Three-dimensional Inversion of GREATEM Data: Application to GREATEM survey data from Kujukuri beach, Japan

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SUMMARY

Studies have shown that Grounded Electrical-Source Airborne Transient ElectroMagnetics (GREATEM) is a promising method for resistivity structures investigating in coastal areas, in addition to inaccessible areas such as volcanoes, mountains and deep forest cover. To expand the application of the GREATEM system, a three-dimensional (3-D) resistivity model that considers large lateral resistivity variations is required. In this paper, we present a frequency- domain 3-D electromagnetic (EM) inversion approach that can be applied to time domain data from GREATEM. In the frequency-domain approach, TEM data were Fourier-transformed using a smooth-spectrum inversion method, and the recovered frequency response was then inverted. To deal with a huge number of grids and a wide range of frequencies in airborne datasets, a method for approximating sensitivities is introduced for efficient 3-D inversion. Approximate sensitivities are derived by replacing adjoint secondary electric fields with those computed in the previous iteration. These sensitivities can reduce the computation time without significant loss of accuracy. Firstly, we verified both of our forwarding and inversion solutions. We then applied this approach to the GREATEM survey data from Kujukuri beach, central Japan. The inverted results of the field data are well fit with the previous study results at Kujukuri area, suggesting the applicability of this inversion approach for constructing 3D resistivity models from the GREATEM field survey data in the future.

Keywords: Airborne EM, GREATEM, 3-D forward modelling, 3-D inversion, Frequency-domain inversion.

INTRODUCTION

Application of airborne electromagnetic (AEM) techniques for groundwater monitoring and modeling has increased steadily in the past decade (e.g., Steuer et al., 2009) owing to advances in AEM systems and processing and in inversion methodologies. However, few studies have applied AEM in areas such as lagoons, wetlands, rivers, or bays, and previous studies have mainly focused on bathymetric data (e.g., Vrbancich and Fullagar, 2007). New applications of AEM survey techniques have been introduced in engineering and environmental fields, particularly for studies involving active volcanoes (Mogi et al., 2009). Real subsurface structures are three-dimensional (3-D) by nature. Although one-dimensional (1D) models based on horizontal layers are adequate in many exploration situations, there are also numerous cases, such as for over thrusts, salt domes, and anticlines, where 3-D modeling is required (Hördt et al., 1992). Inversion of transient electromagnetic (TEM) data for three-dimensional (3-D) distributions of conductivity (or its reciprocal, resistivity) can be done directly in the time domain (Sasaki et al., 2015). Wang et al. (1994) introduced an imaging algorithm based on the concept of back-propagation and the explicit finite-difference time-domain (FDTD) method. Oldenburg et al. (2013) developed an efficient 3-D inversion method for large-scale problems with multiple sources, using matrix-factorization software available on massively parallel computing platforms. As an alternative to time-domain inversion, we tested a frequency-domain approach that employs a Fourier transform technique to recover the frequency response from TEM sounding data and perform inversions in the frequency domain.
The purpose of this study is to develop a 3-D inversion approach that can be applied to TEM data from the Grounded Electrical-Source Airborne Transient electromagnetic (GREATEM). We outline our inversion approach that is based upon the 3-D MT inversion algorithm developed by Han et al. (2008). We have modified this algorithm to fit the airborne EM case. Firstly, we tested the efficiency of the inversion approach and we then applied the inversion method in the frequency domain to GREATEM survey data from Kujukuri beach, central Japan. Finally, we have verified the accuracy of our inverted results by comparing them with the 2-D resistivity models obtained from ATM survey at Kujukuri beach (Mitsuhata et al., 2006).

THE FORWARD SOLUTION

The 3D EM forward-algorithm used in this study draws on a staggered-grid finite-difference (SFD) method (Fomenko and Mogi, 2002; Mogi et al., 2011) that involves the special pre-whitening of a large matrix to improve the stability of computation and a devised solver. It is designed for computer calculation of the electric (E) and magnetic (H) field components resulting from secondary EM fields originating from 3D anomalies inducing the primary EM field on a horizontal multi-layer structure. This approach is used to improve the accuracy of models with a high resistivity contrast over a wider frequency range. The analytical expressions for the E and H fields by the electrical finite-length source for a horizontal multi-layer condition were reported by Ward and Hohmann (1988) and shown as Eqs. 4.184–4.190. Time-domain EM responses were computed by the sine or cosine transformation from the frequency-domain data. The range of computing in frequency domain was 10^5 to 10^{-2} Hz and transient time responses were obtained at 10^{-4} to 1 sec. In forward modelling the second order partial differential equations for scalar and vector potential are discretized on a SFD method.

