

# Inversion of surface and downhole electromagnetic data for a 3D earth

**David Annetts**

CSIRO ESRE

26 Dick Perry Avenue, KENSINGTON, 6151

David.Annetts@csiro.au

## SUMMARY

A program described by Sugeng [1998] which calculates the full domain 3D EM response of an earth modelled by hexahedral finite elements was extended to invert ground survey data using the damped eigenparameter method described by Jupp and Vozoff [1975].

The program is demonstrated using a small numerical model of a SIROTEM-like fixed-loop survey consisting of three surface traverses and a downhole line. Results suggest that provided data are not too noisy, a reasonable model can be recovered if the starting model is quite close to the true model. This suggests that the program might best be used to inverse-model particular features of interest rather than as a starting point for interpretation.

**Key words:** electromagnetic, 3D, inversion, ground, downhole.

## INTRODUCTION

Inversion of electromagnetic data for a one-dimensional earth is commonplace in ground geophysics. However, the earth is not one-dimensional and it is known that significant errors can arise when the earth's underlying dimensionality is not properly modelled. This work does not address the relative merits of inverting particular data sets 1- versus  $n$ -dimensional earths. Rather, it describes a tool by which such merits may be addressed.

In many respects, the work described in this paper can be considered an extension of the work described by Sugeng et al. [2006]. Major differences are that this work describes ground prospecting systems, and that the full computational domain is modelled.

## METHOD AND RESULTS

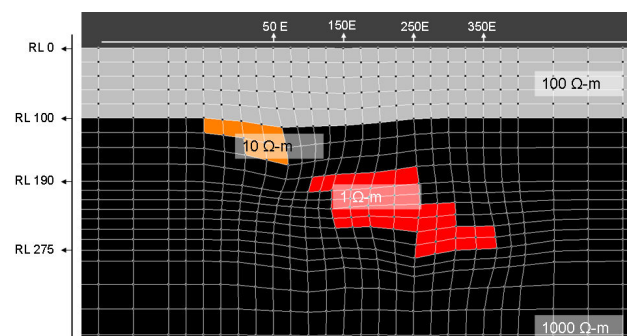
The program *Loki* was described by Sugeng [1998]. It models the 3D response of the earth to using a regular hexahedral finite element mesh. The energising source is restricted to magnetic dipoles, or combinations thereof, but the program is capable of modelling most extant ground EM surveys.

Ellis [1998] concluded that when inverting AEM data for 3D models, the Gauss-Newton method was optimal in terms of convergence and computational efficiency. This method iteratively updates model parameters,  $m$ , to minimise the residual errors,  $r$ . At the  $i$ -th iteration we have

$$m_i = m_{i-1} + J_{i-1}^+ r_{i-1} \quad (1)$$

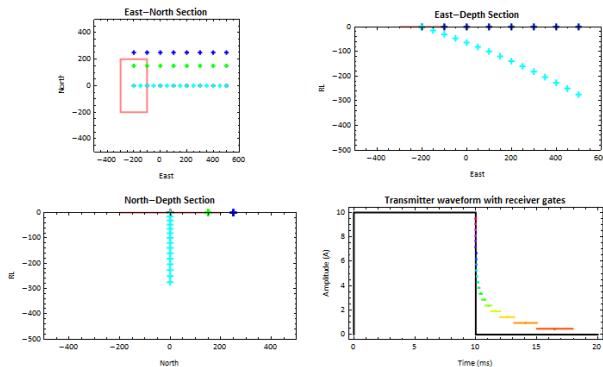
In Equation 1,  $J_{i-1}^+$  is the generalised inverse of the Jacobian matrix or normalised sensitivities. Solution to Equation 1 is based on the damped eigenparameter method described by Jupp and Vozoff [1975] and implemented by Raiche et al. [1985]. The Jacobian is computed using the method domain differentiation described by Wilson et al. [2006].

To test the efficacy of inverting surface and downhole data for a three-dimensional earth, a small survey was set up. The model is illustrated in Figure 1 and consists of a small irregular target zone and an irregular paleochannel both underlying a regolith and hosted by a resistive basement. Figure 1b shows an east-west cross section through +50 N and illustrates the irregular nature of the target. The 3D model consisted of  $49 \times 31 \times 32$  (E x N x Z = 31899) cells.



