Dealing with uncertainty in AEM models (and learning to live with it)

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SUMMARY

Interpreting inversions and modelling airborne electromagnetic (AEM) data is ambiguous. Assessment on the degree of certainty of how representative a selected model is, always reassuring. Geophysicists assessing AEM models are often faced with the conundrum of determining a single 'best', 'right' and geologically sensible model from of all the possible solutions. This paper explores the characteristics of several acceptable models, without being concerned with details of any particular one.

Geoscience Australia's reversible jump (trans-dimensional) Markov chain Monte Carlo (rj-MCMC) is a stochastic algorithm which has enabled the sampling of thousands of plausible models that fit the data at each individual location. Through the statistical analysis of these ensembles of models, a measure of uncertainty and a probable distribution of conductivities at that depth can be derived.

On most occasions, single 'best' solutions fromm deterministic inversions are found to be reasonable representations of the whole suite of models recovered by the MCMC. But the importance of exploring multiples models and their limitations resides on trying to extract what information can actually be determined from the data, information which often cannot be given by a single best model.

Key words: airborne electromagnetics, hydro-geophysics, inversion, groundwater, northern Eyre Peninsula

INTRODUCTION

The northern Eyre Peninsula, located in South Australia is experiencing increasing mineral exploration and development, and access to water is becoming a fundamental requirement to support them. The current understanding of the spatial distribution and groundwater quality of the aquifer systems is very limited. The work presented in this paper is part of a much broaderder report which describes the process of inverting legacy airborne electromagnetic datasets (AEM) originally acquired by mineral exploration companies (Ley-Cooper *et al*., 2015). Accurately inverting the AEM data is one of the crucial steps in obtaining quantitative hydro-geological information from these datasets, which can then be used for groundwater modelling and management.

The AEM modelling literature (e.g. Christiansen *et al*. 2011; Viezzoli *et al*. 2013, Ley-Cooper and Munday 2013) shows there are several inaccurate elements of AEM data acquisition and processing, that get both contribute to the uncertainty of deriving a single model. All models derived from AEM inversions come from the solution of a non-linear problem, hence are non-unique solutions. This means there is the potential to generate a variety of models that fit the measured data and that are theoretically correct.

A VTEM system (Whitely *et al*., 2004) was flown on the west coast of the northern Eyre Peninsula in South Australia (between Elliston and Venus Bay) (Figure 1). In this area, the water supply comes from sources of local isolated freshwater lenses located in the Quaternary aquifers. Over extraction of ground water and natural coastal saltwater wedges have important boundaries that need to be delineated, targets achievable with AEM data. This example is used to illustrate the conundrum that geophysicists are often faced with when dealing with uncertainty from inverting AEM data and selecting a single 'Best' (most representative) model.

Interpreting inversions of AEM data is intrinsically ambiguous, this ambiguity can be exacerbated with extra unknowns such as ill-defined system specifications (often an issue when dealing with legacy data). When assessing and interpreting modelling results, the key concern is to try and determine which of models are representative. Interpreters will always want to explore as many acceptable models as possible and not worry about the particularities of any given one, whilst retaining some certainty regarding their selection.

METHOD

Layered-earth deterministic inversion

The data was inverted using Geoscience Australia's sample-by-sample time domain (GALEISBSTDEM) inversion algorithm, which has the capability of solving for layer conductivities, layer thicknesses, and the AEM system's geometry parameters (Brodie, 2010). The algorithm is a 1D inversion which has inbuilt an iterative gradient-based optimization function.

The inversion is deterministic, which means that every time the program is executed, it looks for a 'best' model, it outputs a single optimum solution. The program attempts to fit the measured data given the imposed regularization constraints and established permissible noise levels.

For the dataset in Figure 1, information about the altitude of the transmitter and the instrument's noise levels were not available. Therefore a series of trials and assumptions regarding noise-levels and system geometry were used to derive a set of what appear to be adequate and credible parameters. This was assessed through multiple inversions from a single line of data (from a flight line in the northwest part of this survey; highlighted in red on the map in Figure 1).

