Geological uncertainty and geophysical misfit: How wrong can we be?

**SUMMARY**

Geophysical inversion employs numerical methods to minimise the misfit between three-dimensional petrophysical distributions and geophysical datasets. Inversion techniques rely on many subjective inputs to provide a solution to a non-unique problem, including use of an a priori input model or model elements (a contiguous volume of the same litho-stratigraphic package) and inversion constraints. Inversions may produce a result that perfectly matches the observed geophysical data but still misrepresents the geological system. A workflow is presented that offers multiple starting models to inversion: (1) simulations are performed to create a model suite containing a collection of geologically possible models; (2) uncertainty analysis is then performed using stratigraphic variability to identify low certainty model regions and elements; (3) ‘Geodiversity’ analysis is then conducted to determine the geometrical and geophysical extremes within the model space; (4) geodiversity metrics are then simultaneously analysed using principal component analysis to determine which models exhibit common or diverse geological and geophysical characteristics, enabling the selection of models that are subjected to geophysical inversion. The Ashanti Greenstone Belt, southwestern Ghana in west Africa is used as a case study to test the value of this workflow. Analysis of inversion results are performed that finds a correlation between regions of geological uncertainty and geophysical misfit. This correlation strongly suggests that geological uncertainty can be used as a powerful geological constraint to optimise inversion processes and produce geologically reasonable models.

**Key words:** Inversion, Uncertainty, 3D modelling, Geological constraints, Ashanti Greenstone Belt.

**INTRODUCTION**

Inverse problems can result in solutions that infer parameter values and describe the undetermined system, however inverse problems do not offer unique solutions (Tarantola and Valette, 1982). Inverse problems are common in geoscience as knowledge of all parameters is rarely known. One such inverse problem is attempting to resolve the 3D geological architecture from a geophysical dataset. Knowledge of the parameters essential to formulating a forward problem (complete descriptions of shape, location and physical nature of all geological structures) are not explicitly known in geology (Frodeman, 1995). An inverse solution is required to determine the petrophysical distributions observed in the geophysical response. Different schemes are available to solve geophysical inverse problems. The least-squares method uses an iterative algorithm in which, at each iteration, the unknown problem parameters (petrophysical distributions) are estimated and evaluated against the measured geophysical field via forward modelling to provide a solution (Fullagar et al., 2000; Jessell, 2001; Oldenburg, 1974; Tarantola and Valette, 1982). The a priori input for inversion is a starting model consisting of a selection of petrophysical properties and/or geological surfaces (Boschetti and Moresi, 2001; Fullagar et al., 2000). Estimation of rock property distributions produces a calculated response that is measured against the natural, or observed, geophysical response. Mathematical methods, such as residual misfit and fit to data metrics, are used to assess whether the estimated rock property distribution adequately reflects the observed geophysical response. Estimation of rock property distributions are performed using an objective function, however constraints such as petrophysical rock properties, are used to reduce the number of possible solutions (Boschetti and Moresi, 2001) but are prone to sampling errors (Worthington, 2002).

Inversion parameters, such as fit to data thresholds; the structure of the starting model; the inversion scheme itself and what results constitute an adequate ‘answer’ are all chosen by the operator. Geometrical constraints can be employed to stop large changes at shallow depths (Fullagar et al., 2008). A ratio can also be assigned to ensure that a unit cannot transgress set upper and lower depth bounds when modified by inversion (Fullagar et al., 2008). Geological constraints reduce the number of potential inverse solutions, but rely on the interpretation of geological information that contains its own uncertainty and error. This means that attempts to reduce the number of possible solutions through imposing geological constraints are achieved at the expense of adding further subjectivity.

Typical inversion procedures operate with a single a priori model, precluding other possible geological scenarios from being tested against the observed geophysical field (Boschetti and Moresi, 2001). The assumption that the a priori model supplied to inversion input is the best geological solution, which is flawed, as geological and geophysical problems are often as poorly parameterised as each other (Jessell et al., 2010; Thore et al., 2002).

This paper outlines a workflow that presents multiple geological realisations from the same input dataset and subjects them to inversion. Figure 1 shows how the process is tested on (1) a model constructed to represent the Ashanti Greenstone Belt (AGB), southwestern Ghana in West Africa; (2) model uncertainty is calculated and (3) detected using...
techniques from Lindsay et al., (2012); (4) & (5) geodiversity metrics are used to characterise the geophysical and geometrical model space possibilities (Lindsay, 2013) (6) principal component analysis (PCA) is used to analyse all geodiversity metric results to identify a selection of representative a priori models; (7) execution of basement-style inversion and (8) assessment of inversion with respect to model uncertainty.