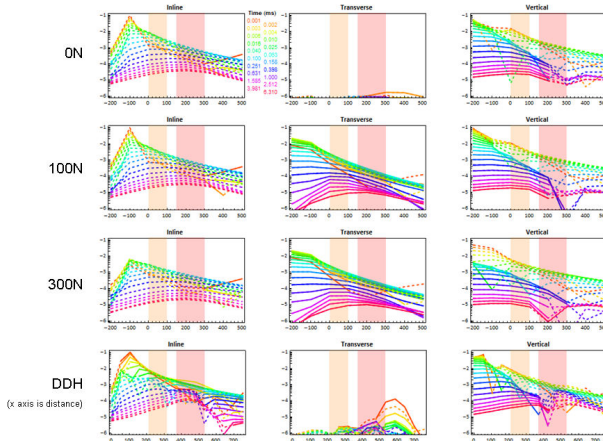
**Figure 1. Model cross section through 0N. Resistivities are shown on shaded portions.**

The four survey lines are illustrated in section and plan views Figure 2 together with the transmitter waveform and receiver gates.



**Figure 2.** Survey lines used for the model study in plan and section view together with the transmitter waveform and receiver gates. The prospecting system is similar to a SIROTEM system, though  $B$  rather than  $\partial B/\partial t$  is plotted here.

Figure 3 compares forward modelling responses for all lines. Shaded regions indicate surface projections of the paleochannel (orange) and the target zone (red). The target's response is most apparent in the vertical component response for all lines except 300N. A transverse component response is expected for lines 0N and DDH because the target is slightly asymmetric towards the north. Simultaneous forward modelling for all lines took some 35 minutes on a desktop PC.



**Figure 3.** Comparison of forward modelling responses for each of the survey lines. The DDH response is plotted against downhole distance rather than station. Dashed lines indicate negative amplitudes and times are referenced to transmitter turn-off using the SIROTEM convention. Vertical axes are logarithmic and plotted in units of nT.

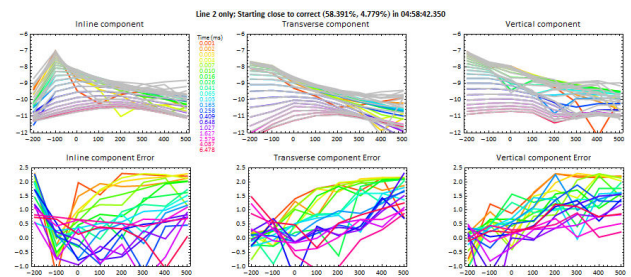
## Inversion results

Results for three scenarios are discussed with reference to Line 100N. The first scenario has a starting model very close to the correct model, and might be the case when confirming other modelling results. Results are plotted in Figure 4 and show that errors, for the most part, are very low.

Error is defined in Equation 2 as point-symmetric RMS error

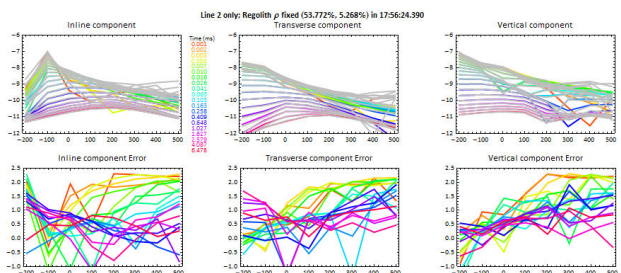
$$e = \frac{1}{N} \sqrt{\sum_N \frac{(d_{field} - d_{model})^2}{(d_{field}^2 + d_{model}^2)/2}} \quad (2)$$

which weights error across all stations and times equally. This is the quantity that is plotted in Figures 4, 5 and 6, and though it is objective, it has the disadvantage of producing a high value when model and field responses are visibly similar which is the case in Figures 4 and 5.



