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Performance of the Double Absorbing Boundary Method when Applied to the 3D Acoustic Wave Equation

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SUMMARY

The double absorbing boundary (DAB) is a new high-order absorbing boundary condition for the scalar acoustic wave equation. It suppresses scattered waves at the edge of a boundary layer in computational domain boundary by using destructive interference analogous to a noise-cancelling headphone. This method has advantages in that it addresses some of the shortfalls in existing boundary conditions, such as the need for tuning in Perfectly Matched Layers or complex formulations at corners such as in high-order absorbing boundary conditions. We extend the original formulation of the DAB to three dimensions and higher-order stencils. Through numerical simulation we test the performance of the DAB by comparison with a reflecting boundary. We find that the DABC is a broadband attenuator with a power attenuation of 20-30dB using only six boundary cells. Increasing the order of the method improves accuracy for wavelengths less than 10 cells, whereas increasing the layer width does not improve accuracy. The method shows promise as a robust and computationally efficient boundary condition for seismic applications.

Key words: double absorbing boundary method, finite difference, acoustic wave equation, seismic modelling.

INTRODUCTION

The computational efficiency of seismic acoustic wave equation modelling can be greatly improved if the computational domain is decreased such that the domain is only as large as the region of interest. Undesirable reflections from the boundary can interfere with desirable wave propagation effects in the model, hence the need for an absorbing boundary condition (ABC) to prevent energy from propagating back into the domain interior.

Absorbing boundary techniques

Specifying an absorbing boundary that is accurate, stable, efficient, broad-spectrum, easy-to-implement, and robust to a wide variety of incident wave angles and model types remains an active topic of research. In recent years two major classes of effective absorbing boundaries have been developed, artificial boundaries and artificial layers. In the artificial boundary regime, such as the ABC (e.g., Clayton and Engquist, 1977), the outer boundary of the model is modified to dampen incoming waves. In later formulations starting with Collino (1993), the absorbing boundary was extended to high-order accuracy through the use of auxiliary variables. In artificial layers such as the perfectly matched layer or PML (Berenger, 1994) waves propagating through a boundary layer are dampened by modified wave equations. Since then numerous improvements of both methods have appeared in the literature. Both types of methods were shown to be equally effective (Rabinovich et al. 2010).

The ABC and PML approaches both have their advantages and drawbacks. High-order ABCs can be implemented up to any desired level of accuracy up to the discretisation error. A drawback of the PML is that given a fixed layer width it has no clear definition of convergence due to the mapping from a perfect analytical formulation to a discrete problem (Hagstrom, 2014). Unlike the PML, high-order ABCs are complex to formulate at corners. A high-order ABC doesn't require tuning, whereas parameters in the PML, such as the layer thickness and damping coefficients, require tuning to effectively dampen waves for a given problem (Givoli, 2008).

In this paper we test the accuracy of the double absorbing boundary (DAB) method. The method aims to achieve the best of both ABC and PML methods while avoiding the drawbacks. The result is an absorbing layer that aims to be effective with very few layer points. We extend the DAB from its original one-dimensional formulation to three dimensions, and modify it to work with higher-order stencils. Using numerical tests we determine the spectral response of the double absorbing boundary with favourable results.

The Double Absorbing Boundary method

The Double Absorbing Boundary (DAB) (Hagstrom et al. 2014) is a recent variant on the ABC technique. It implements a high-order ABC over a layer Ω_M with M interior cells, as seen in Figure 1.

The incoming wavefield is read on the inner boundary Ω_I and a modified advection equation is used to create a secondary wave Φ_1 that is propagated across the layer Ω_M. High-order accuracy can be achieved by applying the modified advection equation again to Φ_1 to generate Φ_2 and so on, up to order N+1, where N is the order of the DAB method. In the outer boundary region Ω_B the secondary waves are added using a modified advection equation such that they attempt to cancel the incoming wavefield at the boundary.