THE INVERSION METHOD

We have modified the 3-D inversion method developed by Han et al. (2008) to fit the airborne EM case. The EM inverse problem is nonlinear with respect to subsurface electrical properties and is generally solved iteratively. The EM inverse problem can be solved as

$$\Delta \mathrm{d} = \mathrm{A}\Delta \mathrm{m} \tag{1}$$

in which $\boldsymbol{\Delta d}$ is a vector of differences between observed and predicted data, $\boldsymbol{\Delta m}$ is a model correction vector, and $\boldsymbol{A}$ is a sensitivity matrix. The objective function $\boldsymbol{\Phi}$ (Sasaki, 2004) is applied to minimise model roughness as following

$$\Phi = \left\|W_d(A\Delta m + \Delta d\right\|^2 + \lambda^2 [\left\|Rm^{k+1}\right\|^2 + \alpha^2 \left\|m^{k+1} - m_b\right\|^2] \tag{2}$$

in which $\boldsymbol{W_d}$ is a diagonal matrix the elements of which are the reciprocal of measurement uncertainties, $\boldsymbol{R}$ is a second-order difference operator quantifying model roughness, $\boldsymbol{m^{k+1}}$ is the (k+1)-th model. To minimise $\boldsymbol{\Phi}$, $\mathbf{m_b}$ can be either a base model or the model of the previous iteration, $\boldsymbol{m^k}$, and $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$ are trade-off parameters; $\boldsymbol{\alpha}$ is fixed at 0.5 and $\boldsymbol{\lambda}$ is selected to yield a model with minimum data misfit at each iteration (Han et al., 2008). Minimising $\boldsymbol{\Phi}$ in Eq. (2) leads to:

$$\begin{bmatrix} W_d A \\ \lambda R \\ \lambda \alpha I \end{bmatrix} \begin{bmatrix} m^{k+1} \end{bmatrix} = \begin{bmatrix} W_d(Jm^k + \Delta d \\ 0 \\ \lambda \alpha m_b \end{bmatrix} \tag{3}$$

in which $\boldsymbol{I}$ is the identity matrix, the iteration is continued until either a specific number of iterations are reached or a root-mean-square (RMS) misfit measure is reduced to an acceptable level. The ***RMS*** data misfit (Han et al, 2008) can be modified as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N/2} \{[ln(A^o / A^p]^2 + w^2(P^o - P^p)^2\}}{N}} \tag{4}$$

in which $\boldsymbol{A}$ and $\boldsymbol{P}$ are the amplitude and phase of $\boldsymbol{Hz}$, respectively, superscripts $\boldsymbol{o}$ and $\boldsymbol{p}$ indicate 'observed' and 'predicted', respectively, and $\boldsymbol{w}$ is a constant controlling the relative importance of the real part to the imaginary part (Sasaki, 2004). An optimum value of $\boldsymbol{\lambda}$ in solving Eq. (3) is searched to minimise R at each iteration (Sasaki, 2004). Three forward modellings were performed at each iteration to find the optimum value of λ.

Approximate sensitivities are derived by replacing adjoint secondary electric fields with those computed in the previous iteration. The sensitivity of the z-component of electric fields, $\boldsymbol{E_z}$, at a measurement site with respect to the conductivity of the m-th block, $\boldsymbol{\sigma_m}$, having a volume of $\boldsymbol{V}$ is given as

$$\frac{\partial H_z}{\partial \sigma_m} = \frac{1}{-i\omega\mu} \int_V E_z \ . \ \tilde{E}_z \, dV \tag{5}$$

$\tilde{E}$ denotes the electric fields produced by the z-directed unit magnetic dipole ($\boldsymbol{Mz}$) located at the measurement site. The sensitivities for the starting homogeneous half-space are used in the first few iterations after that; e.g. at the third iteration the rigorous sensitivity matrix is generated. The rigorous sensitivity matrix is updated in the subsequent iterations using the Broyden's method (Loke and Barker, 1996). These sensitivities can reduce the computation time without significant loss of accuracy.

