Inverting Dynamic Elastic Moduli of a Granular Pack to Get Shear Modulus of the Grain

Zubair Ahmed*
Curtin University and DET CRC
Department of Exploration Geophysics
GPO Box U1987, Perth WA 6151
zubair.ahmed@curtin.edu.au

Maxim Lebedev
Curtin University and DET CRC
Department of Exploration Geophysics
GPO Box U1987, Perth WA 6151
M.lebedev@curtin.edu.au

Mahyar Madadi
Curtin University and DET CRC
Department of Exploration Geophysics
GPO Box U1987, Perth WA 6151
mahyar.madadi@curtin.edu.au

Yulia Uvarova
CSIRO and DET CRC
ARRC CSIRO
26 Dick Perry Ave.
Kensington, WA 6151
Yulia.Uvarova@csiro.au

SUMMARY

Elastic moduli of rocks derived from its powder is a new concept and can be applied in practical geophysics studies. To develop this concept, we make ultrasonic velocity measurement on granular packs of quartz sand. We calculate dynamic elastic moduli from that measurement and invert afterwards to find the shear modulus of quartz. The inversion technique follows Extended Walton Model that relies on the grain's contact surface condition between infinitely rough and perfectly smooth. We use different coordination numbers from previous studies (for different samples) in the process of forward modelling and inversion. Forward models have good match with the laboratory measurements both in bulk and shear moduli of the granular pack. Our overall inverted results for the shear modulus are stable and close to actual shear modulus of quartz. However, the coordination numbers that has better match in forward modelling a little bit overestimates shear modulus. On the contrary, the coordination numbers that predicts the higher effective moduli of the pack is giving closest result. As the experiment set up and procedure are simple and robust, this technique can be extended and run in very rigorous situation such as at hard rock drilling rig site to get the elastic properties of the penetrated rocks in real time, where the effective elastic moduli of a grain can be represented as a statistical averaging of elastic moduli of hard rock minerals. This information can be helpful for planning and monitoring the ongoing drilling procedure. It can also be a replacement of solid cores that are missing or damaged for elasticity study.

Key words: Extended Walton Model, granular media, shear modulus, ultrasonic measurement, coordination number

INTRODUCTION

Knowledge of elastic moduli of rocks are important in many research fields including exploration geophysics and especially in seismology to understand the elastic wave propagation. Moreover, in drilling site, drillers and engineers also require to know the mechanical properties of the subsurface rocks for planning and monitoring. Although surface seismic or wireline logging can provide some of the elastic properties, direct laboratory measurement on the core samples can give more precise information. Core samples themselves have some limitations as well. Collecting and analysing of core samples are expensive and time consuming process. Additionally, core samples can be missing or broken that could lead to a gap in understanding of the elastic properties. To overcome these issues, our approach is to find the elastic moduli of the rock from its drill cuttings or powders that come up during drilling. This approach can be time and cost effective and a replacement of core samples for elasticity study.

So far we have found a number of models in literatures that predict the elastic moduli of the granular media. Hertz (1882) derived effective bulk moduli of a granular pack using normal stiffness applying normal compression of two identical spheres. Mindlin (1949) improved the contacts mechanics theory by adding shear stiffness and derived effective shear modulus of the granular pack. Walton (1987) derived equations for effective moduli for two extreme cases where the grain contact's surfaces are a) perfectly smooth having no friction and b) severely rough having infinite friction over the contact points. Predictions for the bulk moduli from Hertz-Mindlin and Walton models are similar and are confirmed in laboratory however shear moduli are over-predicted (Bachrach and Avseth, 2008). Duffut et al. (2010) established models for shear moduli to solve this problem and got consistent results at low confining stress for zero contact friction case. Jenkins et al. (2005) presented a combination of Walton rough and smooth model to get the effective moduli of a granular pack that has a mixture of rough and smooth surface grains. A compilation of effective granular medium models from different studies can be found at Wang and Nur (1992) and Mavko et al. (2009). Fawad et al. (2011) experimented on some samples of different textures (grain size, shape, sorting and mineralogy) of the granular packs and found noticeable correlation between texture and ultrasonic velocities. Sain (2010) identified that grain sorting index has a crucial impact on coordination number (CN) which is defined as the average number of contacts a grain can have with its neighbours in the pack.

