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Global Colour Model of QCD and Its Relationship to the NJL Model, Chiral Perturbation Theory and Other Models*

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Abstract

The Global Colour Model (GCM) of QCD is a very successful model. Not only is it formally derivable from QCD but under various conditions it reduces to the NJL model and also to Chiral Perturbation Theory, and to other models. Results presented include the effective gluon propagator, the difference between constituent and exact quark propagators, various meson and nucleon observables, a new form for the mass formula for the Nambu–Goldstone mesons of QCD, and the change in the MIT bag constant in nuclei.
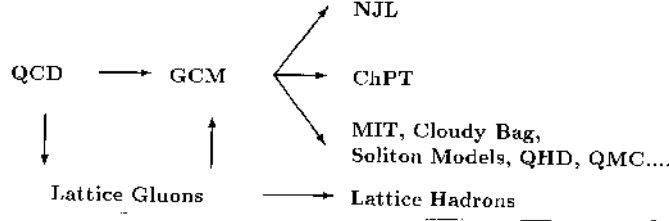
1. The Global Colour Model (GCM)

The GCM can be formally derived from QCD [1]. The GCM is in turn easily related to a number of the more phenomenological models of QCD as indicated in Fig. 1. QCD and models for QCD amount to the calculation of correlation functions:

$$\mathcal{G}(..,x,...) = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}\mathcal{A}....q(x)..... \exp(-S_{qcd}[A,\overline{q},q]) \,. \tag{1}$$

At low energies we expect that only hadronic correlations are directly observable, and we expect (1) to be transformable into a hadronic form, equivalent to the effective action description of low energy nuclear physics:

$$\int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}A \exp(-S_{qcd}[A,\overline{q},q] + \overline{\eta}q + \overline{q}\eta)$$

$$\approx \int \mathcal{D}\pi\mathcal{D}\overline{N}\mathcal{D}N... \exp(-S_{had}[\pi,...,\overline{N},N,..] + J_\pi[\overline{\eta},\eta]\pi + ..) \,. \tag{2}$$

Fig. 1. Relationship of the GCM to QCD and other models.

If in (1) we formally do the gluon integrations, after fixing a gauge (ghosts not shown), we get

$$\int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}A \exp(-S_{qcd}[A,\overline{q},q] + \overline{\eta}q + \overline{q}\eta) = \int \mathcal{D}\overline{q}\mathcal{D}q \exp(-S[\overline{q},q] + \overline{\eta}q + \overline{q}\eta), \quad (3)$$

leaving an action for quarks of the form

$$S[\overline{q},q] = \int \overline{q}(-\gamma.\partial + \mathcal{M})q + \tfrac{1}{2}\int j_\mu^a(x)j_\nu^a(y)D_{\mu\nu}(x-y)$$

$$+ \frac{1}{3!}\int j_\mu^a j_\nu^b j_\rho^c D_{\mu\nu\rho}^{abc} + ..., \quad (4)$$

where $j_\mu^a(x) = \overline{q}(x)(\lambda^a/2)\gamma_\mu q(x)$, and $D_{\mu\nu}(x)$ is the exact pure gluon propagator

$$D_{\mu\nu}(x) = \int \mathcal{D}A A_\mu^a A_\nu^a \exp(-S_{qcd}[A,0,0]). \quad (5)$$

A variety of techniques for computing $D_{\mu\nu}(x)$ exist, such as Dyson–Schwinger equations (DSE) [2], lattice simulations [3, 4] and, as discussed in other papers in this volume, monopole and instanton modellings. Dropping the $D_{\mu\nu\rho}^{abc},..$ defines the GCM. This is equivalent to using a quark–gluon field theory with the action

$$S_{gcm}[\overline{q},q,A_\mu^a] = \int \left(\overline{q}(-\gamma.\partial + \mathcal{M} + iA_\mu^a\frac{\lambda^a}{2}\gamma_\mu)q + \tfrac{1}{2}A_\mu^a D_{\mu\nu}^{-1}(i\partial)A_\nu^a \right), \quad (6)$$

