Joint Inversion Through A Level Set Formulation

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SUMMARY

Geophysical data processing is a highly quantitative field that involves modelling, inversion and visualization. In most cases a geophysical experiment is conducted to collect data that are sensitive to a particular physical property of the earth. The data is processed and inverted to generate an earth model of the physical property in question. To better understand the structure of the earth, different experiments are conducted using a variety of imaging modalities. For example, from seismic, gravity and electromagnetic experiments we may obtain information about the earth's elastic, density and conductivity characteristics. Usually the data of each experiment are inverted separately to generate an ensemble of earth models. However, since the inversion process of each geophysical modality is typically carried out independently, most inversion algorithms do not utilize the information obtained through other modalities.

In this research we propose to jointly invert the data obtained by two physical experiments since the information contained in each model can be used to correct the other model. In many of the cases the two models share the important structures, therefore, edges occur in the same locations. In order to exploit this information, we propose using a level set formulation of the problems. Assuming that both models take two known discrete values, we can then use a single level set function for both models together. This can be later extended to multi-level set functions and with unknown values. By using this formulation we are able to improve inversion results of both problems.

Key words: Joint inversion, level-sets, geophysical inversion.

INTRODUCTION

In order to learn about the properties of the earth a set of geophysical experiments are performed. Usually, several physical experiment are conducted on the same object. These multiple experiments are used to better learn the different properties of the object.

Consider the problem of inverting two data sets obtained by two physical experiments

$$F_1(m_1) + \grave{o}_1 = d_1$$
$$F_2(m_2) + \grave{o}_2 = d_2$$

We would like to jointly invert the data, that is, we believe that information that is contained in the first model is relevant for the second model and vice-versa.

In some cases, it is possible to find an empirical relation between m_1 and m_2 which we write as $m_2 = g(m_1)$, and obtain an augmented system for m_1.

$$F_1(m_1) + \grave{o}_1 = d_1 \quad \text{and} \quad F_2(g(m_1)) + \grave{o}_2 = d_2$$

Nonetheless, such relation is difficult to obtain for most problems. Another option is to find a connection between m_1 and m_2 for example, Gallardo and Meju (2011) proposed using $c(m_1, m_2) = \nabla m_1 \times \nabla m_2 = 0$, which implies that m_1 and m_2 can share edges.

However, we have found that in many case, such relation does not yield the appropriate result. This is because a feasible solution to $c(m_1, m_2) = 0$ are models, m_1 and m_2 such that m_1 is constant where ∇m_2 is significant.

An alternated relationship requires using mutual information (Wells *et al.* 1996). Mutual information measures the information that m_1 and m_2 share. The Mutual Information function is defined by

$$\mathrm{MI}(m_1, m_2) = H[m_1] + H[m_2] - H[m_1, m_2],$$

where the H is the entropy of the density of m. This function finds how values of m_1 correspond with values of m_2.

However, one of the problems in using this method is that MI has many local minima. Using it as a part of the joint inverse optimization can lead to the wrong models.

METHOD AND RESULTS

Here we propose a method that makes similar assumptions, we assume that

- m_1 and m_2 take a number of discrete values
- The gradients of m_1 and m_2 are aligned.

Rather than using a cross gradient formulation we propose to use a level-set formulation. For simplicity, assume that both m_1 and m_2 take two known discrete values. Then we can write

$$m_1 = H_1(\psi(x)) \quad \text{and} \quad m_2 = H_2(\psi(x))$$

where H_1 and H_2 are defined as

$$H_1(t) = \begin{cases} a_1 & \text{if } t > 0 \\ b_1 & \text{else} \end{cases} \quad \text{and} \quad H_2(t) = \begin{cases} a_2 & \text{if } t > 0 \\ b_2 & \text{else} \end{cases}$$

This can be extended to multi-level sets and with unknown values but for now, we keep the formulation for a single level set function.
Assuming that this formulation holds, the problem then simplifies to finding a single level set function $\psi(x)$ such that

$$F_1(H_1(\psi)) + \grave{o}_1 = d_1 \quad \text{and} \quad F_2(H_2(\psi)) + \grave{o}_2 = d_2$$

By using this formulations, the problems are coupled through the assumption that a single level set function is used for both models.