CN is an important parameter in effective elastic moduli predictions and is a long standing problem. Several authors tried to find CN as functions of pressure and porosity following different techniques. Makse et al. (2005) obtained CN as a function of pressure for both two and three dimensional cases from simulation results and found a critical minimal value of 4 and 6 respectively. Garcia and Medina (2006) used numerical simulation and got different CN in loading and unloading cycles as a function of pressure and porosity. Zimmer et al. (2007a) rewrote the trend of CN as a function of porosity from the data compiled by Murphy (1982). Dutta et al. (2010) inverted one measurement on known sample and established separate equations for CN to predict P and S wave velocity.

Despite intensive research activity in this area, experiments are limited. Moreover, direct inversion of shear modulus of the individual grain from such experiment has not been reported. To fill this gap, in this paper, we are presenting our results of calculation of bulk and shear moduli of a granular sand pack and the results of inversion of this data in order to obtain the shear modulus of the individual grain.

THEORY

Walton (1987) derives effective elastic moduli of homogenous and elastically isotropic medium of a granular pack having the grains of same spherical size and elastic property under hydrostatic pressure. His derivation is under assumptions that a pair of grain's contact exhibit normal and shear deformation simultaneously and slippage does not occur in partial area but over the whole contact area. He considers two special cases for the grains' contacts surface: perfectly smooth and infinitely rough. Jenkins et al (2005) combine these two cases incorporating a coefficient α which is a fraction of the contact that resembles effective contacts that are perfectly adhered among the grains (i.e. α=0 if the the grains are perfectly smooth and α=1 if the grains are infinitely rough). They express the equations in terms of shear modulus and Poisson's ratio of the constituent grains. We rewrite effective moduli of the granular pack following Jenkins et al. (2005) in terms of Lamé constants of the constituent grain as:
effective bulk modulus,

$$K_{eff} = \frac{1}{6}\left[\frac{3C^2(1-\phi)^2P}{\pi^4B^2}\right]^{\frac{1}{3}} \tag{1}$$

and effective shear modulus,

$$\mu_{eff} = \frac{1}{10}\left[\frac{3C^2(1-\phi)^2P}{\pi^4B^2}\right]^{\frac{1}{3}}\left[1+\frac{3\alpha B}{2B+A}\right] \tag{2}$$

Where,$A = \frac{1}{4\pi}\left(\frac{1}{\mu} - \frac{1}{\mu+\lambda}\right)$, $B = \frac{1}{4\pi}\left(\frac{1}{\mu} + \frac{1}{\mu+\lambda}\right)$, and P, ϕ, C and α are hydrostatic pressure, porosity, coordination number and fraction of the contacts respectively.

Walton built these models taking the ratio between average incremental stress and average incremental strain which corresponds to static elastic moduli (static moduli). Zimmer (2003) found that the static and dynamic elastic moduli of a granular media can differ from each other. To make the experiment setup simple and readily applicable to the rig site, we are considering the effective elastic moduli from the models are also compliant with the dynamic moduli. We are also considering the grains have the same spherical size and the same elastic properties.

In terms of P and S wave velocities, dynamic bulk and shear modulus of granular pack is expressed as

$$K_{eff} = \rho_S(1-\phi)\left(V_P^2 - \frac{4}{3}V_S^2\right) \tag{3}$$

and

$$\mu_{eff} = \rho_S(1-\phi)V_S^2 \tag{4}$$

where,V_P, V_S and ρ_s are primary wave velocity, shear wave velocity and grain density respectively. Combining equations (1) and (3) we can get B as

$$B = \frac{1}{6\sqrt{2}}\left[\frac{CP^{\frac{1}{2}}}{\left[\rho_s\left(V_P^2-\frac{4}{3}V_S^2\right)\right]^{3/2}\pi^2(1-\phi)^{\frac{1}{2}}}\right] \tag{5}$$

