Recovery of 3D IP distribution from airborne time-domain EM

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SUMMARY

Conventional IP is not the only technique that is sensitive to chargeable material. Any electromagnetic method applied in the presence of chargeable material will be affected. Unfortunately, the effects are often hard to recognize in the data. For the particular case of coincident loop time-domain EM data, negative transients - soundings with a reversal in sign of the received fields - are diagnostic of chargeable materials. This property can also be extended to center loop systems, including many airborne systems. Negative transients are commonly observed in airborne TEM systems, such as Fugro's AeroTEM system or Geotech's VTEM system.

We develop an inversion methodology to attempt to recover a three dimensional distribution of chargeability from observations of negative transients in airborne time-domain electromagnetic data. Forward modeling of chargeable targets is performed directly in the time domain, and the sensitivity of these data to the presence of chargeable material is derived. The methodology is applied to a synthetic data set. Areas of future work and potential problems are discussed.

Key words: Induced Polarization, Airborne Time-Domain Electromagnetics, Inversion

INTRODUCTION

Chargeability is commonly considered to be a diagnostic physical property when exploring for disseminated mineralization (Seigel et al., 2007). Recent work has also extended its application to a number of other fields including hydrocarbon exploration, ground- water studies, and numerous environmental applications.

A chargeable material processes an electrical conductivity that varies as function of frequency. The frequency dependence of the conductivity of geologic materials is commonly parameterized using the Cole-Cole model (Pelton et al., 1978)

$$\rho(\omega) = \rho_0 \left[1 - \eta\left(1 - \frac{1}{1 + (1\omega\tau)^c}\right)\right] \qquad (1)$$

In this expression, ρ_0 is the zero frequency resistivity, η is the intrinsic chargeability, τ is a characteristic time constant which defines the frequency at which peak dispersion is observed and c is the frequency dependence, defining how rapidly the dispersion takes place. In the special case where c = 1, the Cole-Cole model simplifies to the Debye model.

The presence of chargeable material is traditionally mapped using the induced polarization technique (Bleil, 1953). Current is injected into the ground through a pair of transmitter electrodes and the voltage response of is measured across a second pair of receiver electrodes. The presence of chargeability is indicated by a slowly decaying voltage after the transmitter current has been interrupted. These data are commonly inverted to recover either 2D or 3D models of the chargeability distribution of the ground (D. W. Oldenburg & Li, 1994; Li & Oldenburg, 2000).

Although this method has been successfully applied in the mineral exploration industry for a number of years, its application is not always practical. Exploring on a reconnaissance scale can be limited by the time required to place the transmitter and receiver electrodes. The technique can also fail in certain geologic environments. A highly resistive overburden may make it impossible to inject enough current to excite a measurable IP response.

Conventional IP is not the only technique that is sensitive to chargeable material. Any electromagnetic method applied in the presence of chargeable material will be affected. Unfortunately, the effects are often hard to recognize in the data. For the particular case of coincident loop time-domain EM data, negative transients - soundings with a reversal in the sign of the received fields - are diagnostic of chargeable materials. While early papers speculated that particular conductivity distributions or magnetic effects could cause this effect, Weidelt (1982) showed that for a coincident loop system with a step-off primary field, the measured secondary field must be of a single sign over non-polarizable ground regardless of the subsurface distribution of conductivity. This property can also be extended to centre loop systems, including many airborne systems (Smith & Klein, 1996). Negative transients are commonly observed in airborne TEM systems, such as Fugro's AeroTEM system or Geotech's VTEM system. An example, taken from a VTEM survey performed over the Mt. Milligan deposit in British Columbia is shown in Figure 1. The location of the maximum negative response corresponds closely to the location of a known, near surface IP anomaly.

