

High-fidelity Adaptive Curvelet Domain Primary-Multiple Separation

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SUMMARY

In this paper, we propose an adaptive implementation for separating multiples from primary events in seismic data and subsequently removing the embedded multiples from noisy seismic data using the curvelet transform. Because of the sparseness of the curvelet coefficients of seismic data, the optimization problem is formularized by incorporating L1- and L2-norms, based on the framework of Bayesian Probability Maximization. Iterative softthresholding can be used for solving the above optimization problem. By making use of least-square matching filtering, we precondition the multiple models to match the actual multiples in the seismic data prior to the separation step.

Moreover, in order to meet the challenges faced by various types of data complications, we develop a frequency regularized adaptive curvelet domain separation approach. This flexibility overcomes the varying effectiveness of separation methods for different frequency bands in responding to the noise and model inaccuracy control. Accordingly, the high adaptability of this extension leads to its higher separation fidelity than existing curvelet domain separation implementations. We demonstrate the applications of our approach on synthetic and field data examples by comparing them with the results from the conventional least-square separation method.

Key words: Primary-multiple Separation, Curvelet, Adaptive Separation, Least-square.

INTRODUCTION

Multiple attenuation plays an important role in the preprocessing of seismic data, and has a significant impact on obtaining high quality seismic images. Generally, the process involves two steps: the prediction of multiples and the separation of the primaries and the multiples. Considerable effort has been spent on the prediction of multiples in the last two decades. Methods such as Surface Related Multiple Elimination (SRME) are used routinely in the industry. Aside from long-period multiples that are effectively handled by SRME, short-period multiples generated by a shallow seafloor and internal multiples generated by subsurface interfaces of high impedance contrast have also attracted attention from the research community (Hargreaves, 2006; Wang, et al, 2011; Wang, et al, 2012). Apart from the advancement in multiple prediction, an effective strategy for separating the multiples from the primaries is equally important. One of the most widely accepted separation strategies is the L2-norm based leastsquare separation method (LS) (Verschuur and Berkhout, 1997). It allows for a degree of inaccuracy in the prediction of the multiples, including traveltime, amplitude and spectrum errors. However, a compromise has to be made between the preservation of primaries and the attenuation of multiples, especially in places where primary and multiple events cross one another or overlap. For this reason, curvelet-based separation methods have been attracting increasing attention in recent years. They have the advantage of minimizing the damage to the primary events due to the compatible nature of the curvelet transform to seismic data (Herrmann, et al., 2008). Nevertheless, among various implementations of curvelet approaches, the non-adaptive domain separation implementations may encounter numerical divergence if the predicted multiples are very different from the multiples in the data; and the adaptive implementations either only correct for limited misalignment between the predicted and actual multiples, or suffer from high computational cost due to the use of curvelet matching filtering (Saab et al., 2007; Neelamani, et al., 2010).

In this paper, we present our adaptive implementation for primary-multiple separation. Our approach achieves a high degree of robustness and fidelity in primary-multiple separation by overcoming the difficulties that have afflicted previous methods.

FRAMEWORK OF CURVELET DOMAIN SEPARATION

Curvelet domain separation is applied for removing multiples from noisy seismic data. It involves the curvelet transform and a process for simultaneously separating the multiples and the primaries from each other. The curvelet transform is a multiscale and multi-dimensional transform (Candès et al., 2006), which can be written as:

$$C(j,\vec{k},l) = \int_{\mathbb{R}^2} D(t,x) \varphi_{j,\vec{k},l}(t,x) dt dx$$
(1)

where $c(j, \vec{k}, l)$ is the curvelet coefficient indexed by its frequency band j, dip l and time-space displacement \vec{k} , and D(t, x) is the 2D seismic sample at time t and position x; $\varphi_{j,\vec{k},l}(t,x)$ is the curvelet basis. Both \vec{k} and l increase in dyadic order for every other j, hence the term "multi-scale". In contrast to the time-space or frequency basis, a curvelet is localized in both frequency and time-space, as shown in Figure 1. In seismic data, most events are either linear or curved in shape within a small spatiotemporal window, hence the needle-like curvelets form a suitable and natural basis for representing the data, which leads to the sparseness of the curvelet coefficients of the data. There exists a way to exploit this sparsity to separate multiples from primaries by using Bayesian Probability Maximization (BPM) (Saab et al., 2007).

The conventional implementation of BPM is equivalent to the LS method. This is because the prior probability distribution of data and model is preset to Gaussian in BPM, and solving BPM results in extracting the power indices of the distribution function to formularize a quadratic summation form. In the curvelet domain, the L1-norm was introduced in the optimization problem as the sparse coefficients follow a steeper distribution than Gaussian (Saab et al., 2007). An iterative soft-thresholding algorithm was used to solve this optimization problem (Daubechies, et al., 2004).

In this paper, noting that the convergence of the iterative solver used by Saab relies on an initial estimation of the predicted multiples that is sufficiently close to the actual multiples in the data, we propose the use of least-square matching filtering to bring the amplitude, traveltime and spectrum of the predicted model closer to those of the actual multiples in the data prior to the step of the iterative soft-thresholding optimization. The way we implemented it is to replace the original predicted model M' in Saab's equation by applying the designed matching filter f_{LS} to it, i.e., the adaptive model, as shown in Eq. (2).

$$f(P_c, M_c) = ||P_c||_{1,w_1} + ||M_c||_{1,w_2} + ||C^{-1}M_c - f_{LS} * M'||_2^2 + \eta ||C^{-1}(P_c + M_c) - D||_2^2$$
(2)

where P_c and M_c denote the primaries and multiples in the curvelet domain; D and M' are the data and the predicted multiple model in time-space domain, respectively. C denotes the forward curvelet transform and C^{-1} the inverse. $w_{1/2}$ is proportional to the curvelet coefficients of the initial guess of model and data, and subscripts "1, $w_{1/2}$ " and "2" denote the element-wise weighted L1-norm, and L2-norm, respectively. η is the overall noise control parameter. In this case, the iterative soft-thresholding algorithm can still be applied for solving Eq. (2). This implementation is termed as the Adaptive Curvelet Domain Separation method (ACDS).