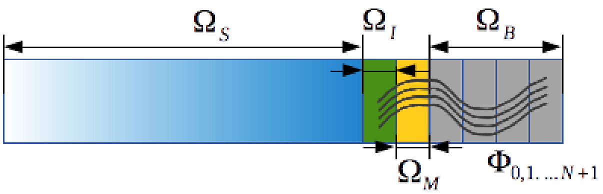

Figure 1. Physical layout of the DAB applied to the right boundary of a 2D grid. Cancellation waves in variables $\Phi_{1...N+1}$ are generated at the inner boundary at Ω_I and are propagated across the layer Ω_M. The cancellation waves then cancel out the solution at the outer boundary Ω_B.

The numerical implementation of a DAB is similar in many respects to the physical system employed by noise-cancelling headphones. The DAB method can be implemented up to any order of accuracy within the discretisation error and does not require tuning or complex formulations at corners. It also has a fairly straightforward implementation and is effective even with very thin layers. This offers the promise of a high level of computational and memory efficiency.

METHOD AND RESULTS

Our aim is to obtain an estimate of the performance of the DAB as a function of wavelength. In order to accomplish this we simulated a typical seismic source propagating into a constant velocity environment with DAB boundary conditions. We derive a power spectrum of undesirable reflections and compare the spectra to the power spectrum from reflecting boundaries.

We extend the one-dimensional DAB method reported by Hagstrom (2014) to a 3D acoustic medium, and extend the method to high order stencils by extending the boundary region Ω_B to cover the extra boundary cells.

Our source is a Ricker wavelet with a 25 Hz peak frequency. We use a cell width of 8 m and a timestep of 1.2 ms using a Courant number of 0.3. To approximate a "reference wavefield" from an ideal, perfectly non-reflecting boundary, we propagate the source wavefield into a larger 201^3 volume while saving an inner 101^3 cube. We then propagate the source into 101^3 grids with DAB boundaries. The DAB on each boundary has total width of six cells, and we test it using a range of orders N=1,2,3,20. We use a second-order space, second-order time stencil to propagate the auxiliary variables over the boundary layer, and an eighth-order space, second-order time stencil to propagate the wavefield over interior cells. The wavefield is evolved within each 101^3 grid until any reflections from the boundary meet again in the centre at t = 0.4 s.

We obtain a "residual wavefield" at each time step by subtracting the reference wavefield at each timestep. A power spectrum of the residual wavefield is computed by Fourier transform and binning. For each boundary type we average over power spectra from the final 17 time steps and divide the result by the averaged power spectrum from the reflecting boundary residual wavefield. In Figure 2 we show the resulting attenuation from each test. We find that the DAB serves as a broadband attenuator, and realises attenuation between 20-30 dB for all seismic wavelengths between 2 and 40 cells (equivalent to 16-320 m wavelengths for this test). Remarkably, this result was achieved using only one cell in the internal layer Ω_M (six cells in total), and shows promise that the DAB is a computationally efficient absorbing boundary. PML methods are expected to attain higher levels (80 dB+) of power attenuation (Katz, 1994), but at the cost of more boundary points. In future work we will look at how the DAB compares to recent PML formulations.

As Figure 2 shows, increasing the order of the DAB method makes a difference to the attenuation of wavelengths smaller than 10 cells. The difference between the lowest and highest order DAB methods for wavelengths smaller than 10 cells is less than 10 dB.

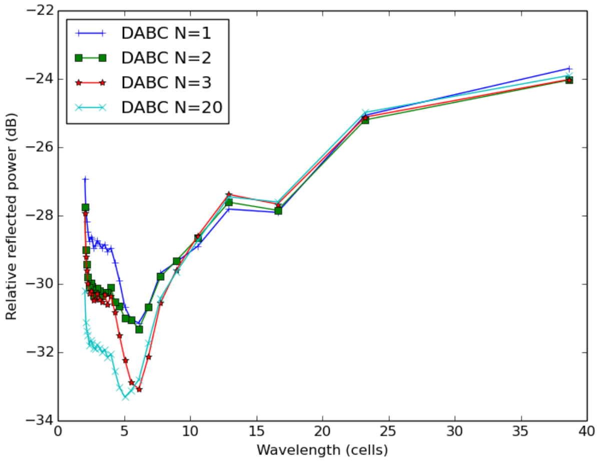

Figure 2. Attenuation of the DAB as a function of order N and seismic wavelength. The entire width of the boundary region (including boundary cells) is only six cells wide.

