Modeling of seismic waves in layered media and the inversion of source parameters

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This paper is organized as follows. After a discussion of the differential equations for wave propagation in the horizontally stratified medium and of the initial and boundary conditions, we derive the displacements on the free surface of the layered medium for plane waves when a point source is located on the s–th imaginary boundary at the depth z_s (physical parameters of the layers s and ($s+1$) are put to be identical). Then, the source will be represented as a single force of arbitrary orientation and a general moment tensor point source. Further, "a primary field" for a point source will be introduced. Method for the solution of the direct seismic problem is considered based on the matrix method of Thomson-Haskell. The tensor represents a superposition of three single couples without moment along the x, y, z-axes and three double couples in xy, xz, yz-planes. Further, we give the results for the field of displacements on the free surface. The far-field displacements are:

$$\begin{pmatrix} u_z^{(0)} \\ u_r^{(0)} \end{pmatrix} = \sum_{i=1}^{3} \int_0^\infty k^2 I_i L^{-1}\left[M_i g_i\right] dk\,,\; u_\varphi^{(0)} = \sum_{i=5}^{6} \int_0^\infty k^2 J_i L^{-1}\left[M_i g_{i\varphi}\right] dk \tag{1}$$

$$I_1 = \begin{pmatrix} J_1 & 0 \\ 0 & J_0 \end{pmatrix},\; I_2 = \begin{pmatrix} J_0 & 0 \\ 0 & J_1 \end{pmatrix}, I_3 = I_2\,.$$

$$\mathrm{g_i} = \begin{pmatrix} g_{iz} \\ g_{ir} \end{pmatrix},\; J_5 = J_0\,,\; J_6 = J_1\,.$$

The near- field displacements are:

$$\begin{pmatrix} u_r^{(0)} \\ u_\varphi^{(0)} \end{pmatrix} = \frac{1}{r}\cdot\left(\int_0^\infty k\cdot J_1(kr)\cdot L^{-1}\left[\begin{pmatrix} M_1 \\ -M_5 \end{pmatrix}\cdot\left(g_{1r} + 2g_{5\varphi}\right)\right]dk + \right.$$

$$\left. + \int_0^\infty \left(kJ_0(kr) - \frac{2J_1(kr)}{r}\right)\cdot L^{-1}\cdot\left[\begin{pmatrix} -M_4 \\ M_6 \end{pmatrix}\cdot\left(g_{3r} + 2g_{6\varphi}\right)\right]dk\right) \tag{2}$$

$$u_z^{(0)} = \frac{1}{r}\cdot\int_0^\infty kJ_1(kr)\cdot L^{-1}\cdot\left[M_4\cdot g_{3z}\right]dk\,,$$

where

$$\begin{gathered} M_1 = M_{xz}\cos\varphi + M_{yz}\sin\varphi\,, \\ M_2 = M_{zz}\,, \\ M_3 = \cos^2\varphi\cdot M_{xx} + \sin^2\varphi\cdot M_{yy} + \sin 2\varphi\cdot M_{xy}\,, \\ M_4 = -\cos 2\varphi\cdot M_{xx} + \cos 2\varphi\cdot M_{yy} - 2\sin 2\varphi\cdot M_{xy}\,, \\ M_5 = M_{yz}\cos\varphi - M_{xz}\sin\varphi\,, \\ M_6 = \sin 2\varphi\cdot M_{xx} - \sin 2\varphi\cdot M_{yy} - 2\cos 2\varphi\cdot M_{xy} \end{gathered} \tag{3}$$

The results of this direct problem (1-3) we use in the inversion of source parameters. The inverse method relies on inverting for components of the moment tensor and a determination of an earthquake source-time function.

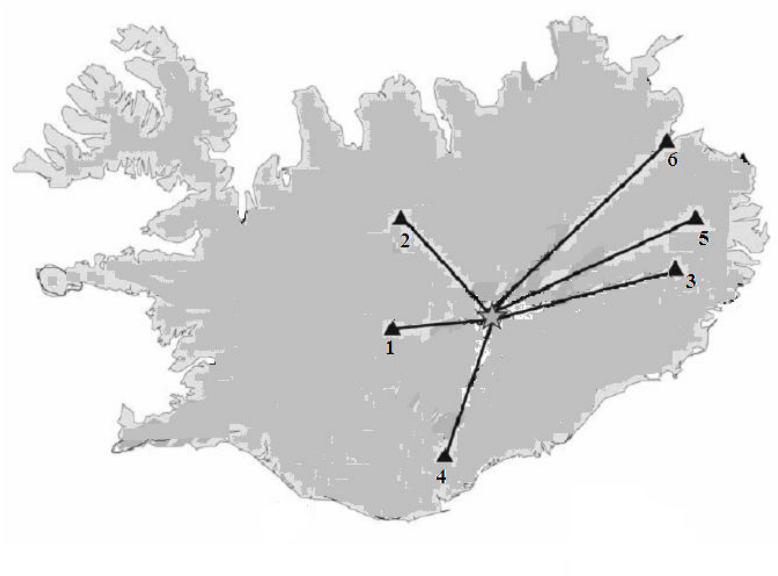


Fig.1 Triangles are locations of 6 seismic stations used in the moment tensor inversion. An earthquake is shown with the star.