VERIFICATION OF THE INVERSION METHOD

We verified the efficiency of our developed 3-D inversion approach using a synthetic model similar to that is used in Sasaki et al. (2015) as shown in Figure 1. The model consists of two conductive (5 Ω-m) targets with dimensions of 400 × 800 × 500m buried in a half-space with a resistivity of 100 Ω-m. The tops of the conductive bodies are 200 and 400 m below the earth's surface, respectively. The grounded wire is 3 km long and oriented along x= -1500. All forward responses were computed using a 54 × 59 × 32 mesh with a minimum cell size of 50 × 50 × 10 m. TEM responses were calculated from 35 frequency responses from10^5~10^{-2} Hz. The data is contaminated with Gaussian noise, whose standard deviation is equal to 3% of the datum magnitude. For the inversion, the earth is discretized into 98,496 (54 × 57 ×32) inverse blocks with the smallest being 50 × 50 × 20 m in size, and a 100 Ω-m uniform half-space was used as the initial and reference models. The forward responses and sensitivities were computed using a 58 × 61 × 36 mesh with a minimum cell size of 50 × 50 × 20 m, which is coarser than the mesh used to generate the synthetic data. Figure 2 shows the resistivity images obtained from the frequency domain inversion of the transient magnetic-field response from the synthetic model (Figure 1), with the final data misfit of 3.5 %. As we can see in this figure, both of the shallower and deeper conductive targets (5 − 10 Ω-m) are recovered in a resistive host structure (50 − 200 Ω-m). Those targets appear to reflect areas of conductors in Synthetic model (Figure 1) which can confirm the accuracy of the inverted results.

INVERSION OF GREATEM FIELD DATA

We applied the inversion approach to GREATEM survey data from Kujukuri coastal plain, central Japan (Ito et al., 2011; Abd Allah et al., 2013). In their studies eleven flight lines spaced 200 m apart were carried out. The GREATEM measures the time-derivative of the vertical magnetic field from a grounded wire. The magnetic field responses were recorded with the current both 'off' and 'on.' Only current 'off' time data were used because they were of better quality than those for when the current was 'on'. The waveforms were digitized through a 24-bit AD converter at a rate of 80 microseconds (μs), and 20,000 sets of data were recorded during one cycle of 1.6 seconds (s). For flight safety, the bird was flown at ~100 m height, and the measured data were subsequently processed to reduce noise. The sensor altitude was monitored using a Global Positioning System (GPS) device attached to the bird, and the sensor height above the ground (terrain clearance) was obtained by taking the difference between the sensor altitude and the topographic elevation interpolated from a 50-m-grid digital elevation map provided by the Geospatial Information Authority of Japan (Abd Allah et al., 2013). To invert TEM data of GREATEM in the frequency domain, we used a Fast Fourier transform technique. This technique determined the complex field values at 35 frequencies from 10^5~10^{-2} Hz at each station, as in Mitsuhata et al. (2001). The transformed frequency responses are divided by frequency responses characteristic of the instruments to obtain the actual frequency response of subsurface which then was inverted.

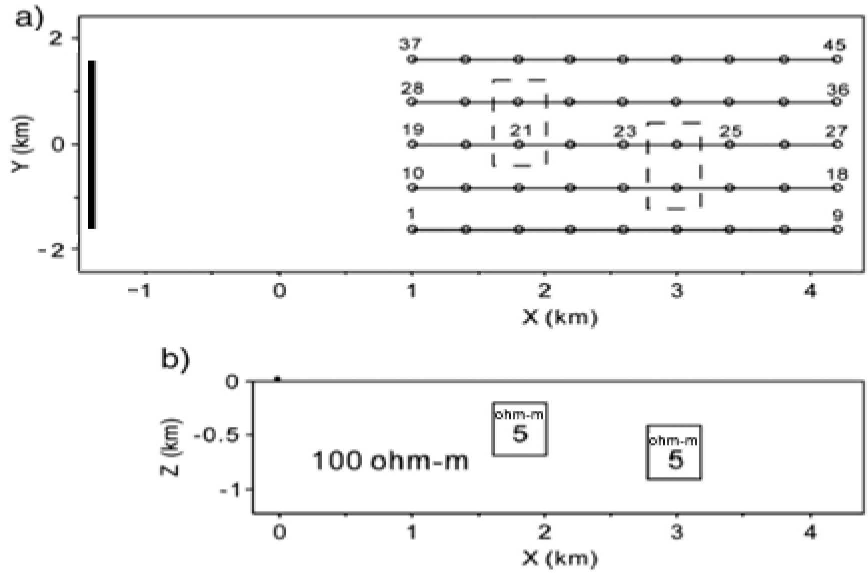

Figure 1. (a) Plan view and (b) vertical cross-section of a 3-Dmodel used to generate the data for inversion. The open circles show the receiver locations for the ground survey. The thin lines show the airborne flight lines that are 150m above the surface. The thick line shows a 3-km grounded source cable. Modified from Sasaki *et al.* (2015).

