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Reciprocity principle in finite difference modelling of waves in elastic media.

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SUMMARY

Reciprocity principle has been used in a number of seismic applications. This principle relates the two wave fields with interchanged source and receiver locations, where the radiation patterns of the source and receiver are interchanged as well. In extending this principle to be used in real-world scenarios where radiation patterns vary in different locations, a number of experiments to determine the validity of this principle were conducted. Given the proliferation of the numerical modelling in today's geophysical data processing and imaging, the verification of validity of the reciprocity theorem for the modelling algorithms is important. We found that the reciprocity principle is not upheld for some instances of finite difference modelling due to the implementation of the free surface boundary condition. In the case of absorbing boundary conditions however, good reciprocity relation can be achieved.

Key words: reciprocity, seismic modelling, finite difference

INTRODUCTION

Reciprocity principle has been used in many areas of physics describing relationships between potentials satisfying either Laplace's equation or the wave equation. In seismic application, the reciprocity principle states the intuitive notion that, interchanging the positions of the source and receiver will yield the same response. This principle is valid only if we interchange not only the locations of the source and the receiver, but also the corresponding radiation patterns. To apply directly this principle to the field data is seldom possible, since the assumption of having the same radiation patterns of the source and receiver is rarely satisfied. In the simplest form the reciprocity principle in elasticity can be expressed as

$$G_{ij}(x_1, t; x_2) = G_{ji}(x_2, t; x_1) \qquad (1)$$

where $G_{ij}(x_1, t; x_2)$ represents Green's function corresponding to a point force source in direction j at location x_2 and recorded at location x_1 at time t in direction *i*. The theoretical derivation of this principle dates back to Maxwell (1864). For a derivation of the reciprocity principle is shown, among others, in the book by Aki and Richards (2002) or a paper by Achenbach (2006).

The use of reciprocity principle in seismology can be found in many texts on seismic wave propagation. Examples are Knopoff and Gangi (1959), and Gangi (1970), who derived the reciprocity theorem in seismological setting and showed it validity in experiments. White (1960) used the reciprocity relationships to obtain low-frequency radiation patterns of shear and compressional waves from relatively complex sources. Gupta (1965; 1966) used the reciprocity principle to obtain the radiation patterns of P and SV waves from horizontal and calculated the polar radiation patterns of P, SV and SH waves.

Berrhill (1984) and Mulder (2005) show that the principle of reciprocity can be used in datuming the source and van Borselen et al. (2013) use this principle in performing deghosting of marine seismic data. Fokkema and van den Berg (1993) provide an extensive overview of acoustic reciprocity and its application in seismic processing and interpretation which includes wavefield decomposition, removal of surface related wave phenomena, boundary imaging, domain imaging and seismic inversion.

Numerical modelling plays pivotal role in today's geophysical data processing and imaging. As such, the verification of validity of the reciprocity theorem for the modelling algorithms is important. In this paper, we investigate the degree of validity of this principle under real world scenarios where the radiation pattern varies in different locations using finite difference modelling. The study considers elastic medium under different characteristics of radiation patterns giving us a vector problem. The study is done on a synthetic 3D model with complex geological structures that allow for creation of different converted waves that would add to the complexity of the problem of validating the reciprocity principle. To this end, we perform finite element elastic seismic modelling on the constructed model where we compare traces from interchanging of source and receiver locations on all radiation patterns under different boundary conditions.

METHOD AND RESULTS

In this study, SOFI3D finite difference full elastic forward modelling code (Bohlen, 2002) was used to model the wave propagation. This program allows for 3D viscoelastic, elastic and acoustic parallel modelling using Taylor or Holberg coefficients and can be used on a suitable multi-processor supercomputer (e.g. on the iVEC Pawsey Supercomputer).