where $D_{\mu\nu}^{-1}(p)$ is the matrix inverse of $D_{\mu\nu}(p)$. This action has a global colour symmetry, hence the description as the GCM. Hadronisation [1] of the GCM involves a sequence of functional integral calculus changes of variables involving, in part, the transformation to bilocal meson and diquark fields, and then to the usual local meson and baryon fields:

$$\int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}A \exp(-S_{qcd}[A,\overline{q},q]) \quad (7)$$

$$\approx \int \mathcal{D}\overline{q}\mathcal{D}q \exp(-\int \overline{q}(-\gamma.\partial + \mathcal{M})q + \tfrac{1}{2}\int j_\mu^a(x)j_\nu^a(y)D_{\mu\nu}(x-y)) \quad (8)$$

$$= \int D\mathcal{B}D\mathcal{D}D\mathcal{D}^\star \exp(-S[\mathcal{B}, \mathcal{D}, \mathcal{D}^\star]) \qquad \text{(bilocal fields)} \qquad (9)$$

$$= \int \mathcal{D}\pi \mathcal{D}\overline{N}\mathcal{D}N... \exp(-S_{had}[\pi, ..., \overline{N}, N, ..]) \qquad \text{(local fields)} \qquad (10)$$

The derived hadronic action, to low order in fields and derivatives, has the form

$$S_{had}[\pi, ..., \overline{N}, N, ..] =$$

$$+ \int d^4x \left[\frac{f_\pi^2}{2}[(\partial_\mu \pi)^2 + m_\pi^2 \pi^2] + \frac{f_\rho^2}{2}[-\rho_\mu \Box \rho_\mu + (\partial_\mu \rho_\mu)^2 + m_\rho^2 \rho_\mu^2] \right.$$

$$+ \frac{f_\omega^2}{2}[\rho \to \omega] - f_\rho f_\pi^2 g_{\rho\pi\pi} \rho_\mu.\pi \times \partial_\mu \pi - i f_\omega f_\pi^3 \epsilon_{\mu\nu\sigma\tau} \omega_\mu \partial_\nu \pi \partial_\sigma \pi \times \partial_\tau \pi.$$

$$- i f_\omega f_\rho f_\pi G_{\omega\rho\pi} \epsilon_{\mu\nu\sigma\tau} \omega_\mu \partial_\nu \rho_\sigma.\partial_\tau \pi +$$

$$+ \left. \frac{\lambda i}{80\pi^2} \epsilon_{\mu\nu\sigma\tau} tr(\pi.F \partial_\mu \pi.F \partial_\nu \pi.F \partial_\sigma \pi.F \partial_\tau \pi.F) + ... \right]. \qquad (11)$$

This induced effective action is the action of quantum hadro-dynamics (QHD). Being a derivative expansion it should not be used in hadronic loop calculations. For that purpose the non-local form of the hadronic effective action is necessary. The above derivation leads naturally to the dynamical breaking of chiral symmetry (Section 2) and so to the chiral perturbation theory phenomenology (ChPT) (Section 4). The bare mesons are described by Bethe–Salpeter equations (BSE), and the baryons by covariant Faddeev equations (FE) [1]. The GCM thus relates hadronic properties directly to the gluon $D_{\mu\nu}$. It is not known why the GCM truncation is so effective.

2. Dynamical Chiral Symmetry Breaking

The hadronic effective action in (10) arises from expanding the bilocal action in (9) about its minimum via the Euler–Lagrange equations (ELE): $\delta S/\delta \mathcal{B} = 0$ give the rainbow constituent quark DSE in (12) and (13), while $\delta S/\delta \mathcal{D} = 0$ has solution $\mathcal{D} = 0$. Fluctuations are described by the curvatures: $\delta^2 S/\delta \mathcal{B}\delta \mathcal{B}$ gives the ladder BSE for mesons, and $\delta^2 S/\delta \mathcal{D}\delta \mathcal{D}^*$ gives the ladder BSE for diquarks:

$$B(p) = \tfrac{16}{3} \int \frac{d^4q}{(2\pi)^4} D(p-q) \frac{B(q) + m}{q^2 A(q)^2 + (B(q) + m)^2}, \qquad (12)$$

$$[A(p) - 1]p^2 = \tfrac{8}{3} \int \frac{d^4q}{(2\pi)^4} q.pD(p-q) \frac{A(q)}{q^2 A(q)^2 + (B(q)+m)^2}, \qquad (13)$$

$$G(q) = (iA(q)q.\gamma + B(q) + m)^{-1} = -iq.\gamma\sigma_v(q) + \sigma_s(q). \qquad (14)$$