We need to find ψ which minimizes the optimization problem:

$$E(\psi) = \frac{1}{2\sigma_1^2}\| F_1(H_1(\psi)) - d_1\|^2 + \frac{1}{2\sigma_2^2}\| F_2(H_2(\psi)) - d_2\|^2 + \alpha R(\psi)$$

where σ_1 and σ_2 are the standard deviations of the noise vectors $\grave{o}_1$ and $\grave{o}_2$ respectively, and $R(\psi)$ is an appropriate regularization functional.

We test this idea on retrieving two signals of different physical properties of the earth. The first one is seismic tomography, where 3D images are derived from the processing of integrated properties of the medium that rays encounter along their paths. This problem is usually formulated as an inverse problem. The given formulation of the forward problems is,

$$d = Am$$

where d is the observed data, m is the model of the earth, and A is the forward ray-path matrix. For the inverse problem we wish to solve the optimization of

$$\min_m \; E_1(m) = \frac{1}{2}\| Am - d\|^2 + R(m)$$

The second problem we want to solve is direct current resistivity. In this problem, scientists drive a DC signal into the earth and measure the potential created by this signal. The goal of the experiment is to infer about the conductivity of the earth.
The given formulation of the forward problem is $\nabla \cdot \sigma \nabla u = q$, where σ is the conductivity, u is the potential and q are the sources. After discretization we wish to solve the inverse problem by optimization of

$$\min_m \frac{1}{2}\sum_j \| P^{\bullet} u_j - d_j\|^2 + R(m)$$

$$\text{s.t. } A(m)u_j = Q_j \qquad j = 1,\ldots,N_s$$

where, $A(m)$ - a discretization of a parameter dependent differential operator, Q_j is the source, P is the observation matrix, u_j is the potential field, d_j is the data vector and $R(m)$ is the regularization. Therefore we wish to solve the optimization of :

$$\min_m \; E_2(m) = \frac{1}{2}\sum_j \| P^{\bullet} A(m)^{-1} Q_j - d_j\|^2 + R(m) .$$

In this research we combine both problems into one using level sets,

$$\min_\psi \; E_T(\psi) = E_1(m_1(\psi)) + \beta E_2(m_2(\psi)) + \alpha R(\psi)$$

We search for a single solution to match both problems. Also, since we work with two very different signals, α and β are scaling coefficients.

For the Regularization we use regularization proposed by Van den Doel and Ascher (2007),

$$R(\psi) = \int_\Omega \rho(|\nabla \psi|) + \lambda(\psi - \psi_{ref})^2 dx .$$

This provides dynamic regularization which depends on the size of the gradient and the distance from a reference vector. In order to reach the minimum efficiently we use the Gauss-Newton Scheme. Here, the i'th iteration of the level set function is:

$$\begin{aligned} m_{i+1} &= m_i + \mu H^{-1} g \\ g &= J_1^T r_1 + \beta J_2^T r_2 + \alpha R' \\ H &= J_1^T J_1 + \beta J_2^T J_2 + \alpha R'' \end{aligned}$$

where J_1 and J_2 are the sensitivity matrices for E_1 and E_2 respectively, and r_1 and r_2 are the differences between the observed data and the predicted data.
For the level-set function we use:

$$L(\psi) = aH(\psi) + b\big(1 - H(\psi)\big)$$

where H is a Heaviside function and a and b are the two possible values of the signal. For the Heaviside function we use a smooth function in order to avoid problems with the derivative,

$$H(\psi) = \frac{1}{2}\big(1 + \tanh(c\psi)\big),$$

where c determines the slope level of the function.

Results

We test our idea on the two dimensional example of 35 meters deep by 90 meters wide, given below in Figure 1.

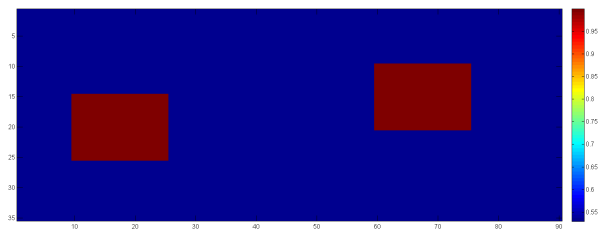

Figure 1: input data 90x35 meters

For the DC resistivity recovery, electrode sources and receivers are placed on the ground at different locations. We simulate injecting current and measuring the voltage on the surface. For this purpose we use the code of Pidlisecky and Knight (2008).
In the tomography problem, 35 sources and 35 receivers are spaced 90 meters apart, and each source (or receiver) is spaced 1 meter away from its adjacent neighbouring sources. All receivers share all sources. ψ is initialized as a constant over the entire area.