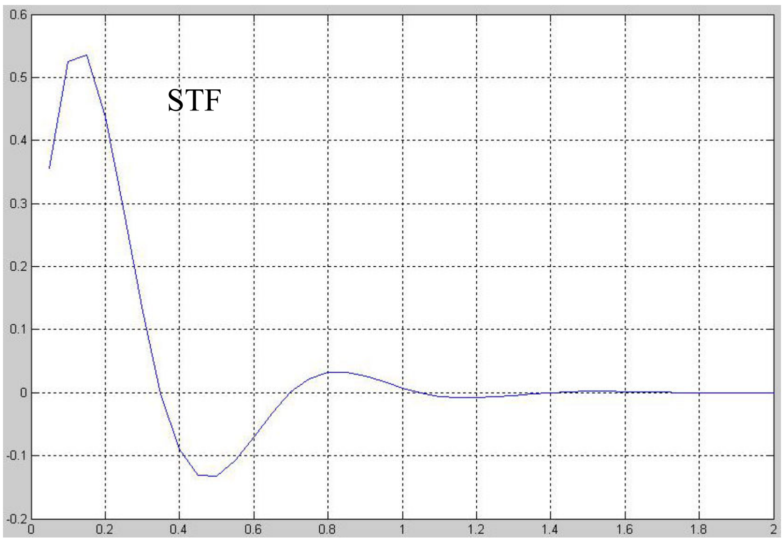


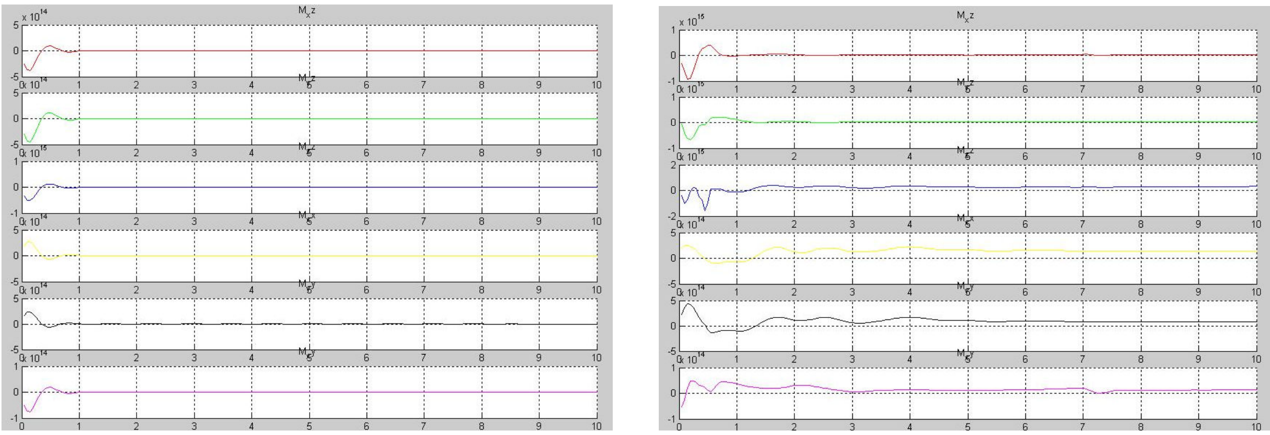
Fig.2 The source time function(STF (t)).

$$M = \begin{pmatrix} 5{,}0*10^{14} & -1{,}4*10^{14} & -7{,}0*10^{14} \\ -1{,}4*10^{14} & 4{,}52*10^{14} & -8{,}3*10^{14} \\ -7{,}0*10^{14} & -8{,}3*10^{14} & -9{,}52*10^{14} \end{pmatrix} \qquad (1)$$