Furthermore, we can derive A from equations (2) and (4) as

$$A = \frac{3\alpha BM}{\rho_s V_S^2(1-\phi)^{\frac{1}{3}} - M} - 2B \tag{6}$$

where, $M = \frac{1}{10}\left[\frac{3C^2P}{\pi^4B^2}\right]^{\frac{1}{3}}$. Now, from A and B, we can find the grain shear modulus as

$$\mu = \frac{1}{2\pi}\left(\frac{1}{A+B}\right) \tag{7}$$

EXPERIMENT SETUP AND METHOD

We place the granular sample inside a cylindrical chamber in which both sides are closed with adjustment cylindrical pistons made from PEEK plastic (Figure 1). A pair of 1 MHz shear wave transducers is attached at the ends of the PEEK cylinders. We use a manual hydraulic pump to control the stress through this piston in vertical direction. Ultrasonic waves generated by source transducer using a wave pulser propagate through cylinders and the sample are recorded by a receiver transducer. The travel times of the waves were acquired using a digital oscilloscope.

We have used moderately to well sorted sand sample (Figure-2) totally composed of quartz so that the conditions for the models namely, same elastic properties of the grains and the same grains size, are met. At first step we demoisturise sample by putting it inside the oven and keeping at 105°C for 6 hours. Then we place sample into the cylindrical chamber and measure the ultrasonic travel time for both primary and shear wave.

We use stresses up to 25 MPa taking an interval of 2.5 MPa with three loading and two unloading cycles shown in Figure-3. We have found that during the second loading phase the sample is decreasing in length with increasing stress but at the third loading phase, sample inside the chamber is not experiencing any change in length (Figures-4a and 4b). The reason is that the sample is not regaining its previous length during the 2nd unloading cycle followed by the compaction during 2nd loading cycle. Therefore, we prefer to use the data measured at 2nd loading cycle in the calculation.

From the laboratory measurements of P and S wave velocities, porosity and density with confining stress, we calculate dynamic bulk moduli using Eqs (3) and (4). To build a forward model we fit that data with Extended Walton Model from Jenkins et al. (2005) using the coordination numbers from the existing literatures. We apply the same procedure for dynamic shear moduli using an appropriate fraction of contact parameter, α. Then we run the inversion using the same α to get the shear modulus of the grain.

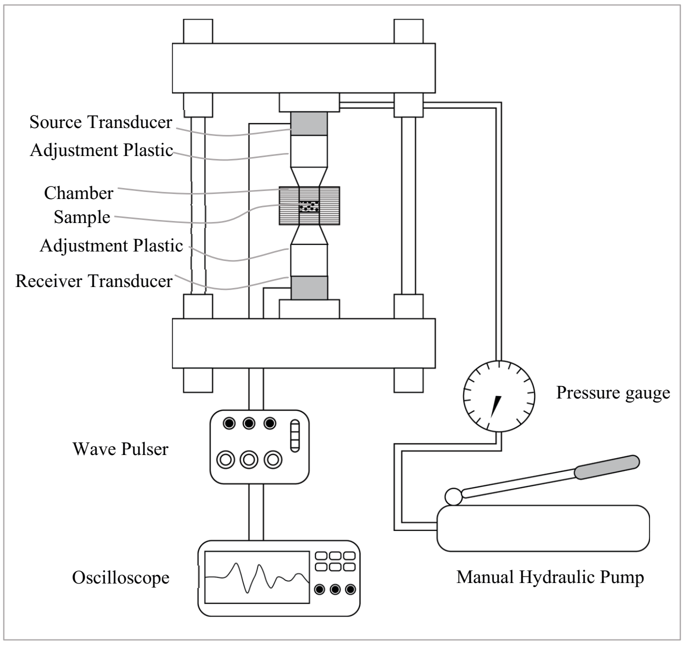

Figure-1: Basic equipment used in the experiment.

Figure-2: Micro-CT of Esperance Beach quartz sand.