Figure 2 shows the data observed by using the ray-path tomography matrix on the true data.

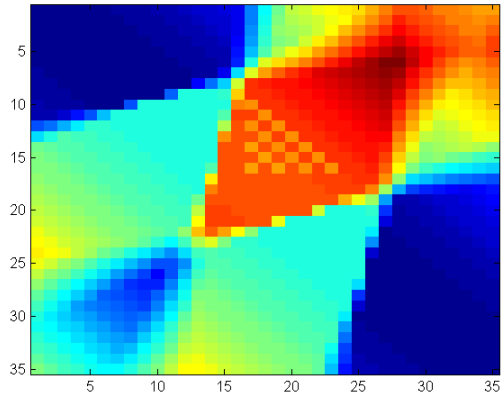

Figure 2: Observed data using tomography

In Figure 3 show the results of recovering the model by minimizing the tomography problem by itself using the level set function. Here, we see that rectangles are recovered but with noise.

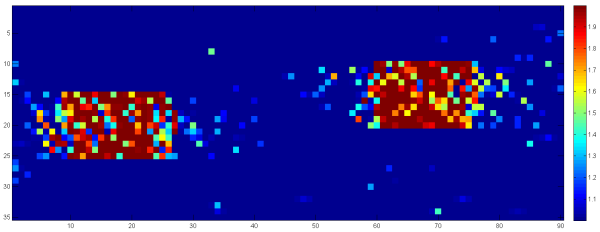

Figure 3: The predicted data result of the ray tomography problem.

Figure 4 shows the recovered model of solving the minimization of the DC resistivity problem alone. Since in this problem all measurements are performed above the surface, as we go deeper into the ground we have less and less information. Therefore, the bottom part of the rectangles is not recovered so accurately.

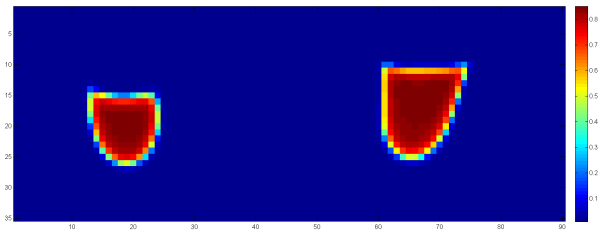

Figure 4: The predicted data results of the DC resistivity problem.

Finally, Figure 5 shows the results of jointly inverting both problems using the single level set formulation. Here, combining both problems into one optimization problem, results in retrieving the correct edges for both rectangles, while removing the noise shown in the tomography problem. In this example we demonstrate how these two problems correct each other allowing the final results to reach a better prediction than solving each of the problems separately.

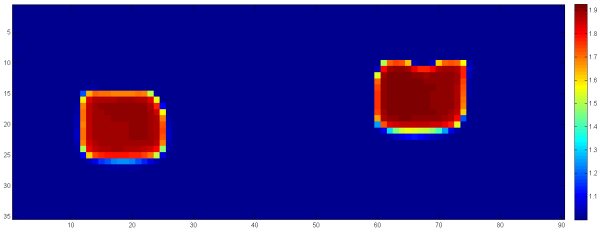

Figure 5: Results using joint inversion of both problems using a single level set function

CONCLUSIONS

In this paper we present a novel method for joint inversion of two different physical experiments of the same object. We assume that the signals are piecewise constant and take a number of discrete values. We also assume that both signal share the same edges. Using these assumptions we are able to use a single level set function for both signals, which takes different levels for each signal. This method uses a stronger constraint than previous joint inversion methods. By doing this both optimization problems become one, correcting each other, reaching a single solution which can be better than solving each problem separately. We test our idea on jointly recovering signals for ray tomography and DC resistivity of the same object. Our results show that the joint inversion allows both inversions to reach a better solution than each one separately. This way each model is able to influence the other, reaching a compromise between the two which is more accurate than solving separately.

Next, we intend to solve similar problems with multiple level sets, allowing more robustness to the model. Also as a part of the minimization scheme we intend to retrieve the levels themselves as well as the curve. Finally, we wish to test our method on field data, in order to prove its robustness.

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