Table1

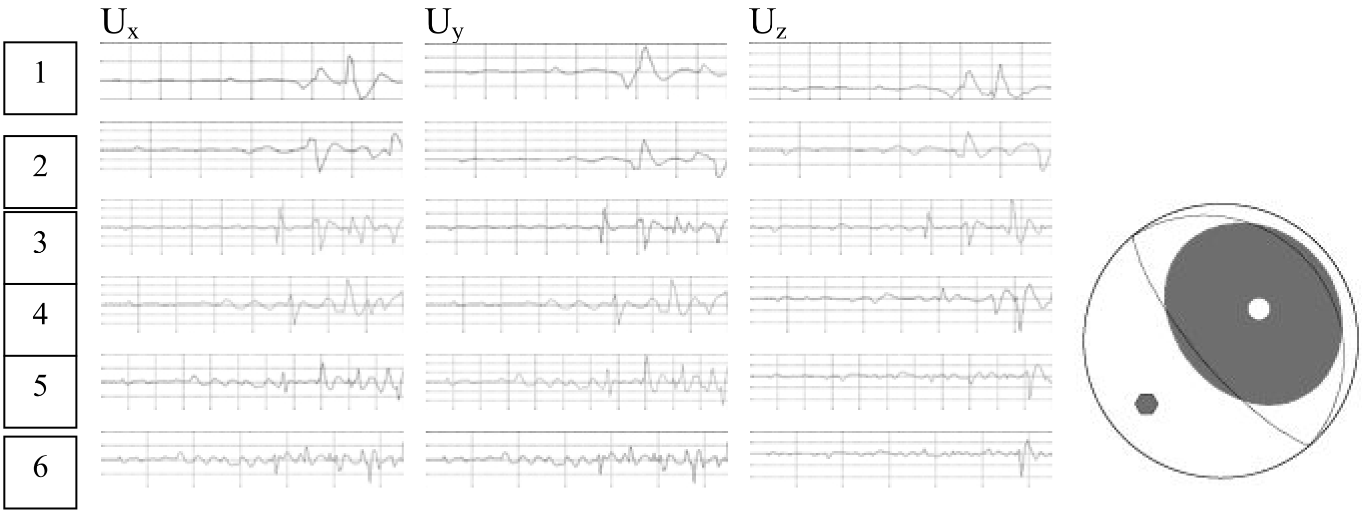
Structural model

Номер шару	V_P, m/c	V_S, m/c	H, m	ρ, kg/m^3
1	3540	2044	3000	$2{,}1*10^3$
2	4930	2846	2000	$2{,}4*10^3$
3	4930	2846	∞	$2{,}4*10^3$

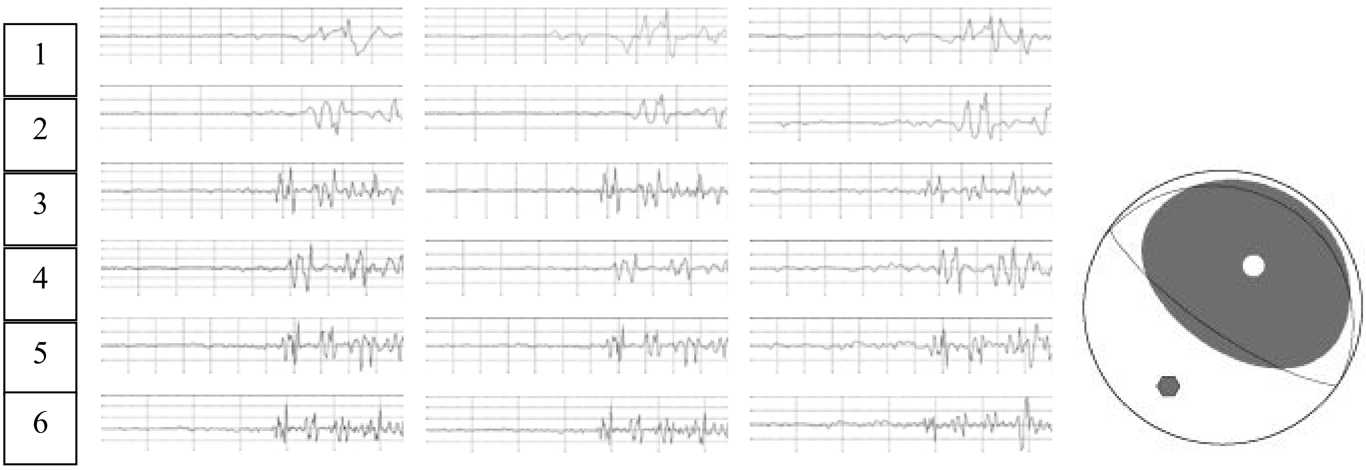


a) b)

Fig.3. Time functions $M_{ij}(t)$ to the six moment tensor components obtained from a) STF(t) (Fig.2) and seismic tensor Mij (formula 1); b) inversions of the records shown in Fig.4,a



a)



b)

Fig.4. Three component displacement seismograms is shown for six stations (Fig.1)

and seismic moment tensor solution a)by using functions $M_{ij}(t)$ on Fig.3.a b) by using functions $M_{ij}(t)$ on Fig.3.b.