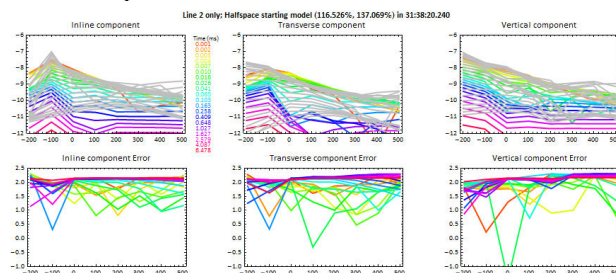
**Figure 4.** Inversion profiles for Line 100N using an initial model which is close to correct. The horizontal axis is distance and both vertical axes are logarithmic, response magnitude is in units of nT while error is plotted as a percentage. Field data are plotted as gray traces; the inverted response is plotted as colour.

The second illustration of Loki follows from the first. The starting model in Figure 5 assumes that an interpreter has determined host and regolith resistivities. They have also determined positions of the paleochannel and target and approximate resistivities. Data here are inverted only for paleochannel and target resistivities. Chief differences between data in Figures 4 and 5 are seen towards the later channels where errors are lower.



**Figure 5.** Inverted profiles for Line 100N using the correct values for both host and regolith resistivity, inverting only for paleochannel and target resistivity.

Results of the final demonstration of Loki are illustrated in Figure 6. Here, a 100  $\Omega$ -m halfspace was used as the initial model. Errors are high for all components, and the inversion has objectively failed. An indication as to why this inversion has failed might be found in the transverse component response. This component typically has a significant magnitude when conductive bodies are traversed off their central axes. However, for the starting halfspace model, this component has a low amplitude response. Because factors that significantly affect this component of response, such as strike length and strike angle are poorly resolved from a single profile, the inversion is effectively becalmed, or trapped in a low-gradient part of solution space. This suggests that more data are needed, a stark statement of the underdetermined nature of the problem.



**Figure 6. Inverted profiles for Line 100N using a halfspace starting model. High errors and dissimilarity between field and model responses indicate that the inversion has failed.**

## CONCLUSIONS

The program *Loki* was extended to provide the capability of inverting data. As a capability demonstration, a small model was set up modelling a SIROTEM-like survey consisting of three surface traverses and a downhole line. The model consisted of a small hill, a variably-weathered regolith with paleochannel and an irregular target. It was found that when initial models were quite close to correct models, then model data could be inverted with a low error. However, when no information was available, such as when the initial model was a resistive halfspace, convergence could not be achieved. Inversion results are promising, and suggest that a reasonable

approach might be to derive background information using 1D methods and to use 3D methods to invert for structure in regions of interest.

## ACKNOWLEDGMENTS

This work could not have proceeded without the considerable groundwork completed during AMIRA's P223F project. Sadly, the project finished before work by Art Raiche, Fred Sugeng and Glenn Wilson could be completed. This work merely dots 'i's and crosses 't's.

## REFERENCES

- Ellis, R. G. (1998), Inversion of airborne electromagnetic data, *Explor. Geophys.*, 29(2), 121-127.
- Jupp, D. L. B., and K. Vozoff (1975), Stable Iterative Methods for the inversion of Geophysical Data, *Geophys. J. R. Astr. Soc.*, 42, 957-976.
- Raiche, A. P., D. L. B. Jupp, H. Rutter, and K. Vozoff (1985), The joint use of coincident loop transient electromagnetic and Schlumberger sounding to resolve layered structures, *Geophysics*, 50(10), 1618-1627.
- Sugeng, F. (1998), Modeling the 3D TDEM response using the 3D full-domain finite-element method based on the hexahedral edge-element technique, *Exploration Geophysics*, 29(4), 615-619.
- Sugeng, F., A. Raiche, and G. Wilson (2006), An efficient compact finite-element modelling method for the practical 3D inversion of electromagnetic data from high contrast complex structures, in *18th workshop on Electromagnetic Induction in the Earth*, El Vendrell, Spain.
- Wilson, G. ., A. P. Raiche, and F. Sugeng (2006), 2.5D inversion of airborne electromagnetic data, *Exploration Geophysics*, 37(4), 363-371.