We developed a simple discretised 3D numerical model, which is suitable for SOFI3D program, representing a rugged near surface geology containing two planar reflectors, one tilted and one horizontal, to be used in modelling program. We then populate the density, P-wave and S-wave velocities in the model based on typical properties of a regolith cover. Figure 1 shows a slice of the model populated with the P-wave velocity. Preliminary considerations such as spatial and temporal sampling conditions are take into account as shown in Equations (2) and (3), which are used to calculate the spatial grid spacing, *dh*, and the time step, *dt*. This is done to avoid numerical artefacts and instabilities during the modelling run.

$$\mathrm{dh} \le \frac{\mathrm{V}_{\mathrm{S}_{\min}}}{\mathrm{n} * \mathrm{f}_{\max}} \qquad (2)$$

$$\mathrm{dt} \le \frac{\mathrm{dh}}{\mathrm{h} * \sqrt{3} * \mathrm{V}_{\mathrm{P}_{\max}}} \qquad (3)$$

The model used in this paper represents volume of 200 x 200 x 200 meters with material parameters of global maximum P-wave velocity v_{p_max} = 5500m/s and global minimum S-wave velocity v_{s_min} = 1500m/s. The source wavelet is the Ricker wavelet with centre frequency of 50 Hz giving us a maximum frequency, f_{max} of 100 Hz. Using equation (2) and (3), for the 6th order Holberg FD operator we will have *n*=4.77 and *h*=1.283482: we use a grid spacing of 1m with 200x200x200 gridpoints with a time step of 0.08ms. Note that the Y-coordinate translates to depth and the X- and Z-coordinates are along the two horizontal directions.

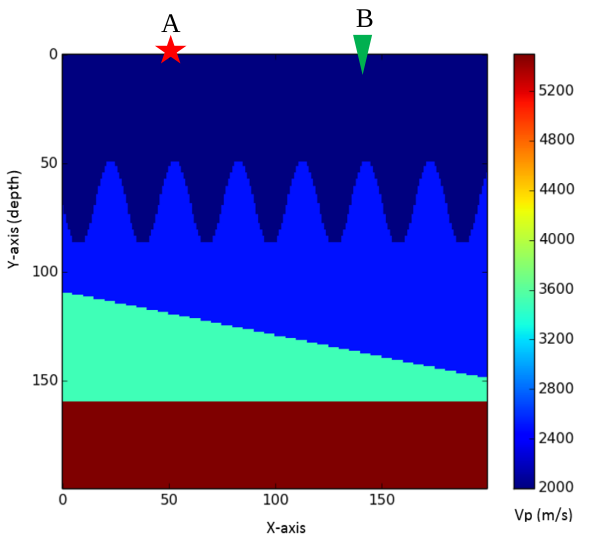

Figure 1. 2D XY slice of the P-wave velocity model. Model is made of grid spacing of 1m spanning a volume of 200mx200mx200m. Local maximum velocity is 5500 m/s and local minimum velocity is 2000 m/s.

The acquisition geometry used in this test consist of a point source and point receiver. SOFI3D modelling code allow the source to be activated in *x*-, *y*- and *z*-direction individually and the receiver records in all three directions. In testing the reciprocity principle, the source and the receiver location will be interchanged. The traces recorded are compared in pairs where the source and receiver location are switched while keeping the radiation direction at the location fixed which gives a total number of 18 traces (9 pairs). For example, we compared traces recorded in X-direction from location B with source at location A with Y-direction force with the trace recorded in location A in Y-direction with source at location B with force in X-direction. This is configuration is illustrated in Figure 2.