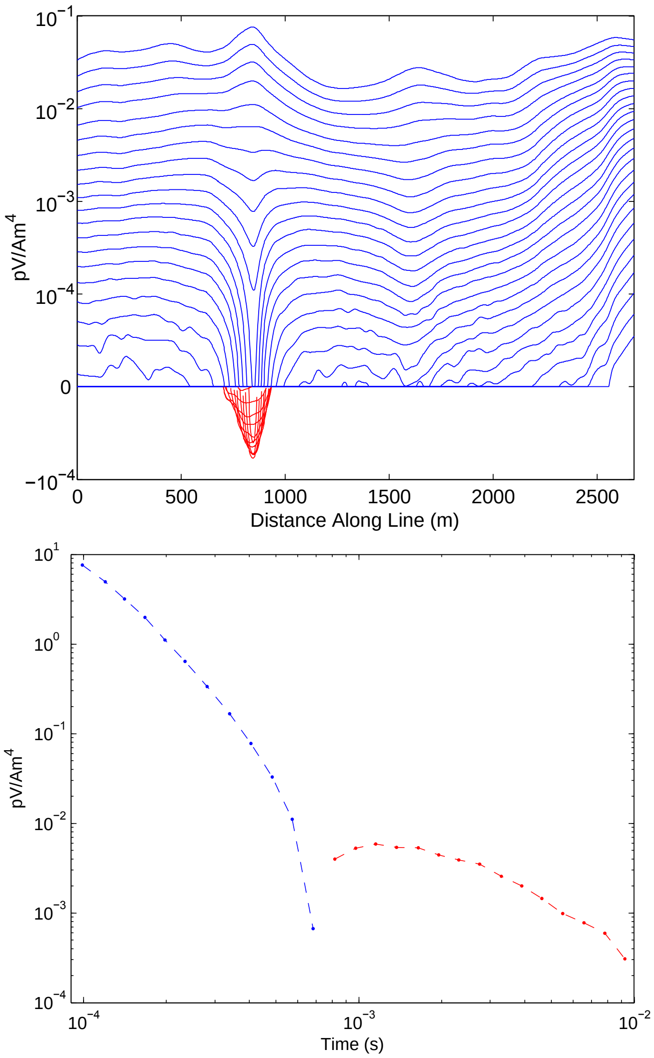

Figure 1. Example of a negative transient observed in a VTEM survey at the Mt. Milligan deposit in British Columbia

Despite the fact that negative transients can be related to the presence of chargeable material, relatively little has been done to try and interpret them directly. (Smith et al., 2008) used the presence of negative transients to map the presence of tailings around a mine site. (Beran & Oldenburg, 2008) applied an over-determined inversion routine to fit IP effected TEM data to a simple layered model exhibiting Cole-Cole dispersion properties. (Kozhevnikov & Antonov, 2010) applied a similar methodology, solving the over-determined problem to recover Cole-Cole parameters of either a uniform half-space, or a uniform two-layer model.

In this work, we develop a methodology to forward model the electromagnetic response of a three-dimensional chargeable earth directly in the time domain. These equations are then used to derive the sensitivity of the time domain response to the Debye chargeability parameter. Finally, we attempt to recover the three-dimensional distribution of chargeable material from synthetic airborne electromagnetic data containing negative transients. Areas of future work, and potential problems are discussed.

METHODOLOGY

Maxwell's equations in the frequency domain, assuming a $e^{i\omega t}$ time dependence are

$$\vec{\nabla} \times \vec{E} + i\omega\mu\vec{H} = 0 \tag{2a}$$

$$\vec{\nabla} \times \vec{H} - \vec{J} = \vec{S} \tag{2b}$$

The current density, J, is related to the electric field, E, through Ohm's law

$$\vec{J} = \sigma(\omega)\vec{E} \tag{3}$$

where σ is the electrical conductivity, σ = 1/ρ. Assuming a conductivity distribution exhibiting Debye dispersion, Ohm's law can be rewritten as

$$\sigma_0\vec{E} + \tau\sigma_0 i\omega\vec{E} = \vec{J} + \tau(1-\eta)i\omega\vec{J} \tag{4}$$

Equation 4 is easily transformed back to the time domain. Along with Maxwell's equations in the time domain, this results in the time dependent system

$$\vec{\nabla} \times \vec{e} + \mu\vec{h}_t = 0 \tag{5a}$$

$$\vec{\nabla} \times \vec{h} - \vec{j} = \vec{s} \tag{5b}$$

$$\sigma_0\vec{e} + \tau\sigma_0\vec{e}_t = \vec{j} + \tau(1-\eta)\vec{j}_t \tag{5c}$$

DISCRETEIZATION

The system 5 can be discretized in time (assuming $s_n = 0$ for n > 0) using a backwards Euler method with time stepping Δt = 1/k giving

$$\vec{\nabla} \times \vec{e}_{n+1} + \mu k\vec{h}_{n+1} = \mu k\vec{h}_n \tag{6a}$$

$$\vec{\nabla} \times \vec{h}_{n+1} - \vec{j}_{n+1} = 0 \tag{6b}$$

$$\sigma_0(1+k\tau)\vec{e}_{n+1} - \sigma_0 k\tau\vec{e}_n = (1+k\tau(1-\eta))\vec{j}_{n+1} - k\tau(1-\eta)\vec{j}_n \tag{6c}$$

We discretize in space onto an orthogonal staggered grid and use the finite volume approach with material averaging of resistivities (Haber & Ascher, 2001). Physical properties are placed at cell centres, magnetic fields are placed on cell edges and electric fields and current densities are placed on cell faces (Figure 2).