Note that the quark propagator in (14) is a constituent quark propagator [and appears in the exponent in (9)], and not the exact quark propagator, as would arise from (1), and which would satisfy the Slavnov–Taylor identities (STI). In the GCM and related phenomenologies the quarks in the bare hadrons in (11) are further dressed, for example by mesons, in the functional integrations of (10). Imposing the STI in (12) and (13) could thus result in double counting problems. The exact quark propagator is relevant to the question of absolute quark confinement. The constituent quark propagator has the running mass $M(q) = B(q)/A(q)$. The constituent quark mass arises as the most likely value of $M(q)$ in, say, the BSE. It will depend on the bare current mass and to some extent even the hadron state.

3. The Nambu–Jona–Lasinio Model (NJL)

The NJL model is a special case of the GCM. Formally the NJL model [5] is the contact interaction limit of the GCM: $D_{\mu\nu}(x-y) \to g^2\delta_{\mu\nu}\delta(x-y)$ or $D(p) \to g^2$. But in the constituent quark DSE the contact limit is undefined because it leads to divergences in (12) and (13). A cutoff Λ is then always introduced in NJL computations, which is equivalent to using the gluon propagator $D(p) \to g^2\theta(q^2 - \Lambda^2)$. Hence the NJL model is the GCM but with a box-shaped $D(p)$, rather than a 'running' $D(p)$. The GCM bosonisation in (8) arises from a generalised Fierz identity [1]

$$j_\mu^a(x)j_\mu^a(x) = \overline{q}(x)\frac{M_m^\theta}{2}q(x)\overline{q}(x)\frac{M_m^\theta}{2}q(x) - \overline{q}(x)\frac{M_d^\phi}{2}\overline{q}(x)^{cT}q(x)^{cT}\frac{M_d^\phi}{2}q(x), \; (15)$$

where $\{M_m^\theta\} = \{\sqrt{\tfrac{4}{3}}K^a \otimes 1 \otimes F^c\}$ and $\{M_d^\phi\} = \{i\sqrt{\tfrac{2}{3}}K^a \otimes \epsilon^\rho \otimes H^f\}$ involve generators of the Dirac, colour and flavour symmetries. The use of this identity gives the generalised NJL model.

4. Chiral Perturbation Theory (ChPT)

When the quark current masses $\mathcal{M} \to 0$ $S[\overline{q}, q, A_\mu^a]$ has an additional global $U_L(N_F) \otimes U_R(N_F)$ chiral symmetry: writing $\overline{q}\gamma_\mu q = \overline{q}_R\gamma_\mu q_R + \overline{q}_L\gamma_\mu q_L$ where $q_{R,L} = P_{R,L}q$ and $\overline{q}_{R,L} = \overline{q}P_{L,R}$ we see that these two parts are separately invariant under $q_R \to U_R q_R, \overline{q}_R \to \overline{q}_R U_R^\dagger$ and $q_L \to U_L q_L, \overline{q}_L \to \overline{q}_L U_L^\dagger$. Its consequences may be explicitly traced through the GCM hadronisation. First the ELE $\delta S/\delta\mathcal{B}=0$ have degenerate solutions. In terms of the constituent quark propagator we find

$$G(q; V) = [iA(q)q.\gamma + VB(q)]^{-1} = \zeta^\dagger G(q; \mathbf{1})\zeta^\dagger, \qquad (16)$$

where

$$\zeta = \sqrt{V}, V = \exp(i\sqrt{2}\gamma_5 \pi^a F^a) \tag{17}$$

and consequently the fluctuations $\delta^2 S/\delta\mathcal{B}\delta\mathcal{B}$ have massless Nambu–Goldstone (NG) BSE states.

