3}$$

where K_{fl} and K_m are the bulk modulus of the pore fluid and grain material, respectively. The shear rigidity (G) of a suspension is zero since the fluid is load bearing. Marion et al. (1988) found that in suspended sediments the velocity varies negligibly with porosity and is accurately approximated by Wood's (1955) equation

$$V_P = \sqrt{\frac{K_R}{\rho}}, \tag{4}$$

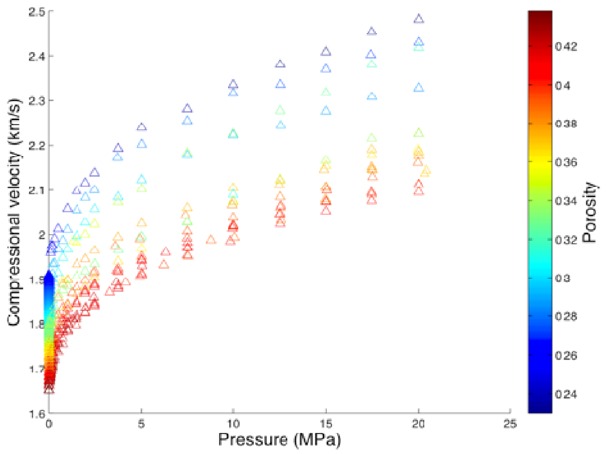
where ρ is the bulk density. Figure 1a shows velocity-pressure measurements, coloured by porosity, for unconsolidated samples from Zimmer (2003). Wood's bound has been plotted on the y-axis to show that velocity approaches that of a suspension at zero effective pressure. Studies by others (e.g. Prasad, 2002) confirm this result. This suggests that, provided we can estimate the critical porosity of a given rock sample, we can use the critical porosity as a zero effective pressure constraint in the data fitting process.

Critical porosity is closely correlated to rock texture. Textural controls include sorting, grain size, grain shape (roundness and sphericity), and fabric (packing and grain orientation). Studies by Beard and Weyl (1973), and Scherer (1987) suggest that grain sorting is the primary control on critical porosity, with well sorted sediments typically having higher porosities. Figure 1b shows porosity-sorting data for 48 unconsolidated samples from Beard and Weyl, (1973). Scherer (1987) generalized this data to develop a relationship between sorting and critical porosity (also plotted below)

$$\phi_c = 20.91 + \frac{22.9}{S_0}, \tag{5}$$

where S_0 is the Trask (1931) sorting coefficient, calculated from grain size distribution data. We use this method to estimate the critical porosity of samples from Zimmer (2003), then input these values into Equations (3) and (4) to calculate the zero effective pressure constraints.

a)



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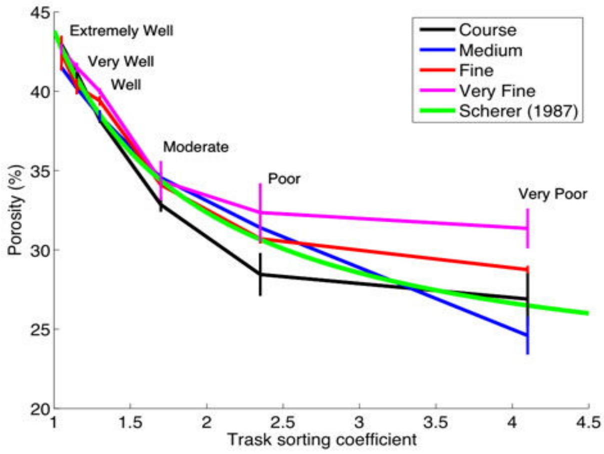


Figure 1. a) Plot of compressional velocity versus effective pressure, coloured by porosity, for unconsolidated samples from Zimmer (2003). Dry data is water saturated by Gassmann fluid substitution. Wood's bound for suspensions is plotted on the y-axis to show that velocities tend to that of a suspension at zero effective pressure. b) Plot of critical porosity versus Trask sorting coefficient for 48 unconsolidated samples from Beard and Weyl (1973). Data shows well sorted samples have higher critical porosities than poorer sorted sediments. Also plotted is Scherer's (1987) relationship.