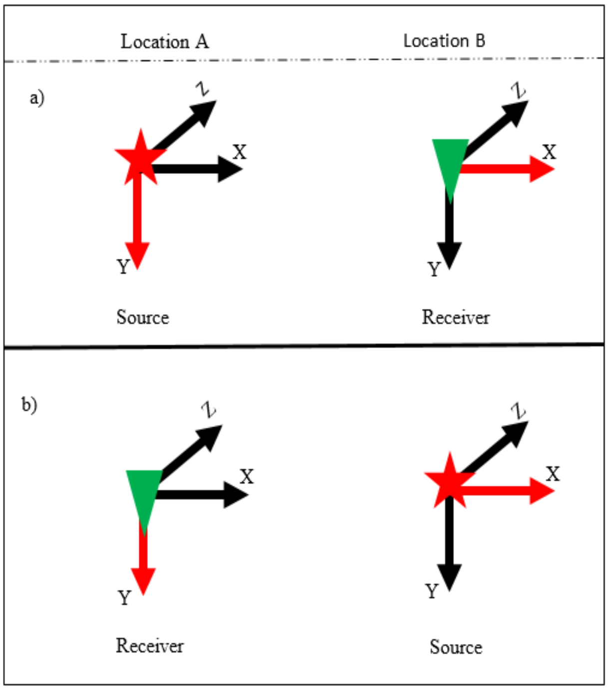

Figure 2. Different configurations for sources and receivers that were used for reciprocity test. The red arrow indicates the direction of force fired and recorded. Trace recorded by configuration in a) was recorded in X-direction at location B while having a source in Y-direction at location A and trace from configuration b) was recorded in Y-direction at location A with a source in X-direction at location B.

The numerical experiments were performed by having the source and receiver at the surface with an offset of 40m. In validating the principle, a 2-layer model was used for simplicity where the reflector is 100m away from the surface. In the first experiment, the source and receiver was placed at the surface of the 2-layer model with the free surface boundary condition. The reciprocity response is shown in Figure 3 where difference in magnitude, phase shift and change in polarity can be easily observed. In the second experiment, the source and receiver are planted at a 50m depth while maintaining the source and receiver distance to the reflector distance and the corresponding response is shown in Figure 4. It is noted that a better reciprocity response was obtain however some traces do exhibit a small change in magnitude and phase.

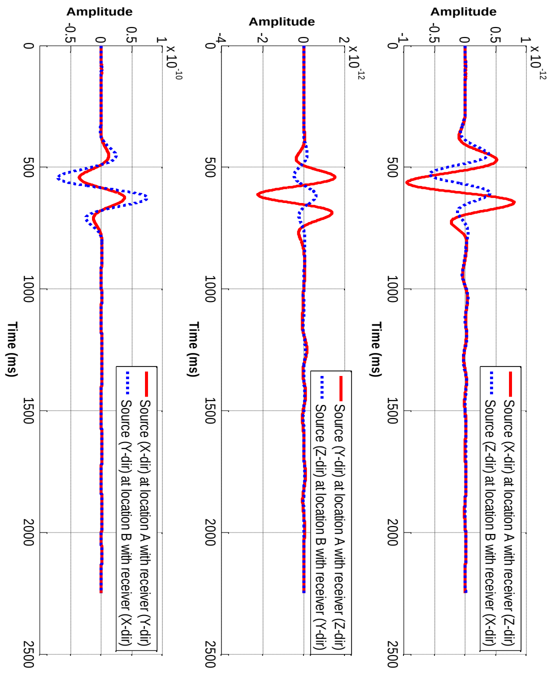

Figure 3. Reciprocity response of the 2-layer model with free surface boundary condition. From examples in (a), (b) and (c), difference in magnitude, polarity and phase are observed.

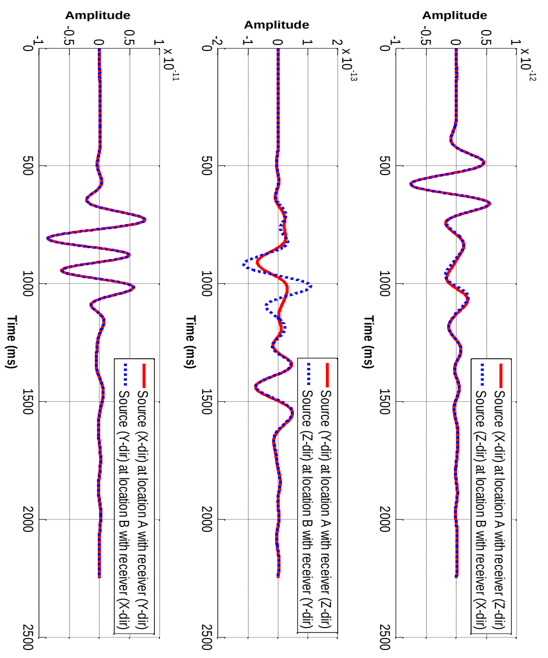

Figure 4. Reciprocity response from source and receiver buried at depth of 50m with free surface boundary condition. From response (b) small difference in magnitude was observed whereas in response (c) a small phase shift was observed in between time 750ms-1100ms.