In the hadronisation (10) new variables are forced upon us to describe the degenerate minima (vacuum manifold):

$$U(x) = \exp(i\sqrt{2}\pi^a(x)F^a) \,. \tag{18}$$

$$V(x) = P_L U(x)^\dagger + P_R U(x) = \exp(i\sqrt{2}\gamma_5 \pi^a(x)F^a) \,. \tag{19}$$

The NG part of the hadronisation gives

$$\int d^4x \left(\frac{f_\pi^2}{4} tr(\partial_\mu U \partial_\mu U^\dagger) + \kappa_1 tr(\partial^2 U \partial^2 U^\dagger) + \frac{\rho}{2} tr([\mathbf{1} - \frac{U + U^\dagger}{2}]\mathcal{M}) + \right.$$

$$\left. + \kappa_2 tr([\partial_\mu U \partial_\mu U^\dagger]^2) + \kappa_3 tr(\partial_\mu U \partial_\nu U^\dagger \partial_\mu U \partial_\nu U^\dagger) + ... \right) \,. \tag{20}$$

This is the ChPT effective action [6], but with the added insight that all coefficients are given by explicit and convergent integrals in terms of A and B, which are in turn determined by $D_{\mu\nu}$. The higher order terms contribute to $\pi\pi$ scattering. The dependence of the ChPT coefficients upon $D_{\mu\nu}$ has been studied in refs [7, 8, 9]. The hadronisation procedure gives a full account of NG–meson–nucleon coupling.

5. The MIT, Cloudy Bag and Soliton Models

While the GCM hadronisation in (11) and (20) is the main result, at an intermediate stage one obtains [11] extended meson quark-meson coupling type models (QMC) [12]. Applying mean field techniques to this GCM quark–meson coupling effective action leads to soliton type models, which have been studied in detail in [13] and the significance of the extended mesons demonstrated. From the soliton models a futher ansatz for the soliton form leads to the MIT and cloudy bag model (CBM). In particular the GCM expression for the MIT bag constant may be extracted:

$$\mathcal{B} = \frac{12\pi^2}{(2\pi)^4} \int_0^\infty sds \left[\ln\left(\frac{A^2(s)s + B^2(s)}{A^2(s)s} \right) - \frac{B^2(s)}{A^2(s)s + B^2(s)} \right], \tag{21}$$

which is based on the energy density for complete restoration of chiral symmetry inside a cavity. This bag constant is for core states as no meson cloud effect is included.

With a mean field description of the pion sector via $\sigma(x)$, which describes the isoscalar part of $\sigma(x)V(x)$, where $\sigma(x)$ is a 'radial' field multiplying the NG

boson field $V(x)$

$$\mathcal{B}(\sigma) = \frac{12\pi^2}{(2\pi)^4} \int_0^\infty s ds \left[\ln\left(\frac{A^2(s)s + \sigma^2 B^2(s)}{A^2(s)s} \right) - \frac{\sigma^2 B^2(s)}{A^2(s)s + B^2(s)} \right], \quad (22)$$

which reduces to (21) when $\sigma = 1$, being the non-perturbative field external to an isolated nucleon core, and $\sigma < 1$ describing a partial restoration of chiral symmetry outside of the core. Using the gluon propagator discussed in Section 6 we obtain the plot of $\mathcal{B}(\sigma)/\mathcal{B}(0)$ shown in Fig. 2. Dressing of the nucleon core by mesons is partly described by a reduction in σ in the surface region, causing a reduction in the nucleon mass.
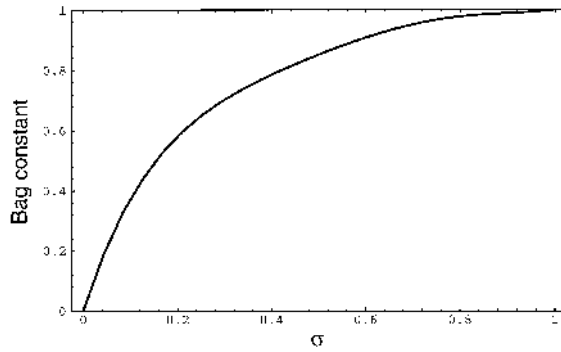
Fig. 2. Variation of the MIT bag constant $\mathcal{B}(\sigma)/\mathcal{B}(1)$ with respect to σ. Partial restoration of chiral symmetry is described by $\sigma < 1$.