An increase in the DAB layer width shows no improvement in attenuation. In Figure 3 we plot the power spectra from waves encountering a DAB boundary with the order N=1 and number of interior layer cells M=1,2,3,20. At small layer widths we need a negligible difference in attenuation. For large layer widths where M=20, there is a significant reduction in attenuation. This is presumably due to the low order accuracy of propagating cancellation waves across the DAB layer.

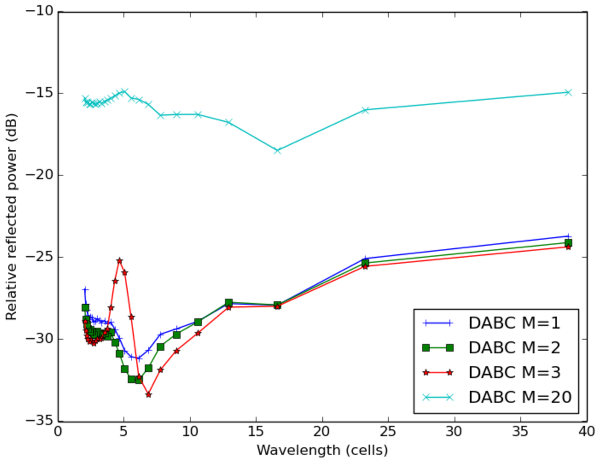

Figure 3. Attenuation of the DAB as a function of M, the number of interior cells in the DAB layer. The order of the method is kept at N=1. Increasing the number of interior layer cells has a negative effect on the attenuation, presumably due to limitations of the second-order

algorithm for propagating cancellation waves across the layer.

In Figure 4 we plot slices of the 3D wavefields as they propagate into the boundary regions. In the left column is the wavefield interaction with the reflecting boundary at times t=0.24 and t=0.4. The right column shows the wavefield interaction with the DAB boundary (M=1,N=3). As seen in the plot in the bottom right corner of Figure 4, the DAB boundary exhibits minor corner reflections around 10% of the amplitude of the incoming wave. Possible solutions to this problem will be discussed in future work.

CONCLUSIONS

We test a new absorbing boundary condition that combines the best aspects of traditional boundary methods. To test the method we propagate a Ricker wavelet source into different DAB implementations and compare the residual reflections from each boundary with the residual from a reflecting boundary. We found that the double absorbing boundary is a good broadband attenuator with a 20-30 dB decrease in reflected power. Higher levels (80 dB+) of power attenuation are expected in tuned PML methods. In the near future we expect to compare the DAB with recent PML formulations using a similar analysis.

Tests show that increasing the power of the method shows a (<10 dB) increase in attenuation for shorter wavelengths. Increasing the width of the DAB layer appears to have no significant effect at smaller layer widths and shows a negative effect for larger layer widths. We suspect this is due to the accuracy of cancellation wave propagation across the layer. We also observed minor corner reflections from the DAB boundary. In forthcoming work we hope to address these spurious reflections and test the performance of the DAB and the latest PML methods using heterogeneous models.

In conclusion, the DAB method performs well in attenuating reflections given a limited number of layer cells. Given that the method also does not need tuning it shows promise as a robust and flexible boundary layer for use in seismic applications.