The data was inverted solving for a 5, 12 and 30-layer models. For the 30-layer case, the thickness of each layer is fixed and the values of conductivity are resolved by the inversion. The same line was then inverted with different smoothness constraints, a parameter that determines the level of vertical variation or transition between subsequent layers. Results are compared as stack in Figure 2.

The recovered sections show distinct vertical variations. The most striking contrast is between the joltier less-smooth bottom section E) and the very smooth slowly varying section on panel C).

The option of solving for both layer-thickness and conductivity poses a higher degree of difficulty when choosing a representative model. The top two panels A) and B) in Figure 2 show results now solving for a (less-layer model) 12-layer and a 5-layer model. As in the previous example with the smoothness variations panels C), D) and E), there are also some differences between the derived models, but the overall fits suggest all sections are equally valid.

One of many common features for all sections is the conductive wedge on the right-hand side of the section. The line is mapping the front of a saltwater intrusion in the southernmost part of the profile (Figure 2). The differences between the sections are many but most notable, is one between 2000-3000 m from Location 2. At this location the presence of a less conductive feature (potentially a fresh water lens) seems to be mapped best in D) and E) but is barely present on the top sections C) which has smoother constraints imposed and on A) and B) sections from the few-layered models.

Markov chain Monte Carlo MCMC stochastic inversion

As previously mentioned, in the modelling of AEM data requires several decisions (such as starting models, smoothness constraints, regularization, number of layer selection) that need to be taken. These, plus inaccurate system description and processing all get compounded and contribute to the uncertainty of deriving a single model solution.

In this context the same line highlighted in Figure 1 has been chosen to illustrate one way of dealing with uncertainty. The line was inverted using Geoscience Australia's reversible jump (trans-dimensional) Markov chain Monte Carlo (rj-McMC) algorithm. The algorithm inverts each sample independently, and calculates the expected theoretical AEM system response over a given 1D layered-earth conductivity model.

The principle behind the rj-McMC inversion algorithm is to construct an ensemble of thousands of 1D conductivity models at each AEM survey location. Each model is described by layer-conductivity and thickness values. It follows MCMC sampling rules and uses prior information. The number of layers used to describe each model is a free parameter, allowing for significant flexibility in the model parameterization. By allowing the data to estimate the necessary number of layers, errors associated with over- or under-fitting data and incorrect assumptions about model structure reduced.

Through the statistical analysis of the ensemble we get insights of the level of uncertainty of inversion results and non-uniqueness of the derived models. A more detailed description of method and can be found in Malinverno (2002), Bodin and Sambridge (2009), Bodin et al. (2009), and Minsley (2011). The particular Trans-dimensional rj-MCMC algorithm used for this work is described in Brodie and Sambridge (2012), and Brodie and Richardson (2013).

RESULTS

With over 2000 sounding locations along the line and 20 000 models per location, Figure 3 is a summary mean conductivity-depth section imaged by stitching the mean model from each location. It final construction comes from the analysis of 40 million models. On the top panel the mean and average misfit at each location are plotted as a profile in black and red respectively.

The conductive wedge on the right-hand side of the section, a common feature in all the previous inverted sections (Figure 2), is also present but now portrayed as a thicker body. This shows the range of models that identify the wedge as an interface starting at ~50 m depths. The presence of the less conductive features at Location 2, which was described as a potentially a fresh water lens in the interpretation of Figure 2, seems to be better mapped and its extent better defined in Figure 3.

To further inspect the MCMC results and compare its results to those derived from the deterministic inversions, the same four locations flagged in section in Figure 2 and Figure 3, have been plotted as individual soundings in Figure 4, and are analysed in greater detail. The four locations (one in each panel in Figure 4) show a range of probable distributions of conductivities as a function of depth. These summary models represent:

a) The grey shading is derived from 20 000 models
b) The 10th percentile conductivity and the 90th percentile conductivity (black lines), i.e. 10% of models have a lower conductivity and 10% of models have a higher conductivity respectively
c) Mode or most frequently occurring conductivity (dark green)
d) Mean or average log-conductivity (orange)
e) Median or 50th percentile conductivity (dark blue)

f) The cyan, pale, green and pink curves correspond to the 30, 5 and 12-layer deterministic models from GALEISBSTDEM inversion

The plots shows that that the deterministic inversions accurately recover the geo-electrical structure, are within the range of credible models. They appear to be mostly just as valid as the mean or mode models, and can be generally deemed to be representative of the whole suite of models at these locations.

