Joint inversion of seismic traveltimes and gravity data using petrophysical constraints with application to lithology differentiation

Jiajia Sun
Colorado School of Mines
1500 Illinois Street
Golden CO 80401
USA
jsun@mymail.mines.edu

Yaoguo Li
Colorado School of Mines
1500 Illinois Street
Golden CO 80401
USA
ygli@mines.edu

SUMMARY

Multiple geophysical data collected over the same area but based on fundamentally different physics usually contain complementary information about the subsurface. Joint inversion combines the complementary information by integrating all the geophysical data into a single inversion scheme. Thus, models resulting from joint inversion are more likely to represent the subsurface better than models derived from a single type of data. In this study, we consider joint inversion of seismic traveltimes and gravity data, and present a new joint inversion algorithm that uses petrophysical information as constraints. Using a synthetic example, we show that this new method can effectively build the available petrophysical information into inversion and improve the definition of both structure and physical properties. We also show that this method can deal with the situation where only partial petrophysical information about the subsurface is available. An important component of our method is applying fuzzy c-means (FCM) clustering algorithm to the recovered physical property distribution to generate a lithology map that is consistent with both the observed geophysical data and the a priori petrophysical information.

Key words: Joint inversion, petrophysics, lithology differentiation, gravity, seismic traveltime

INTRODUCTION

Joint inversion of different geophysical data sets has recently received considerable attention. Different geophysical methods are sensitive to different physical properties and their variations on different scales, and joint inversion provides a way to combine these complementary information contained in each data set by integrating all the geophysical data into a single inversion scheme. The integration is achieved by requiring the resulting physical property models to honour all the data simultaneously (Vozoff and Jupp, 1975).

One important consideration with joint inversion strategy is that one needs to find a reasonable relationship between different models so that they can complement each other. This relationship could be some empirical relationship between different physical properties, some correlation measure from statistics (Lelievre et al., 2010), or some structural similarity measure between models of different physical properties (Haber and Oldenburg, 1997; Gallardo and Meju, 2003). In this study, we consider the empirical petrophysical relationship between physical properties, and propose a new inversion method that can effectively build this information into inversion by means of fuzzy c-means (FCM) clustering technique. This new method is also effective when only partial petrophysical information is available.

Generally, different lithologies have distinct ranges of physical properties as well as unique relationships among them. Inference about the subsurface lithology can then be made by grouping physical property values obtained from geophysical inversions into several clusters on the basis of their "distances" from each other (Paasche et al., 2006). FCM clustering technique is a powerful tool to explore the similarity between data (e.g. physical property values in our study) and classify the data under consideration into clusters (i.e. lithologies in this study) (Bezdek, 1981; Duda et al., 2000). Therefore, we also use the FCM clustering to differentiate between different lithological regions.

In the following, we begin with a brief introduction of the FCM clustering algorithm and the formulation of the joint inverse problem. We then demonstrate this new method using synthetic joint inversion of gravity and seismic traveltimes.

METHODOLOGY

Fuzzy c-means (FCM) clustering algorithm

FCM clustering algorithm is an unsupervised clustering method that can organize data into groups based on similarities between data entries (Bezdek, 1981; Duda et al., 2000). Mathematically, FCM clustering algorithm can be expressed as the minimization of the following objective function:

$$\varphi_{fcm} = \sum_{j=1}^{N} \sum_{k=1}^{C} u_{jk}^{q} \left\| y_j - v_k \right\|_2^2 \tag{1}$$

where N is the number of model cells, C is the number of clusters, y_j is the j^{th} data entry (e.g. physical parameter set at the j^{th} cell in our study), and v_k is the center of the k^{th} cluster. u_{jk} is the membership function that measures the degree to which the j^{th} data entry belongs to k^{th} cluster. The parameter, q, also known as fuzzification parameter, controls the degree of 'fuzziness' of the resulting membership functions, and satisfies $q \geq 1.0$. In this study, we set $q = 2.0$, which is widely accepted as a good choice (Hathaway and Bezdek, 2001).

Throughout this paper, we assume the total number of clusters, C, is known. We can use any derivative based method to minimize the objective function (1) with respect to the cluster centers, v_k, and the membership functions, u_{jk}.

In our study, we use the FCM clustering technique to incorporate petrophysical information into joint inversion of crosshole seismic traveltimes and gravity data, and therefore, to encourage the recovered slowness and density distributions to follow the a priori petrophysical information. After completing the inversions, we use FCM clustering method to differentiate between different rock units.

Formulation of inverse problems with petrophysical constraints

The area under investigation is divided into cells, each having a constant density contrast and seismic slowness to be recovered. In the following, $\boldsymbol{m_1}$ is the model vector containing the slowness values, and $\boldsymbol{m_2}$ the density model vector. The crosshole seismic traveltimes are denoted by $\boldsymbol{d_1}$, and gravity data $\boldsymbol{d_2}$.