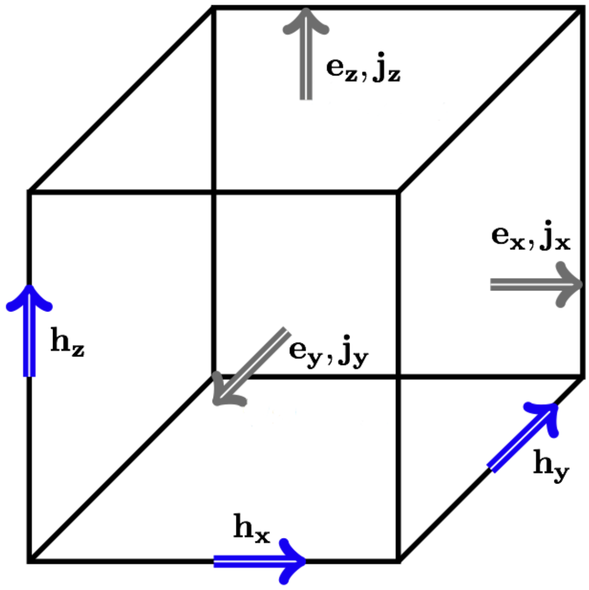

Figure 2. The system is discretized onto an orthogonal staggered grid with magnetic fields placed on cell edges and electric fields and current densities placed on cell faces. Physical properties are defined at cell centres.

This results in the discrete system

$$\mathbf{curl}^\top \vec{e}_{n+1} + k\,\mathsf{diag}\left(\mathbf{A}_c^e \mu\right)\vec{h}_{n+1} = k\,\mathsf{diag}\left(\mathbf{A}_c^e \mu\right)\vec{h}_n \tag{7a}$$

$$\mathbf{curl}\,\vec{h}_{n+1} - \vec{j}_{n+1} = 0 \tag{7b}$$

$$\mathsf{diag}\left(\mathbf{A}_c^f \sigma_0 (1 + k\tau)\right)\vec{e}_{n+1} - \mathsf{diag}\left(\mathbf{A}_c^f k\tau\sigma_0\right)\vec{e}_n = \mathsf{diag}\left(\mathbf{A}_c^f (1 + k\tau(1-\eta))\right)\vec{j}_{n+1} - \mathsf{diag}\left(\mathbf{A}_c^f k\tau(1-\eta)\right)\vec{j}_n \tag{7c}$$

Defining the aggregate variables

$$\mathbf{M} = k\,\mathsf{diag}\left(\mathbf{A}_c^e \mu\right) \tag{8a}$$

$$\mathbf{\Sigma} = \mathsf{diag}\left(\mathbf{A}_c^f \left(\sigma_0 \left(1 + k\tau\right)\right)\right) \tag{8b}$$

$$\mathbf{\Sigma}_\tau = \mathsf{diag}\left(\mathbf{A}_c^f \left(k\tau\sigma_0\right)\right) \tag{8c}$$

$$\mathbf{T} = \mathsf{diag}\left(\mathbf{A}_c^f \left(1 + k\tau\left(1-\eta\right)\right)\right) \tag{8d}$$

$$\mathbf{T}_\tau = \mathsf{diag}\left(\mathbf{A}_c^f \left(k\tau\left(1-\eta\right)\right)\right) \tag{8e}$$

simplifies the system in 7 to

$$\mathbf{curl}^\top \vec{e}_{n+1} + \mathbf{M}\vec{h}_{n+1} = \mathbf{M}\vec{h}_n \tag{9a}$$

$$\mathbf{curl}\,\vec{h}_{n+1} - \vec{j}_{n+1} = 0 \tag{9b}$$

$$\mathbf{\Sigma}\vec{e}_{n+1} - \mathbf{\Sigma}_\tau\vec{e}_n = \mathbf{T}\vec{j}_{n+1} - \mathbf{T}_\tau\vec{j}_n \tag{9c}$$

Thus, for a model of known physical properties and given the initial states of e, h and j we can solve system 9 to forward model the behaviour of the fields.

INVERSION METHODOLOGY

Our inversion will methodology will largely follow that described in (Oldenburg et al., 2013). We apply a Gauss-Newton procedure to recover a model for the Debye chargeability, η. This is done by solving the optimization problem

$$\begin{aligned} \min_{\eta} \quad & \phi(\eta) = \phi_d(\eta) + \beta\phi_m(\eta, \eta_{ref}) \\ \text{sub. to} \quad & 0 \le \eta < 1 \end{aligned} \tag{10}$$

In this expression, φ_d is a measure of the data misfit and φ_m is a regularization term designed to produce small, smoothly varying models. β is regularization parameter that dictates the relative importance of φ_d and φ_m.

The Gauss-Newton step is defined by

$$\left(\mathbf{J}(\eta)^\top \mathbf{J}(\eta) + \beta W_m^\top W_m\right)\delta\eta = -g(\eta) \tag{11}$$

where J(η) is the sensitivity matrix. J(η) is a dense matrix, and in most cases is too large to be formed. Instead, we solve equation 11 using a conjugate gradient method, which only requires the ability to multiply J(η) and J(η)⊤ onto a vector.