The dotted cyan line shows an estimated depth of investigation (DOI), as suggested in Christiansen and Auken (2012), at each location determined from the 30-layer smooth inversion. It is worth noting that the DOI is model dependent and can vary up to hundreds of meters for two same equally-valid models at the same location.

The spread bound between the 10th and 90th percentiles shows the wide range of conductivities from models that fit the data. The range at any given depth of this spread can be viewed as one measure of uncertainty (i.e. the resolution of the conductivity at that depth). It is clear that for these four soundings the resolution at 50 m depth is better than that at 200 m depth, essentially because the spread is wider at depth. The right panel (location 4) shows the broad range of conductivity that can fit the data below ~200 m. The layer can be both resistive (1 mS/m) and conductive (10 000 mS/m), which is a good indication that conductivity values at that depth are not actually resolvable. Also important to note is the mode (dark green curve) at this location 4, below 100 m depth it is outside the 10-90% region. The algorithm seems to be frequently proposing a conductor at depth to fit the data, it is most probably been influenced by too much waiting (inaccurate the noise model) in the late-time gates.

CONCLUSIONS

Results from this work show that over-all, robust deterministic inversions like the GALEISBSTDEM can accurately recover and reasonably represent the geo-electrical structure of the underlying ground. And although on most occasions single 'best' solutions from deterministic inversions are reasonable approximations of the whole suite of models recovered by the MCMC, they unfortunately don't provide full uncertainty quantification.

The GArjMCMC process on the other hand samples thousands of plausible models, and at the expense of being computationally expensive, it enables a broader exploration of the model characteristics consistent with measurements and does not focus on any one particular model. It determines which models are more representative of the data. The high number of models permits a statistical analysis, which is used as a quantification instrument to a measure the degree of uncertainty and the probable distribution of conductivities at that depth.

The importance of exploring a great number of plausible models resides on assessing what information can actually be determined from the data and its limitations, information which frequently cannot be given by a single best-model.

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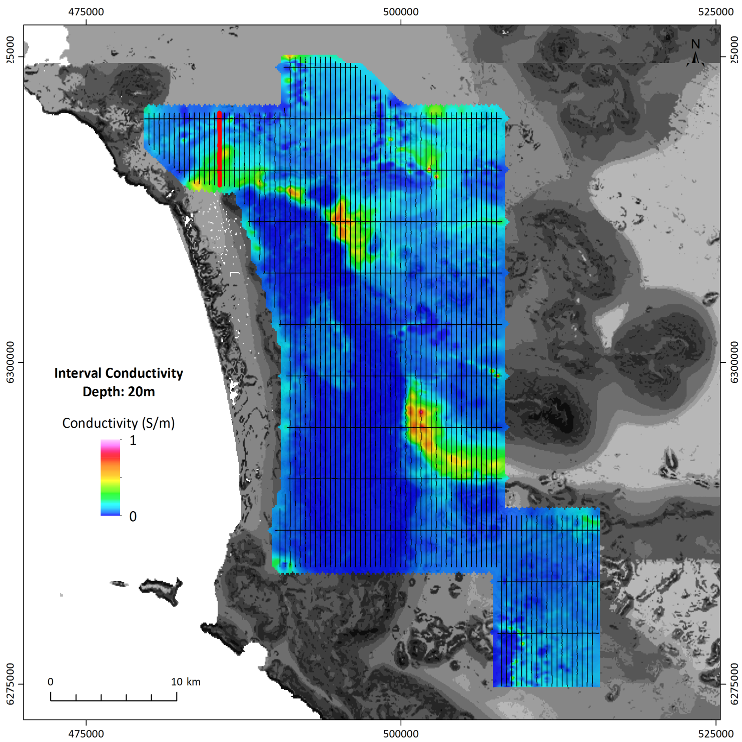

Figure 1 The flight-line distribution and a derived map of inverted conductivities at a depth of 20 m below surface have been draped on a terrain valley bottom flatness index (MrVBF: Gallant and Dowling, 2003). The paler grey colours are indicative of depositional environments such as low flat areas in the landscape (valleys and plains). Dark areas are indicative of erosional features in the landscape where terrain steepens and outcrop is more likely. Darker grey areas are steeper and higher parts of the landscape.