Porosity-depth trends

Mechanical compaction is the dominant porosity-reducing mechanism for non-diagenetic rocks during burial from 0 to 2.5-3km depth. During burial sands are subjected to compressive forces due to increased overburden load. This compressive force results in compaction. It has theoretically been shown that compaction-induced porosity loss follows an exponential trend with depth

$$\phi = \phi_c e^{-cZ}, \tag{6}$$

where ϕ is the porosity at depth Z, and c is a constant (Athy, 1930). However, at shallow burial, grain rotation and reorientation can account for significant porosity loss (Berner, 1980). This added component of porosity loss means Equation 6 is not applicable in the shallow section. To account for this, Dutta et al. (2009) fit a general exponential equation of the form

$$\phi = ae^{bZ} + ce^{dZ}, \tag{7}$$

where a, b, c, and d and fitting parameters. Note: (a+c) is equal to the critical porosity at Z=0. Assuming a linear relationship between depth and effective pressure, we use Equation 7 to fit porosity-pressure data from Zimmer (2003). We also constrain the fit with estimated critical porosity values from Equation 5.

Our new pressure sensitivity model

Since bulk modulus (K) is proportional to $1/\phi$ for unconsolidated sands (Reuss or Lower Hashin-Strikman bound – see Mavko et al., 1998), and the porosity-pressure relationship follows a double exponential trend (Equation 7), we propose a double exponential relationship of the form

$$K = A + Be^{-CP_{eff}} + De^{-EP_{eff}} , \qquad (8)$$

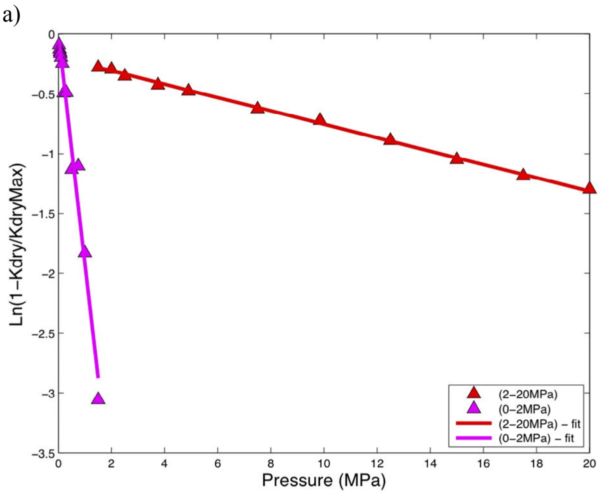
to describe the pressure sensitivity of unconsolidated sands, where A, B, C, D, and E are fitting parameters. Note: (A+B+D) is equal to the zero effective pressure value calculated by substituting Equation 5 into Equation 3. We show that this model, with the corresponding critical porosity constraint, accurately describes the pressure sensitivity of unconsolidated sands. The model also exhibits asymptotic behaviour at high effective pressures, honouring observed laboratory data. The same form of Equation 8 is also fit to shear moduli (G). We have chosen to fit moduli data, rather than velocities, since this helps ensure the velocity-porosity-pressure relationship is physically realizable. It also improves the accuracy of fluid substitution, since the dry-rock moduli and porosity are inputs into Gassmann's equations.

APPLICATION TO LABORATORY DATA

To test our new model, a database of laboratory measurements on unconsolidated samples was assembled from published data. 15 datasets were compiled from Zimmer (2003), covering both sand and glass bead samples. The samples range from moderate to high porosities (26 to 43%) and have a range of grain size distributions and sorting. The database includes measurements of compressional velocity, shear velocity, and porosity, as a function of effective pressure over the range of 0-20MPa.

Figure 2b shows a typical fit of our double exponential model to the bulk (K) and shear (G) moduli for the Galveston Beach sand sample from Zimmer (2003). In general, Equation 8 provides an excellent fit to the data, with accurate results to zero effective pressure. The fitting process involves fitting two exponentials of the form of Equation 2, one for the low effective pressure range (e.g. 0-2MPa) and one for the high effective pressure range (e.g. 2-20MPa). See Figure 2a for further description on the fitting process.

a)



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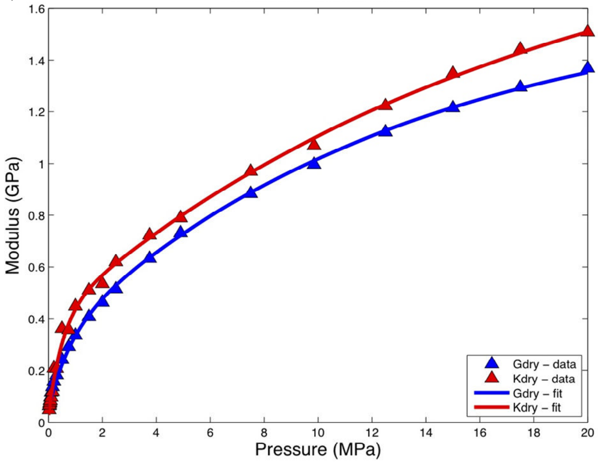
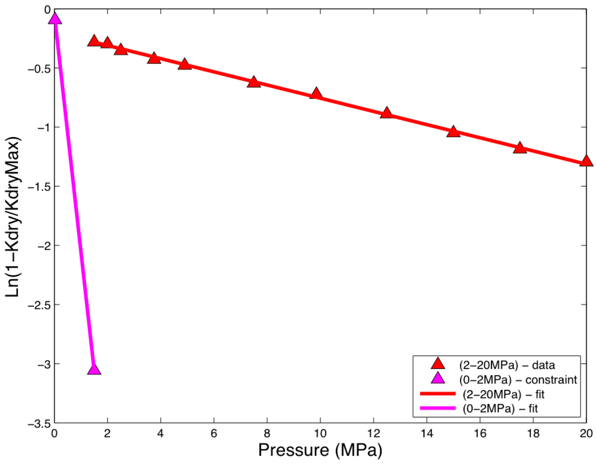