The joint inversion of seismic traveltimes and gravity data is then formulated as an optimization problem that minimizes an objective function

$$\varphi(m_1, m_2) = \varphi_{d1} + \beta_1 \varphi_{m1} + \varphi_{d2} + \beta_2 \varphi_{m2} + \lambda\left(\sum_{j=1}^{N}\sum_{k=1}^{C} u_{jk}^q \left\| y_j - v_k \right\|_2^2 + \eta \sum_{k=1}^{C} \left\| v_k - t_k \right\|_2^2\right) \quad (2)$$

The two φ_d terms quantify how well the observed data can be reproduced by inverted models. φ_{m1} and φ_{m2} measure the amount of structure in the two inverted models. Regularization parameters β_1 and β_2 balance between data misfit term and model structure term. Constants λ and η are determined by numerical experimentation.

In equation (2), $y_j = [m_{1j}, m_{2j}]^T$ represents the slowness and density contrast in the j^{th} cell. $v_k = [v_{1k}, v_{2k}]^T$ is the center for the k^{th} cluster estimated by FCM algorithm. $t_k = [t_{1k}, t_{2k}]^T$ represents a possible pair of velocity and density value determined a priori from rock sample measurements. In this study, we consider the slowness and density as discrete random variables, and thus, their joint distribution is expressed in equation (2) as t_k. The last term measures the distance between cluster centers updated by FCM algorithm, v_k, and target cluster centers determined from a priori petrophysics, t_k. One advantage of this strategy is that it does not compromise the well-behaved convergence of FCM algorithm, while at the same time, guides the search for cluster centers to the desired locations based on a priori petrophysical information.

As mentioned in the Introduction, this new inversion algorithm can deal with the situation where we only have partial petrophysical knowledge about the subsurface. For example, assume that there are three different rock units in the subsurface, and we have petrophysical information for two of them from measurements on rock samples. In this case, C = 3, and the last term in equation (2) would become:

$$\sum_{k=1}^{3} \left\| v_k - t_k \right\|_2^2 = \left\| v_1 - t_1 \right\|_2^2 + \left\| v_2 - t_2 \right\|_2^2 + 0\left\| v_3 - t_3 \right\|_2^2. \quad (3)$$

In other words, cluster centers, v_1 and v_2, for which we have petrophysical information available, are estimated by both FCM algorithm (i.e. $\sum_{j=1}^{N} u_{jk}^q \left\| y_j - v_k \right\|_2^2$, k=1, 2) and the petrophysical constraints (i.e. $\left\| v_k - t_k \right\|_2^2$, k=1, 2). However, the center for the third cluster, v_3, is updated only by FCM algorithm by turning off the third term, $\left\| v_3 - t_3 \right\|_2^2$, since no petrophysical information about the third rock type exists.

SYNTHETIC EXAMPLES

Joint inversion with complete petrophysical information

Figure 1 shows the model set up. The model section extends 900 *m* horizontally and 600 *m* in depth. The model region is discretized into 864 25*m*×25*m* cells. The background slowness is 0.5 *s/km*. There are two slowness anomalies with slowness 0.2 *s/km* above and below the background slowness value. The two anomalous density regions have the same density contrast of 0.4 g/cm^3. We assume there is a vertical borehole on each end of the model region and we position seismic transmitters evenly in one borehole and receivers in the other (Figure 1). To simplify the problem, we calculate the traveltimes at each receiver using straight-ray tracing. We calculate the gravity response every 20*m* on the surface and every 40*m* in the two boreholes. Independent Gaussian noise is added to simulate observed data.

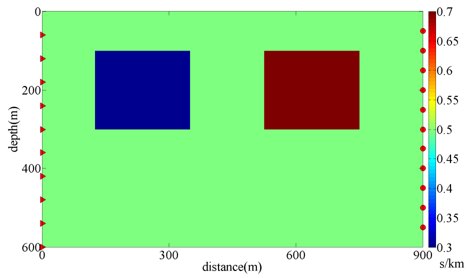

Figure 1. Slowness model and geometry of the synthetic crosshole seismic experiment. Red triangles and circles mark the positions of transmitters and receivers. The corresponding density model has the same structure but an identical density contrast in the two blocks (0.4 g/cm^3) in a zero background.

Figure 2 shows the joint distribution of slowness and density. The three distinct points in the crossplot of slowness versus density indicate the presence of three different lithologies. We assume this petrophysical information is available a priori from rock sample measurements. We now consider jointly inverting seismic traveltimes and gravity data by minimizing function (2) with t_k being equal to $(0, 0.5)^T$, $(0.4, 0.3)^T$ and $(0.4, 0.7)^T$ for k = 1, 2, 3, respectively. In other words, the a priori petrophysical information is built into inversion by letting t_k assume those possible values determined from rock physics, as shown in Figure 2.