METHOD AND RESULTS

A 3D geological model of the AGB (Figure 2) was constructed using a combination of field, geophysical and remotely sensed data. Input field data includes petrophysical measurements, structural observations, lithological descriptions and a revised stratigraphic column produced by Perrouty et al. (2012). Geophysical interpretation of potential field datasets was extensively employed to provide greater geological understanding over the study area. Geophysical datasets include gravity (BGI, http://bgi.omp.obs-mip.fr/ - 4.6km resolution) and aeromagnetic data (Geological Survey of Ghana, line spacing at 200 metres).

Figure 1. Process followed for the testing of the proposed inversion procedure.

Uncertainty simulation is employed in order to understand the effects of different sets of orientation inputs on the result 3D geometry displayed in the 3D model. The orientation measurements that help the shape the strike and dip of geological surfaces (geological contacts or faults) are varied to within five degrees of the original value using psuedo-random equiprobable perturbation. For example, a measurement of 084/62E can be changed to 081/58E or 089/64E and so on, as long as the new measurement is within ± five degrees of the original. The perturbed sets of orientation measurements are used to re-calculate and construct ‘perturbed models’, which, together with the initial model, collectively form a ‘model suite’ (Lindsay et al., 2012). Uncertainty exists in locations showing difference between models, for example, when a particular geological formation displays different geometries across the model suite. Differences are found by discretising the models into stratigraphically attributed voxets, where integers represent units, and find locations where multiple integers (stratigraphic units) exist (Figure 3).

Figure 2. Geology map of the Ashanti Greenstone Belt and inset showing location in West Africa.

Figure 3. Uncertainty contained within the AGB 3D model represented by stratigraphic variability (SV) (Lindsay et al., 2012) where the number of different units at a given location are represented by an integer. SV values <4 have been filtered to emphasise locations with medium to high uncertainty. Most uncertain locations are associated with the units found at depth within the model. The contact between the overlying Tarkwaian units is shown in red.

SV implies that model elements change shape or location from model to model. Unfortunately, SV does not describe how uncertainty affects model geometries. Quantitative analysis of the geometrical and geophysical characteristics exhibited by the model suite allows models to be compared and then ranked...
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against each other. Analyses are performed under a collection of techniques called ‘geodiversity metrics’ (Table 1) (Lindsay, 2013). Geodiversity analysis found that depth for the contact between the Birimian and Tarkwaian units ranged between 6650 m and 8050 m. The contact surface area of the Birimian units also displayed high degrees of variability across the model suite (between 51 & 3847 voxels), reflecting the high degree of uncertainty associated with these units. RMS for the forward gravity response was found to range between 9.46 and 8.86 mGal, indicating that the model suite is a prime candidate for geophysical inversion to reconcile the high level of misfit between the observed and calculated response of the model.

PCA, a multivariate statistical technique, was used to analyse results of geodiversity analysis. The advantage of PCA is that many variables with different measurement types can be simultaneously explored to identify (1) variables that contribute the most variation across the model suite and (2) which models express the most diverse (and common) characteristics (Figure 4). Model space is characterised as well, with diverse models forming the boundaries of model space, and common models forming the barycentre. Models can then be selected for inversion based on common and diverse characteristics, ensuring that model space has been sampled in a representative manner. Hotelling’s T² statistic (Hotelling, 1931) was used to pick the three most diverse (3, 38 & 101) and common models (92, 33 & 59). Note that model 1, the model that was produced with the unperturbed data does not feature in this list. Model 1 was ranked as the fourth most diverse model by PCA, highlighting that producing a single 3D model from modeling workflows misrepresenting the target geology.

INVERSION

A ‘basement’ style inversion was chosen to reconcile model geometry and petrophysical density distributions with the observed gravitational response (Fullagar et al., 2008). A surface representing a geological contact between two units is used as a geometrical input. The upper layer represents the ground surface and the second represents the basement. The inversion process iteratively optimises the geometry of the basement interface, then density properties of the basement layer independently. Inversion is performed on a discretised version of the 3D model, performed by subdividing the model into 1 km by 1 km prisms extending from the Earth’s surface to the base of the model. Each prism is further subdivided using the basement interface into the two layers. The observed Bouguer gravity response is assigned to each basement prism as a property.