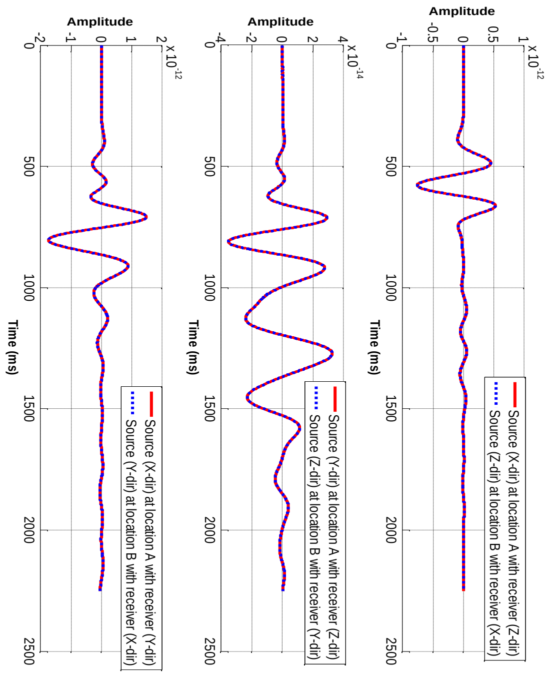

Figure 5. Reciprocity response from 2-layer model with absorbing boundary condition. A good reciprocity relation was obtained.

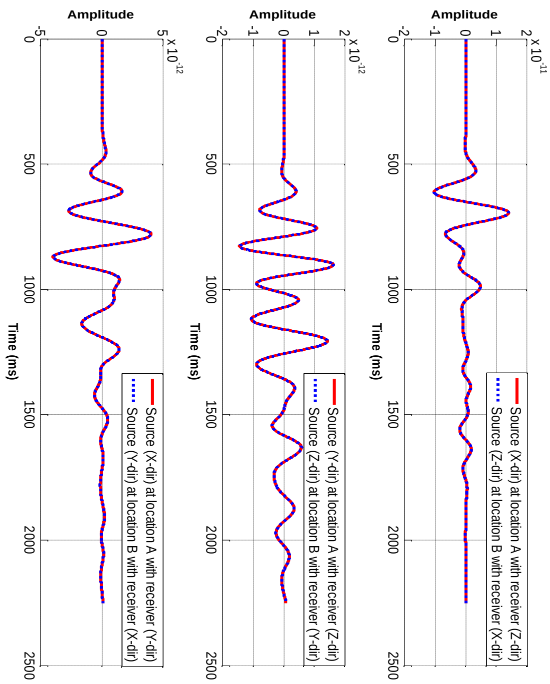

Figure 6. Reciprocity response from 4-layer complex model (Figure 1) with absorbing boundary condition. A good reciprocity relation was obtained.

Absorbing boundary condition was then used on the same model with the same source-receiver configuration. Reciprocity response is shown in Figure 5 where a good match of traces was observed. The experiment was then extended to using a more complex model as shown in Figure 1 and the response is shown in Figure 6.

CONCLUSIONS

In this work we studied the reciprocity principle by studying responses of an elastic medium by means of finite difference numerical modelling. We observed that for surface source and receiver locations, the reciprocity principle was validated with free surface boundary conditions. With the increase in the depth of the source and receiver from the surface, the reciprocity improved, as we observe in Figure 4. In the case of absorbing boundary condition, regardless of the complexity of the model, reciprocity relation is valid.

We attribute this observed discrepancy between the expected theoretical outcomes and the results of the finite difference modelling to the implementation of the free surface condition in the modelling code. Even though we tested this only on one finite difference modelling code, we suspect that similar outcomes could be observed using other codes. Since numerical modelling of seismic wave propagation plays very important role in today's research and industry practice, and the reciprocity is used in a broad range of data processing and imaging methodologies, the outcome of this study should encourage the extra caution when interpreting results from such approaches.

As part of the future research, we will try to establish what specific elements of the finite element modelling exactly causes the observed discrepancy.

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