ACKNOWLEDGMENTS

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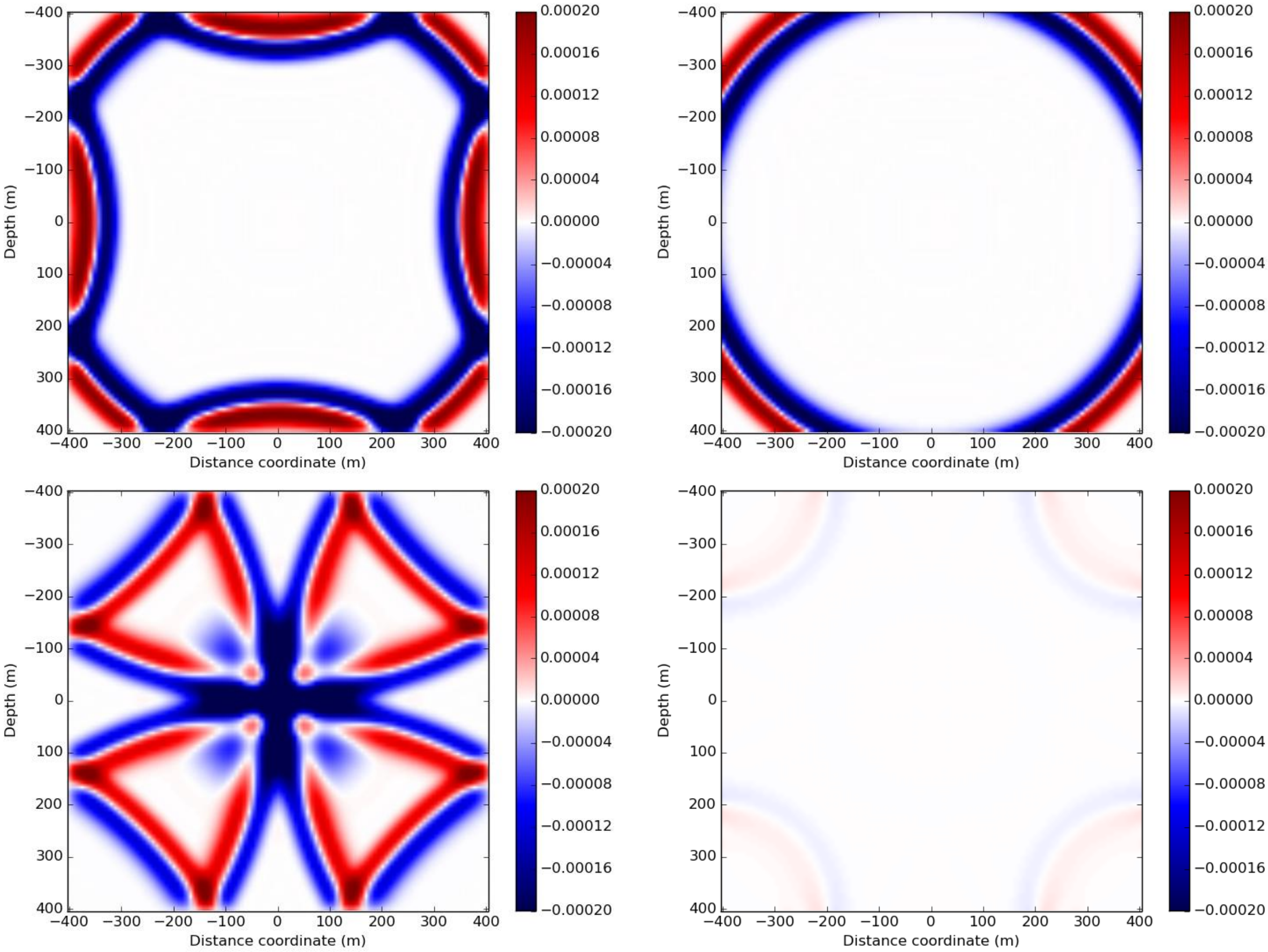

Figure 4. Slices of the 3D wavefield as it interacts with the boundary. In the left column is the wavefield encountering a reflecting boundary at 0.24 seconds (top) and 0.4 seconds (bottom). In the right column is the wavefield encountering a DAB boundary with order N=3 and M=1 interior cells in the DAB (six layer cells in total).