However in nuclei a mean meson field description [14] means that σ is even further reduced outside of the nucleons, and the effective bag constant is further reduced. The σ field, which can model in part correlated $\pi\pi$ exchanges, along with the ω meson field, are believed to be important to a mean field modelling of nuclei. In [15] it has been argued that the reduction of the effective bag constant for nucleons inside nuclei is essential to the recovery of features of relativistic nuclear phenomenology. The GCM thus allows $\mathcal{B}(\sigma)$ and details of relativistic nuclear phenomenology to be directly related to the constituent quark propagator, and in turn to the gluon propagator.

6. Gluon Propagator Separable Expansions

In applications the GCM model amounts to solving a sequence of non-linear and linear integral equations:

$$D_{\mu\nu} \;\to\; G_{const.quark} \;\to\; meson\ BSE\ \&\ diquark\ BS \;\to$$

$$covariant\ Faddeev\ eqns\ for\ baryons \;\to\; hadronic\ observables.$$

In GCM computations one uses what amounts to a mixed metric: a Euclidean metric for the internal quark–gluon fluctuations, and with the hadron momenta in the time-like region of the Minkowski metric (usually but not necessarily in

the centre-of-mass (cm) frame). The constituent quark propagator equations (12) and (13) are solved in the Euclidean region ($s = q^2 \geq 0$), and then $A(q^2)$ and $B(q^2)$ are $O(4)$ invariant. Significantly it was discovered [16] that when low mass BSE states are solved with this mixed metric approach the (on-mass-shell) bound state form factors also show a remarkable degree of $O(4)$ invariance with respect to the relative quark momentum, even though this was not assumed in the numerical solution of the BSE. If we note that an $O(4)$ hyperspherical expansion

$$D(p - q) = D_0(p^2, q^2) + q.pD_1(p^2, q^2) + ... \tag{23}$$

of the gluon propagator, where

$$D_0(p^2, q^2) = \frac{2}{\pi} \int_0^\pi d\beta \, \sin^2 \beta \, D(p^2 + q^2 - 2pq \cos \beta) \tag{24}$$

together with the numerical technique of using a multi-rank separable expansion

$$D_0(p^2, q^2) = \sum_{i=1,n} \Gamma_i(p^2)\Gamma_i(q^2), ..., \tag{25}$$

then this separable expansion automatically generates $O(4)$ invariant BSE solutions when (23) is used, at lowest order, in a BSE. Hence the significant realisation that using a multi-rank separable expansion is ideally suited for GCM computations. This technique then renders the DSE and BSE equations to essentially algebraic form, with only the baryon integral equation computations requiring extensive numerical solution. As shown in [16] this very useful property of $O(4)$ BSE solution invariance appears to be a consequence of having no nearby singularities in the constituent quark propagator, and so appears to be related to the confinement property of the quarks.

It is important to note that the separable technique is introduced only as a numerical technique and only after all integral equations have been formally derived using the covariance of the GCM. If the separable expansion of the gluon propagator were introduced in the defining action of the GCM then the explicit breaking of covariance would block any of the usual momentum-space DSE, BSE,... equations from being derivable.

Implementing the separable expansion [8] we first solve the DSE (12) and (13). Then sums are obtained:

$$B(s) = \sum b_i \Gamma_i(s), ...\sigma_s(s) = \sum_{i=1,n} \sigma_s(s)_i, \sigma_v(s) = \sum_{i=1,k} \sigma_v(s)_i. \tag{26}$$

The gluon propagator is implicitly parametrised in [8] by assuming entire functions (as a means of implementing quark confinement) for σ_s and σ_v, say

$$\sigma_s(s)_i = c_i \exp(-d_i s), \qquad \sigma_v(s) = \frac{2s - \beta^2[1 - \exp(-2s/\beta^2)]}{2s^2} . \tag{27}$$

Then (12) gives

$$b_i^2 = \tfrac{16}{3}\pi^2 \int_0^\infty sdsB(s)_i \frac{B(s)}{sA(s)^2 + B(s)^2} \tag{28}$$

in which $B(s) = B(s)_1 + B(s)_2 + ...$, and $B(s)_i = \sigma_s(s)_i/([s\sigma_v(s)^2 + \sigma_s(s)^2])$.