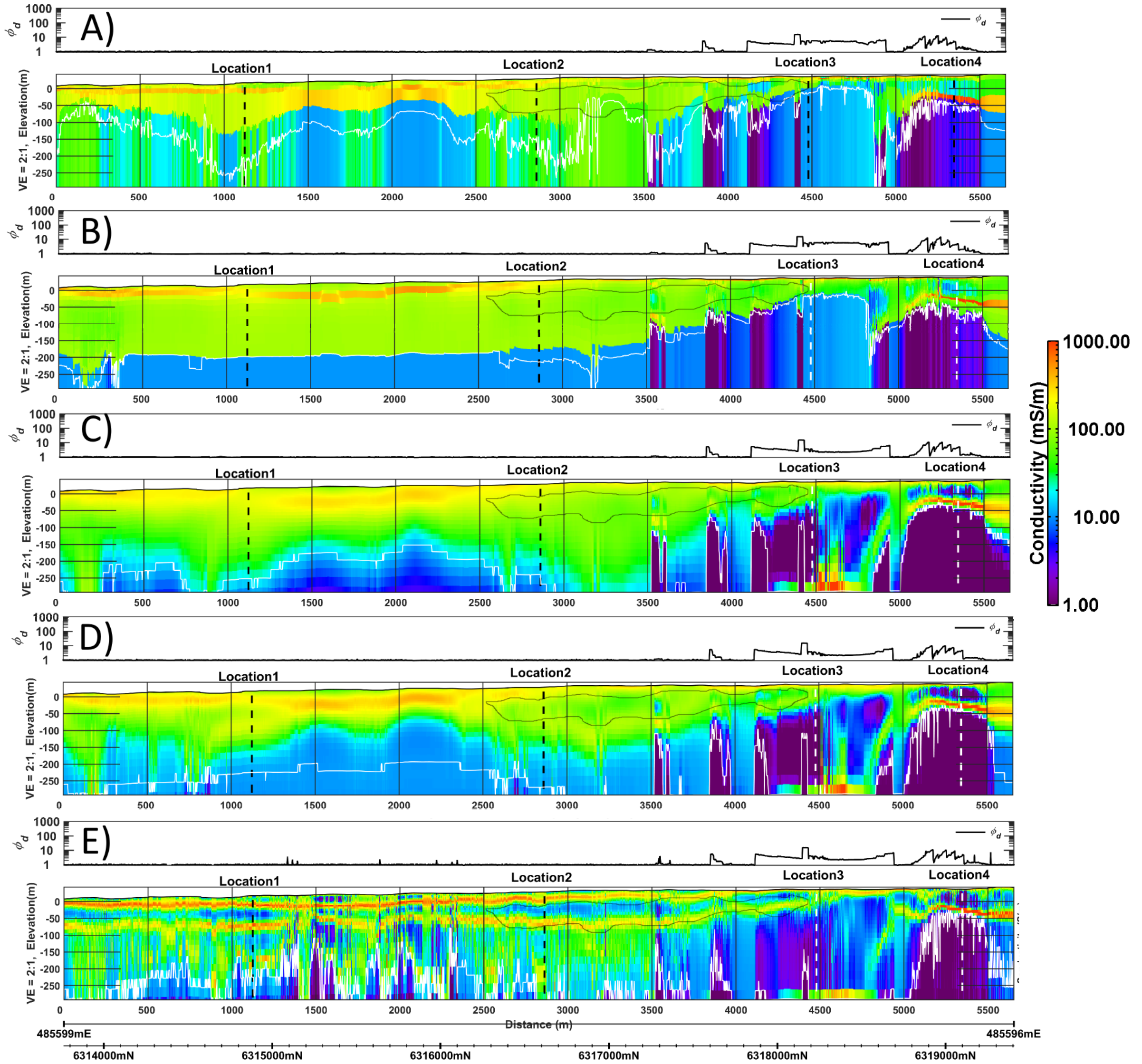

Figure 2 Inverted sections from same line of AEM data; A) Solving for conductivity and thickness of a 5 layer model and B) for a 12 layer model. The three bottom panels have a different vertical conductivity smoothing, C) has the highest smoothness; D) and E) are progressively less smooth. The faint white line at the bottom of each section is an estimated depth of investigation calculated at each location. Every section is accompanied by misfit parameter ϕ_d calculated at each sounding point and plotted as a profile. Numbers closer to 1 show a better correspondence between the proposed model and the measured data.