RESULTS AND DISCUSSION

The inverted 3-D resistivity model of GREATEM field survey data from Kujukuri coastal plain area (Figure 3b) revealed that, the onshore of Kujukuri beach is dominant by a very conductive structure (< 1 Ω-m) which is overlaid by a low resistivity structure (2-10 Ω-m) dominant onshore area. In addition, a high resistive structure (10- 50 Ω-m) that found up to a depth of 400 m. Abdallah et al. (2013) showed that, A high-resistivity landmass (3–25 Ωm) exists down to a depth of ~400 m underlain by a low-resistivity structure (<2 Ωm) which is the characteristic of most of the Kujukuri onshore area. Mitsuhata et al. (2006) performed three different-scale electromagnetic measurements in Kujukuri coastal plain to investigate the distribution of saline groundwater. They reported that on the land side a high-resistivity (~50 Ω-m) surficial zone about 30 m thick is underlain by a low resistivity (~5 Ω-m) zone. They additionally showed that a low-resistivity structure exists behind the highly resistive landmass. Comparing our results with these previous study results as shown in figure 3, the following geological features can be interpreted from current 3-D inversion results.

1- A high-resistivity landmass that consists of highly permeable sand or gravel layers and is part of the recharge area (Mitsuhata et al., 2006) exists down to a depth of ~400 m.

2. A surficial low-resistivity structure exists behind the sandy ridge zone, which is thought to be caused by seawater intruding through and beneath the ridges (Abdallah et al., 2013).

3. A very conductive structure that consists of Quaternary siltstone is characteristic of most of onshore of Kujukuri coastal plain area (Abdallah et al., 2013).

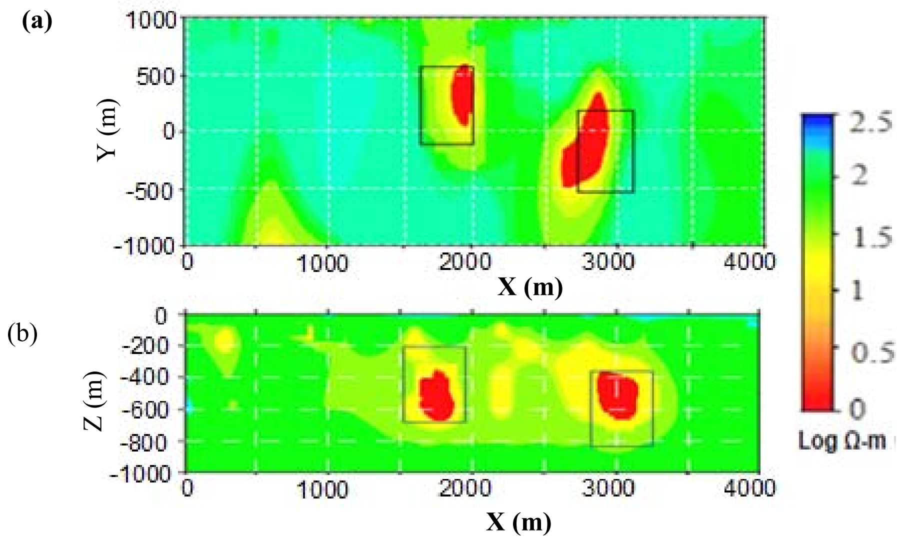

Figure 2. (a) (a) Plan-view map at a depth of 500 m and (b) vertical cross section at y = 200 m of the resistivity models recovered from frequency-domain inversions of the magnetic-field response obtained from the synthetic model shown in Figure 1. The rectangles indicate the locations of conductive bodies.

CONCLUSIONS

Our numerical forward solution is verified by comparing with the 2.5D FEM and 3D SLDM solutions that is shown in Mitsuhata (2000). Furthermore, the inversion method is also verified using a synthetic model similar to that is used in (Sasaki et al., 2015).