Figure 2. a) Log of normalized dry bulk modulus (K) versus effective pressure for the Galveston Beach sand sample from Zimmer (2003). Kmax – the high effective pressure asymptote value, normalizes data from 2-20MPa while Kdry at 2MPa normalizes data from 0-2MPa. Data for each case is then fit in a least squares sense to determine the fitting parameters for the double exponential model. b) Plot of bulk modulus (K) and shear modulus (G) versus effective pressure for the Galveston Beach sand sample, with corresponding fits of Equation 8. The double exponential model accurately fits the data over the full pressure range.

Figure 3 demonstrates the predictive power of the proposed model to low effective pressures. The inclusion of a critical porosity constraint at zero effective pressure enables accurate determination of model fitting parameters at low effective pressures (e.g. in the range of 0-2MPa). Figure 3b shows the corresponding fit of our double exponential model to saturated (Gassmann) compressional velocity data (red curve), predicted using only data from 2-20MPa and the critical porosity constraint. Actual data from 0-2MPa is shown in magenta. The double exponential model accurately predicts the pressure sensitivity to zero effective pressure, where the same prediction using a traditional single exponential relationship (e.g. Yan and Han (2009)), as in Equation 2 (blue curve), fails to predict the data at low effective pressures. This has obvious implications for rocks where core data at very low effective pressures is not available.

a)



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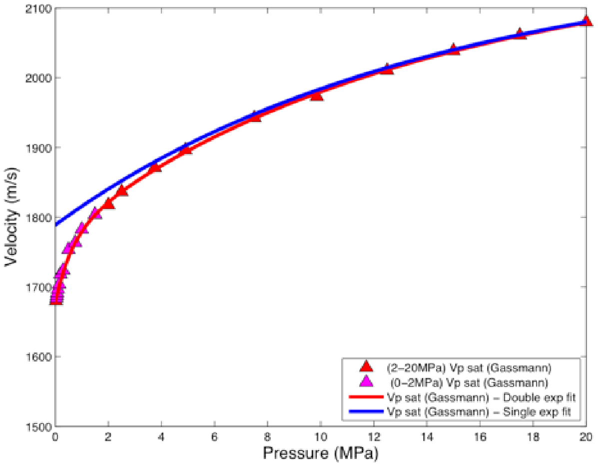

Figure 3. Blind test prediction of data below 2MPa for Galveston Beach sand sample from Zimmer (2003), using critical porosity constraint and our new double exponential model. a) Log of normalized dry bulk modulus (K) versus pressure for data from 2-20MPa (red triangles). The zero effective pressure constraint estimated from grain size distribution data and Equation 3, along with the value at 2MPa, is also plotted (magenta triangle at zero pressure). This critical porosity constraint enables accurate prediction of fitting parameters below 2MPa, even though data is not available in this range. b) Corresponding fit of the double exponential model to saturated (Gassmann) compressional velocity data (red curve), predicted using only data from 2-20MPa and the critical porosity constraint. Actual data from 0-2MPa is shown in magenta. The double exponential model, with its critical porosity constraint, accurately predicts the pressure sensitivity to zero effective pressure. The predicted model using a single exponential relationship, as in Equation 2, is also plotted (blue curve). As we can see, it fails to predict the data at low effective pressures.

CONCLUSIONS

We used compaction theory and the critical porosity concept to develop a new double exponential model to describe the pressure sensitivity of unconsolidated sands. The model accurately describes the behavior of bulk modulus (K) and shear modulus (G) over a wide range of effective pressures, especially near the fract point at zero effective pressure. Our new model also allows for improved calculation of pressure-sensitive velocities with Gassmann fluid substitution. The use of a critical porosity constraint means accurate predictions can be made to zero effective pressure, even if laboratory core data at low effective pressures are not available. Our new model is an alternative to existing velocity-pressure relations for time-lapse seismic studies, pore-pressure prediction and reservoir characterization.

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