Figure 3 shows the slowness model recovered from joint inversion, and Figure 4 shows the recovered slowness model with only seismic traveltime data. We observe in Figure 4 that there are two obvious slowness anomalies, but we also observe serious spurious features in the model. The jointly inverted slowness model in Figure 3, however, shows fewer spurious features, and the spatial extents of the slowness anomalies are much better resolved than in Figure 4.

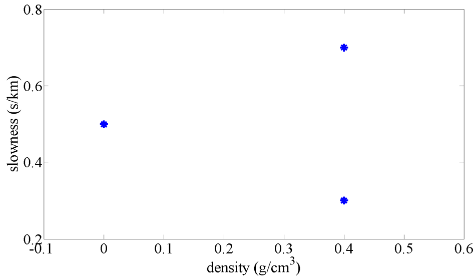

Figure 2. Joint distribution of slowness and density

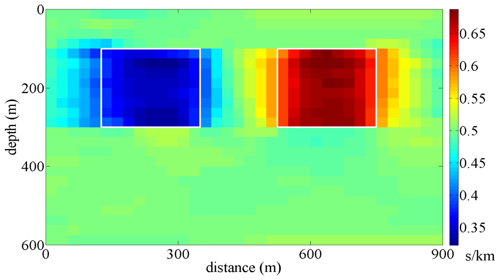

Figure 3. Slowness model recovered from joint inversion.

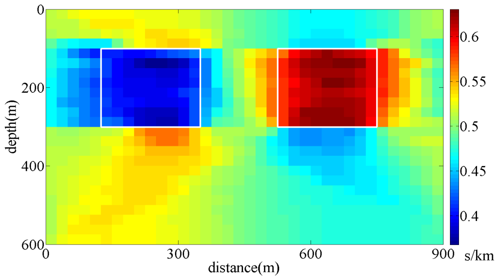

Figure 4. Slowness model recovered without petrophysical constraints.

Figure 5 and 6 show the density model recovered from single inversion and joint inversion, respectively. The density model shown in Figure 5 roughly indicates where the density anomalies are. But the inverted density anomalies are characterized by smooth features, especially smeared boundaries. Furthermore, the recovered density values are lower than the true value of 0.4 g/cm^3. In contrast, the locations and the shape of the recovered density anomalies in Figure 6 are better resolved and more consistent with the true anomalies. Also, the inverted density values at these two locations are nearly 0.4 g/cm^3, which is equal to the true value.

Figure 7 shows the joint distribution of slowness and density values recovered from joint inversion. The three distinct clusters, enclosed by the red ellipses, indicate the presence of three different lithologies in the subsurface. As comparison, Figure 8 shows the joint distribution of slowness and density values estimated from separate inversions. It is obvious that incorporation of additional petrophysical information into geophysical inversion helps differentiate between different rock units. This conclusion is further confirmed by lithology differentiation result (Figure 9) obtained by applying the FCM clustering algorithm to the slowness and density models from joint inversion. Three different colours indicate three different rock types in this cross section. It is clear that the three different lithologies are identified at correct locations and with a reasonable definition of boundaries.

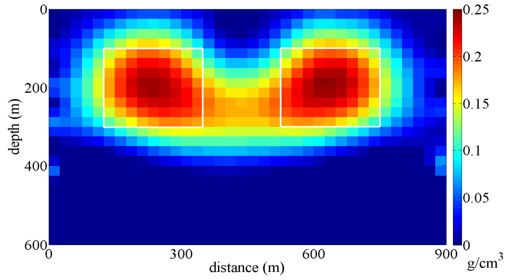

Figure 5: Density model recovered from gravity inversion.

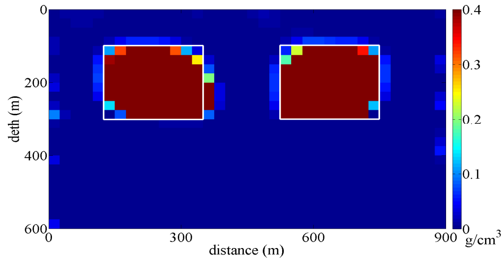

Figure 6: Density model recovered from joint inversion.

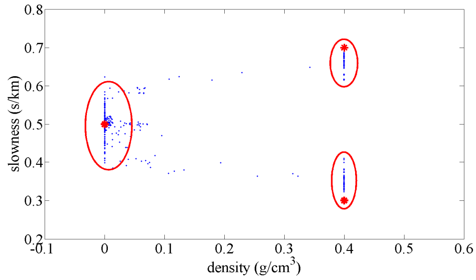

Figure 7. Joint distribution of recovered slowness and density values. The red stars show the distribution of true slowness and density values.