The regularization parameter is chosen using a cooling schedule. β is initially chosen to be large so that $\beta\varphi_m$ dominates the objective function. When the model that solves equation 10 is identified, β is decreased by a constant factor. This continues until the desired level of data misfit is achieved.

SENSITIVITY CALCULATIONS

As mentioned above, the inversion requires the ability to multiply J(η) and $J(\eta)^\top$ onto a vector. The sensitivity matrix can be written as

$$\mathbf{J} = -\mathbf{Q}\hat{\mathbf{A}}^{-1}\mathbf{G}_\eta \tag{12}$$

where Q is a projection matrix projecting the modelled data onto the receiver locations, A is the forward modelling operator, and G_η is given by:

$$\mathbf{G}_\eta = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ k\,\mathsf{diag}\left(\vec{j}_{n+1} - \vec{j}_n\right)\mathbf{A}_c^f\,\mathsf{diag}\left(\tau\right) \end{bmatrix} \tag{13}$$

Multiplying J onto a vector involves, multiplying by the sparse matrix G_η, perform a forward modelling with the product as the source term, and projecting the resulting fields onto the receiver locations

It is important to note that calculations involving J(η) involve the parameters σ_0 and τ. Thus, recovering a model of η will first require an estimate of the other two parameters. This is similar the traditional inversion of IP data, where an estimate of resistivity is required prior to inverting for chargeability.

SYNTHETIC EXAMPLE

To demonstrate this methodology, the response of a single line of airborne time-domain data was simulated above a conductive, chargeable body buried in a relatively resistive background. The block was modelled to have $\sigma_0 = 0.1$, $\tau = 0.1$ and $\eta = 0.1$ and the background was non-chargeable with $\sigma_0 = 0.001$. A cross section through the true model is shown in figure 3.

Ten airborne soundings where simulated along a line passing above the block, 50m above the ground. The transmitter was modelled as a point dipole source, co-located with the receiver. Each data point was contaminated with random Gaussian noise with a standard deviation of 10% of the data value. The inversion was run using the methodology described above and assuming knowledge of the true distribution of σ_0 and τ. The final model is shown in figure 4.

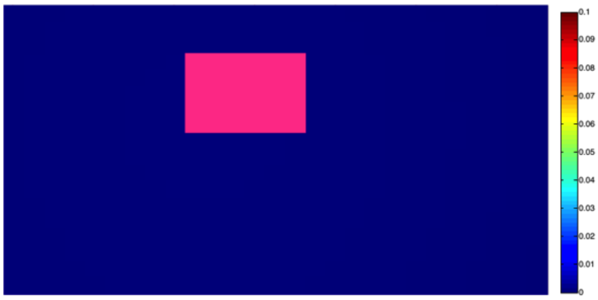

Figure 2. True chargeability model.

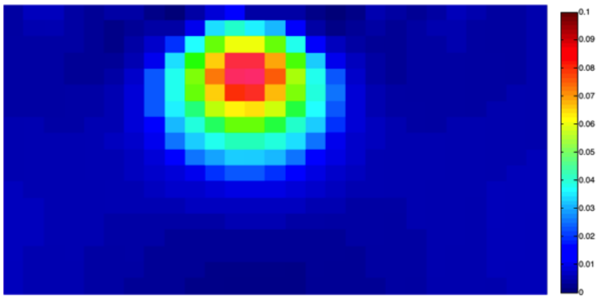

Figure 3. Recovered chargeability model.

CONCLUSIONS

We have developed a methodology for inverting IP affected time domain EM data, and demonstrated it's potential application on a simple synthetic example. Although a successful test, the example we have presented is an ideal case. It does not guarantee that this method will be successful under less ideal conditions. There are still many interesting questions that need to be answered before this work can be applied to real world situations.

The data that were inverted contained distinct negative transients, clearly indicating the presence of chargeable material. More often than not, the IP effects will be subtle and not be so easily recognized. It may still be possible to recover an estimate of chargeability in the absence of negatives, but your success will be determined by how accurately you know the distribution of background conductivities.

The model we have used in developing this technique assumes that all material exhibits Debye dispersion. Time domain data has previously been found to be largely insensitive to the value of the frequency dependence (Smith, 1988). This should be tested within the framework of the inverse problem.

The sensitivities used in the inverse problem assumed a prior knowledge of the three-dimensional conductivity structure in the area and of the value of the time constant expected for the chargeable. In more realistic situations, you would not have this information, and you would have to estimate it prior to inverting for chargeability. It will be important to understand how important the accuracy of these estimates are to the successful recovery of the distribution of chargeability under different conditions. It may also be interesting to investigate the potential of joint inversion to recover multiple parameters simultaneously.

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