Current multiple prediction methods often suffer from the truncation of high-order multiple terms, spectral narrowing and noise contamination. Uniform estimation of model inaccuracy and noise level in ACDS may not meet the complication of practical field data since the modelling error, noise and signal might occupy different frequency ranges. To avoid applying external filtering that would result in a multiple-fold increase in computational cost, we make use of the intrinsic features of curvelets. Noting that the curvelet transform naturally partitions data into different frequency bands, it is feasible to manipulate the curvelets in each frequency band independently. Hereby we propose a new approach termed Frequency-regularized Adaptive Curvelet Domain Separation method (FrACDS), to effectively separate primaries and multiples in the presence of model inaccuracy and noise contamination for each frequency band. The objective function of the optimization problem $F(P_c, M_c)$ can be recast as:

$$F(P_c, M_c) = \sum_i f_i(P_c, M_c) \tag{3}$$

where $f_j(P_c, M_c)$ holds the same expression as Eq. (2) but only with respect to scale *j* of all variables. The flowchart of the overall process is shown in Figure 2.

SYNTHETIC AND FIELD DATA EXAMPLES

Two simple numerical examples, as shown in Figure 3, were first tested to assess the performance of ACDS. In both examples, the multiple events crisscross the horizontal primary, and conventional LS produces compromised results which manifest as residual multiples at the crossing. In contrast, the multiple events are almost completely removed with minimal damage to the primary by ACDS. This is because the primary and the multiple events at the crossing are represented by different curvelet coefficients and hence they are well separated.

In the next example of 2004 BP 2D model shown in Figure 4, we applied LS and ACDS to remove the multiples predicted by reverse time demigration (Billette and Brandsberg-Dahl, 2005; Zhang and Duan, 2012), and compared the migration stacks. The first-order water bottom multiple is completely removed by ACDS but not by LS. By imposing ACDS, the migration swings are significantly attenuated at the top of the salt body on the right. Besides, the upper boundary of the salt body is distorted by LS on the left while is consummately preserved by ACDS. The superiority of preservation of primary events by ACDS is also reflected in the anomaly and the parallel sedimentary areas, as are pointed out by the arrows.

A field data of a 2D line acquired from offshore Australia is shown in Figure 5. In this example, Shallow Water Demultiple (SWD) approach was applied to obtain the surface related shallow-water multiple model (Hung, et al., 2010). Due to the observed moderate noise level of the data shown in Figure 5(a), we applied FrACDS for primary-multiple separation, and compared with the results by LS. From the enlarged insets in Figure 5(c) and (d), the major primary events are better preserved by FrACDS than by LS where residual shallow multiples penetrate. The effect of LS is also limited in attenuating the widespread noise that snaps and smears the image. The strategy we applied in FrACDS is to tolerate the noise level and to subtract the multiple related curvelet coefficients in the noise-intensive frequency bands. Consequently, the separation result by FrACDS presents a clearer image of lower noise level and more weakened residual multiples than that by LS. FrACDS provides the flexibility of the separation strategy leading to high separation effectiveness of real field data, but it also involves more parameters.

CONCLUSION

To summarize, we have improved the curvelet domain separation method for removing multiples from noisy seismic data. Synthetic and field data examples demonstrate that our approach outperforms the conventional LS method in terms of the noise attenuation, the multiple removal and the preservation of primaries. By applying least-square matching filtering to the predicted multiple model prior to the separation step, it has been shown that ACDS is more robust than the current implementations of curvelet domain separation thus is more adaptive to the models predicted by various methods. However, uniform estimation of noise and model inaccuracy throughout the spectrum in ACDS may not meet the challenges faced by various types of data. Hence we further propose a more flexible implementation FrACDS by introducing the frequency-regularized strategy. FrACDS provides high fidelity by assessing the modelling accuracy and noise level in different frequency bands, as we did to attenuate the less credibly model multiples and high-frequency noise in the field data example.

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Figure 1. (a): The curvelet tiling in frequency domain. $\tilde{\varphi}_{j,k,l}(\xi)$ is the Fourier transform of curvelet basis $\varphi_{j,\vec{k},l}(t,x)$ as shown in red for given *j* and *l*. (b): The curvelet in Panel (a) presented in time-space domain with zero displacement. (c): that with non-zero displacement.



Figure 2. The flowchart of ACDS and FrACDS. The step "Frequency-regularization" is solely applicable to FrACDS.



Figure 3. Left and right panels are two numerical examples illustrating LS and ACDS methods. Row (a): multiple contaminated data; (b): multiple models; (c): optimal LS; (d): ACDS results.



Figure 4. (a): Multiple contaminated seismic data; (b): optimal LS result; (c): ACDS result.



Figure 5. (a): multiple contaminated data; (b): SWD model; (c): optimal LS result; (d): FrACDS result. The insets are zoomed in of the rectangle areas.