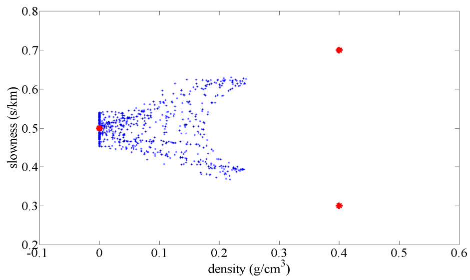

Figure 8. Joint distribution of slowness and density values recovered from separate inversions of seismic traveltime and gravity data. The red stars show the distribution of true slowness and density values.

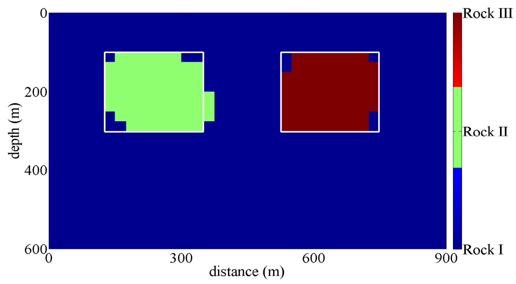

Figure 9. Lithological differentiation result by applying FCM clustering algorithm to jointly inverted models.

Joint inversion with partial petrophysical information

We next assume that we have petrophysical information about two out of three rock types in this area. The petrophysical information we have in this case is summarized in Figure 10.

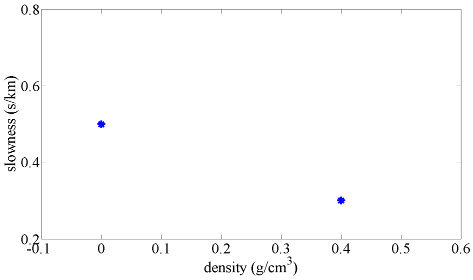

Figure 10: Joint distribution of slowness and density for two of the three rocks.

We now use this partial petrophysical information to constrain joint inversion of seismic traveltimes and gravity data. For brevity, the slowness and density models recovered from joint inversion with incomplete petrophysical data are not shown here. However, the joint distribution of the recovered slowness and density values (Figure 11) is a good indication of our success in using partial petrophysical information to better constrain geophysical models.

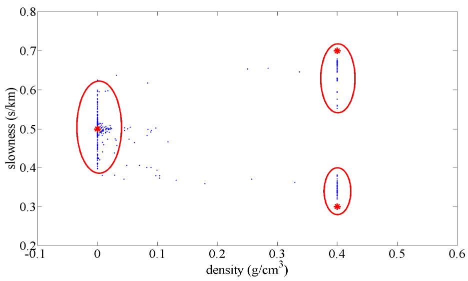

Figure 11. Joint distribution of slowness and density values recovered from joint inversion with partial petrophysical information.

Figure 12 shows the lithology differentiation result. It is interesting to notice that even if no information about the third rock type is incorporated into inversion, we can still identify that rock at the correct location and with good boundaries. The reason is that incorporation of petrophysical information about the other two rock types improves the characterization of these two rocks, and consequently, the third rock type is constrained indirectly.

CONCLUSIONS

Petrophysical information is often available from physical property measurements on rock samples. In this paper, we present a new inversion algorithm that can effectively build the a priori petrophysical information into joint inversions. We demonstrate through a synthetic example that incorporation of such information into joint inversion of multiple geophysical data can greatly improve the inverted images. The inverted models honour both the observed geophysical data and the a priori petrophysical information.

One advantage of our proposed joint inversion method is that it can deal with the situation where only partial petrophysical information about the area is available. We have demonstrated that the inverted models recovered from joint inversion constrained by only partial petrophysical information represent the subsurface structure better than models from single data type inversion.

The inverted models obtained from the proposed joint inversion algorithm are further processed by FCM clustering algorithm to automatically differentiate different lithologies present in the subsurface. Numerical results show that the locations and shapes of different lithologies can be correctly identified by applying FCM clustering algorithm.

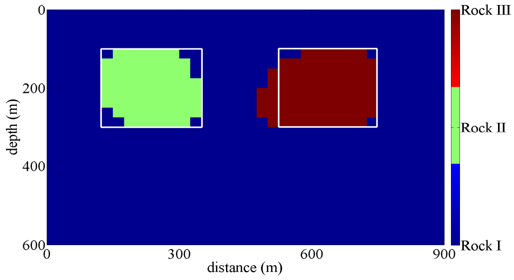

Figure 12: Lithology map resulting from FCM clustering.

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