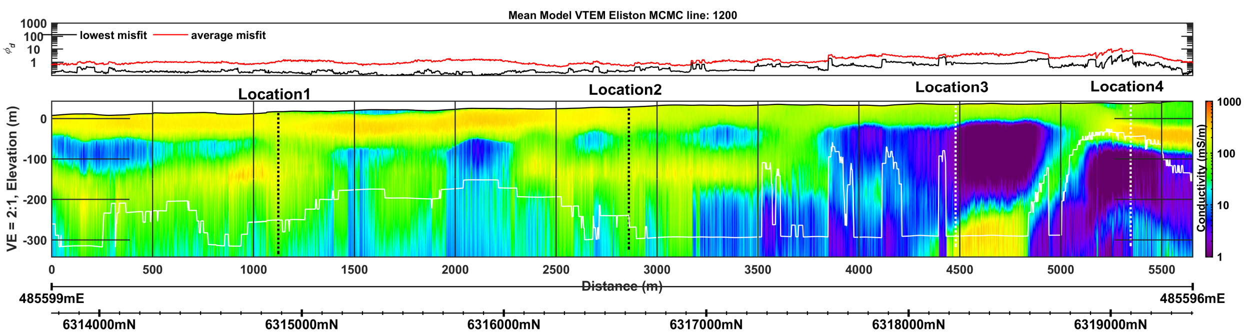

Figure 3 Profile section of mean conductivity for VTEM Line 1200 from Mt Elliston, generated from and MCMC algorithm. Four locations have been selected to further inspect the models. The faint white line at the bottom of the section is the same estimated depth of investigation calculated from the smooth deterministic 30 layer inversion, plotted for reference.

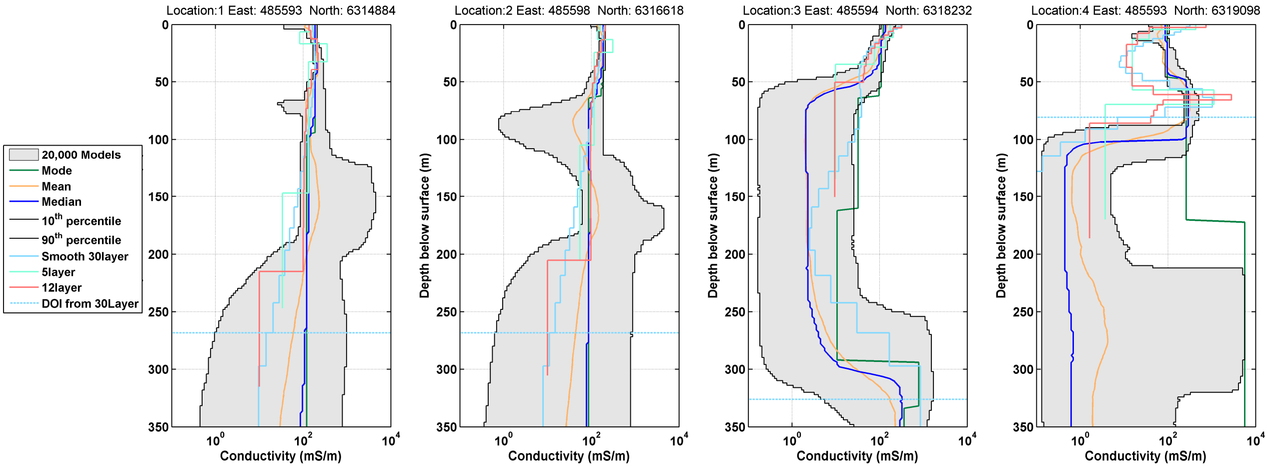

Figure 4 Four locations along flight 1200 allows evaluation of the results from 20 000 models, all of which can emulate and fit the measured data.