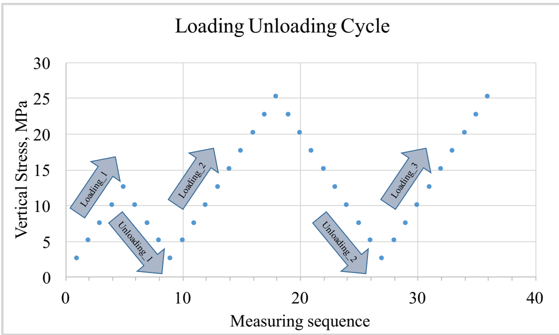

Figure-3: Loading–unloading procedure

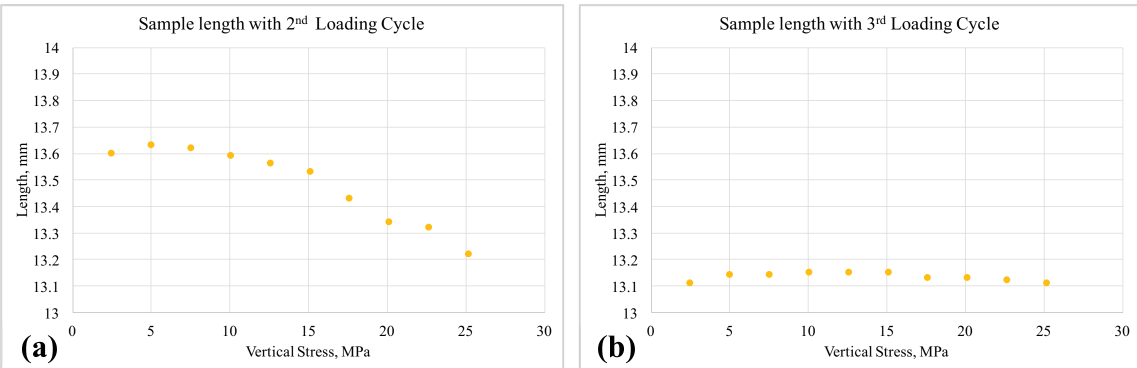

Figure-4: Sample length with increasing stress in (a) 2nd loading cycle and (b) 3rd loading cycle

RESULTS AND DISCUSSION

Figures-5a and 5b show the dynamic effective bulk and shear moduli of the sand pack calculated using different models. Taking the typical elastic parameters of quartz such as shear modulus as 44 GPa and Lamé first parameter, λ as 8 GPa, by using equation (1) we build the models with their respective coordination number adopted from previous studies listed at Table-1. We also use an arbitrary non stress dependent constant coordination number 8. We build the models for shear moduli of the sand pack using equation (2). To get the best fitting with the overall models, we take α as 0.2. We then inverted the dynamic moduli using equations (5), (6), and (7) to get shear moduli (Figure-6a) of the constituent grain. Normal distributions of the inverted results for each model is presented in the Figure-6b.

Although CNs listed in Table-1 are derived using different techniques such as numerical simulation (Makse et al. (2004), and Garcia and Medina, (2006)), inversion (Dutta et al. (2010) from dynamic data, and empirical fitting (Zimmer et al. (2007) of the observations from non-identical grain sizes compiled by Murphy (1982), we are keen to test the affect of those variety of pressure and porosity dependant CNs on our inversion result.

Vertical Stress, MPa	Makse et al., 2004	Gercia and Medina, 2006	Zimmer et al. 2007	Dutta et al., 2010
2.5	7	4	8	8
5.0	7	4	8	8
7.6	7	4	8	9
10.1	7	4	8	9
12.6	7	4	8	9
15.1	7	4	8	10
17.6	7	4	8	10
20.2	7	4	9	10
22.7	7	4	9	10
25.2	7	4	9	10

Table-1: Coordination number from existing literatures.

Model from an arbitrary constant CN 8 throughout the stress range has a very good match with dynamic bulk and shear moduli. Model from Zimmer et al. (2007) has also a very good match with the dynamic bulk moduli as long as it contains CN=8 up to 17.5 MPa. Models from other CN values are either over or under predicted depending on whether the CNs is more or less than 8 respectively.

Inversion shows stable results throughout the stress range for all the CNs. At the same time results are close to each other within the coordination numbers ranging from 7 to 10. Although CN from Dutta et al. (2010) gives overpredicted models in both effective bulk and shear moduli, the inversion result is the closest to the actual shear modulus of quartz. The better fit models from arbitrary CN=8 and Zimmer et al. (2007) have the closer values of shear modulus of 50.5 GPa and 49 GPa respectively.