Surfaces from the selected models representing the base of the Tarkwaian group were chosen as the ‘basement’ interface as it displays a relatively high density contrast between the upper layer and the underlying Birimian Series basement units (Figure 4) and also defines a major unconformity. The Birimian property distributions were heterogeneous and density values (2.55 g/cm³ above interface, 2.7 below interface g/cm³) were assigned according to the petrophysical analysis of Perroux et al., (2012).

Six inversions were executed using with a limit of 100 iterations. Each inversion was deemed successful and produced final RMS errors of 0.20 mGal, except for model 101 where the final RMS error was 0.19 mGal. The results of each inversion are compared using a difference map (Figure 5). Inversion results are compared to (1) confirm whether a convergence on a single solution is found and (2) if not, which regions were difficult to reconcile to the observed geophysical input data.

Figure 6a shows that differences do exist between solutions and signifying that the choice of a priori model is important, as the inversion processes do not converge on a single solution, regardless of input parameters. Further, the difference and uncertainty maps bear correlations. Localised difference anomalies are located near Bogoso, south-southwest of Bogoso east of Damang. These anomalies correlate to uncertain areas shown in the SV map on the right, notably those just east of Bogoso, south-southwest of Bogoso and east of Damang.

UNCERTAINTY CONSTRAINTS

The results here show that model uncertainty depicted by SV can be used to assist inversion in two ways: (1) by focussing inversion on regions that are uncertain and (2) providing additional geological constraints. Focussing inversion on uncertain areas will optimise the algorithm by supplying the locations that require modification, rather than relying on least-squares or stochastic methods to search for where and how modifications should be applied.

CONCLUSIONS

Subjective decisions required for execution of geophysical inversion are reduced by (1) producing multiple geological realisations from input data and (2) guiding the choice of
geophysical input data through geodiversity analysis. A model now has more use than just one a priori reference point for the distribution of petrophysical properties. Geological and geophysical possibilities contained within the model suite are described by geodiversity analysis. PCA helps to select inversion parameters and deconvolve complex interactions between model elements. PCA of geodiversity metrics provided the identification of models 92, 33, 59, 3, 38 and 101.

![Image showing the projection of stratigraphic variability](image)

Figure 6. a) Differences between inversion results for geometry (depth). b) Image showing the projection of uncertainty (stratigraphic variability) onto a horizontal 2D surface and used to correlate model uncertainty with various aspects of inversion modelling.

that were input to inversion. In addition, Model 1 was shown to not be the most representative example of the input data, and was actually found to be highly misrepresentative in terms of the geodiversity metrics employed. Visualisation of stratigraphic variability highlighted sources of uncertainty and SV has been recognised as a possible inversion constraint that can insert more geological information into a dominantly geophysical process. The role of geological input in geophysical inversion can now be increased.

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Table 1: Description of the geodiversity metrics used to characterise the model suite.

<table>
<thead>
<tr>
<th>Name</th>
<th>Subject</th>
<th>Measurement</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometrical geodiversity metrics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>Voxet</td>
<td>Metres$^3$</td>
<td>Volume for each formation</td>
</tr>
<tr>
<td>Depth</td>
<td>Voxet</td>
<td>Metres</td>
<td>Shallowest and deepest occurrence of each formation</td>
</tr>
<tr>
<td>Curvature</td>
<td>Surface</td>
<td>$k_m$: Mean curvature</td>
<td>Average $k_m$ and $k_g$ values for each formation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_g$: Gaussian curvature</td>
<td></td>
</tr>
<tr>
<td>Contact relationships</td>
<td>Surface</td>
<td>Area (metres$^2$)</td>
<td>Contact surface area and contact relationships</td>
</tr>
<tr>
<td>Geological complexity</td>
<td>Voxet</td>
<td>Number of different lithologies</td>
<td>Scalar value representing geological complexity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>around point-of-interest</td>
<td></td>
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<tr>
<td><strong>Geophysical geodiversity metrics</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Root mean square (RMS)</td>
<td>Residual grid</td>
<td>Global measure of geophysical misfit</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>Calculated grid</td>
<td>Global measure of geophysical variability</td>
<td></td>
</tr>
<tr>
<td>Entropy</td>
<td>Residual grid</td>
<td>Global measure of geophysical variability</td>
<td></td>
</tr>
<tr>
<td>2D correlation coefficient</td>
<td>Between observed and calculated grids</td>
<td>Global measure of geophysical covariance – recognises similar patterns</td>
<td></td>
</tr>
<tr>
<td>Hausdorff distance</td>
<td>Observed and calculated grids</td>
<td>Global measure of geophysical misfit – accounts for pattern translation, rotation and dilation</td>
<td></td>
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</tbody>
</table>