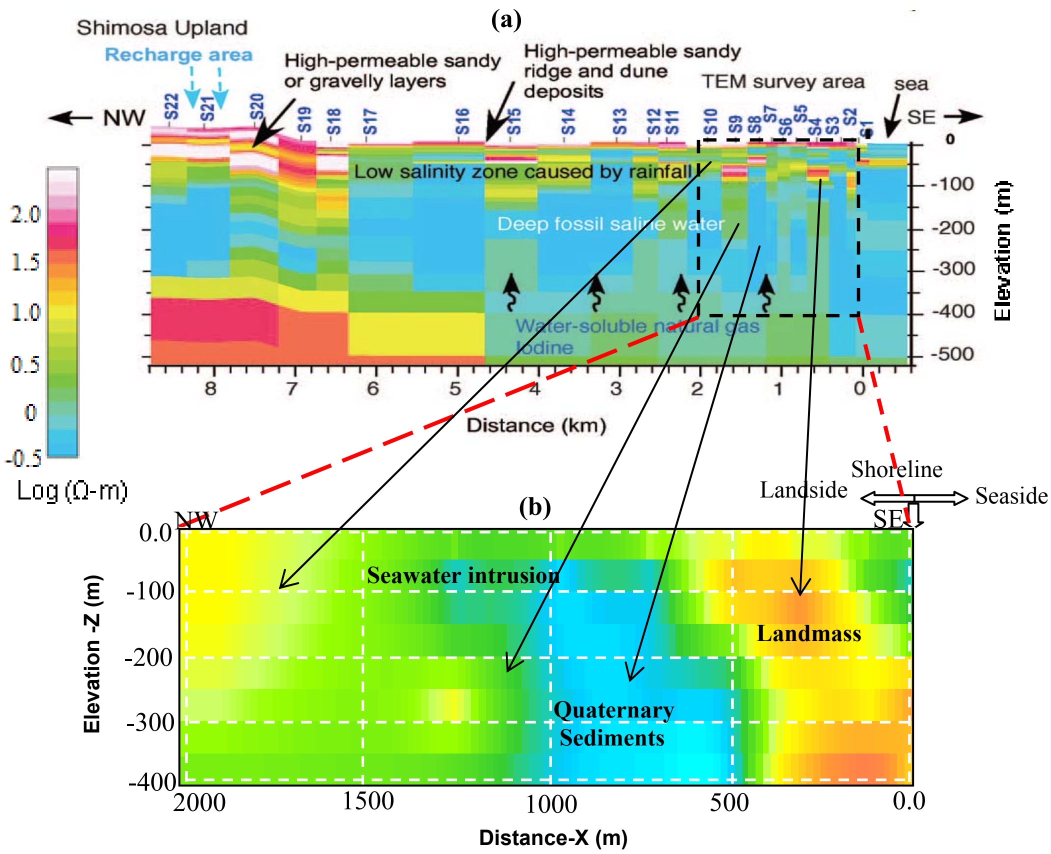

Figure 3. Northwest–southeast resistivity structure profiles comparing (a) the results estimated by 2D inversion of AMT (Mitsuhata et. al., 2006) and (b) the 3-D inversion results along the y= 200 m at Kujukuri coastal plain. Modified from Mitsuhata et. al. (2006).

The inverted results of electromagnetic response from the synthetic model were able to recover conductive zones of potential interest within the resistive region. These conductive zones appear to reflect areas of conductors in Synthetic model. The 3-D inversion results of GREATEM survey data at Kujukuri beach showed that, dominant by a very conductive structure (< 1 Ω-m) which is overlaid by a low resistivity structure (2-10 Ω-m) dominant onshore area. In addition, a high resistive structure (10- 50 Ω-m) that found up to a depth of 400 m. These results are in a good agreement with the previous studies results on Kujukuri area which can confirm the credibility of using the devolved inversion method for constructing 3D resistivity models from the GREATEM field survey data in the future. The computation time required for the frequency-domain inversion is shorter than that required for time-domain inversion, and Fourier transformation takes only a few seconds (Sasaki et al., 2015), hence, the frequency-domain inversion is one of preferable solutions especially long-offset impulse TEM data because of its computational efficiency.

ACKNOWLEDGMENTS

The authors would like to thank the colleagues at Seoul National University for providing the original inversion code that have been modified and used in this study.

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