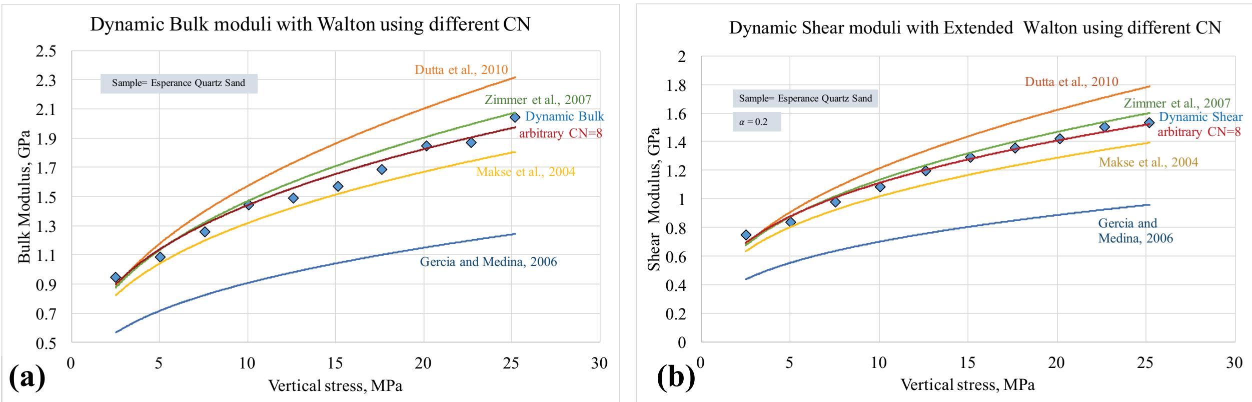

Figure-5: Dynamic elastic moduli of quartz sand pack with Walton model using different coordination numbers (a) dynamic bulk modulus (b) dynamic shear modulus.

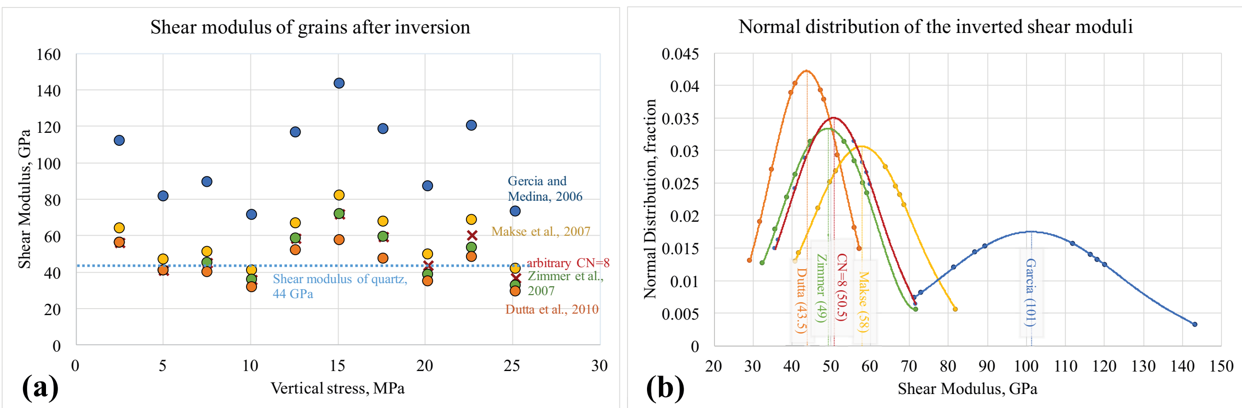

Figure-6: (a) Shear modulus of quartz grain after inversion using different coordination numbers. (b) Normal distribution of the inverted results.

CONCLUSION AND FURTHER STUDY

The inversion technique works well to predict the shear modulus of quartz in the Esperance Beach sand pack sample. We use several coordination numbers (CN) obtained from the literature and found that results of the inversion do not dependent on the CN if CN is within the range 7 to 10.

There are several things that we are considering to improve with this technique by

- restricting the stress range up to 12.5 MPa so that the coordination number will not be stress dependent.
- modifying the steps of the ultrasonic measurement so that the last loading cycle would be followed by a couple of loading and unloading cycles of lesser stress level.
- trying to run same experiment with the same type of sample at least three times to see whether microstructure of the pack can cause a big impact on the results.

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