Having solved the constituent quark non-linear equations (12) and (13) one can then proceed to solve the meson and diquark BSE and finally the bare nucleon–core Faddeev equations [1]. The implicit gluon propagator parameters in (27) are then determined by a best fit to some hadron data: f_π, m_π, m_{a_1}. Subtleties associated with the incorporation of the quark current masses and the reasons for choosing to fit these particular data are discussed in [8]. The gluon propagator parameter values are also given in [8], and various hadron observables are presented, and discussed here in Section 7.

Having determined the best fit the translation invariant form of the gluon propagator may be determined from (24) which, with $q^2 = 0$, gives

$$D(p^2) = D_0(p^2, 0) = \sum_i \Gamma_i(p^2)\Gamma_i(0)$$

$$= \sum_i \frac{1}{b_i^2} B(p^2)_i B(0)_i$$

$$= \sum_i \frac{1}{b_i^2} \frac{\sigma_s(0)_i}{\sigma_s(0)^2} \frac{\sigma_s(p^2)_i}{p^2\sigma_v(p^2)^2 + \sigma_s(p^2)^2} . \tag{29}$$

However, the form of the extracted gluon propagator will clearly depend on the forms assumed in (27), and futhermore, while a unique best fit was obtained the fit is somewhat flat with respect to variation of some of the parameters. To help more accurately resolve the form of the effective gluon propagator that arises in the GCM current work is underway [17] that constitutes a hybrid lattice–GCM calculation. In this a multi-rank separable expansion is constructed and its parameters determined by fits to both some hadron data and as well to the form for the gluon propagator that arises from lattice computations [3, 4], except for the infrared (IR) region of the gluon propagator. This amounts to probing the deep IR properties of the gluon propagator by means of low energy hadronic data. Results will be reported elsewhere.

7. Meson and Nucleon Observables

Table 1 shows various hadronic observables computed using the separable expansion technique of Section 6. Of particular relevance here are the $\pi\pi$ scattering lengths which arise from the ChPT effective action (20) with the parameters given by the explicit GCM forms [7]. The constituent quark masses arise from the value of the constituent quark running mass $M(s)$ at the value corresponding to the dominant s value in the BSE. This can only be determined after the BSE solution is known. Various diquark masses are shown. Diquarks are extended quark–quark correlations in baryons, and arise naturally from the

Table 1. **Hadronic observables**

Observable	Theory	Expt/Theory
Fitted observables		
f_π	$93 \cdot 00$ MeV	$93 \cdot 00$ MeV
a_1 meson mass	1230 MeV	1230 MeV
π meson mass	$138 \cdot 5$ MeV	$138 \cdot 5$ MeV
K meson mass (for m_s only)	496 MeV	496 MeV
Predicted observables		
$(m_u + m_d)/2$	$6 \cdot 5$ MeV	$6 \cdot 0$ MeV
m_s	135 MeV	130 MeV
ω^0 meson mass	804 MeV	782 MeV
a_0^0 $\pi - \pi$ scattering length	$0 \cdot 1634$	$0 \cdot 21 \pm 0 \cdot 01$
a_0^2 $\pi - \pi$ scattering length	$-0 \cdot 0466$	$-0 \cdot 040 \pm 0 \cdot 003$
a_1^1 $\pi - \pi$ scattering length	$0 \cdot 0358$	$0 \cdot 038 \pm 0 \cdot 003$
a_2^0 $\pi - \pi$ scattering length	$0 \cdot 0017$	$0 \cdot 0017 \pm 0 \cdot 003$
a_2^2 $\pi - \pi$ scattering length	$-0 \cdot 0005$	Not measured
r_π pion charge radius	$0 \cdot 55$ fm	$0 \cdot 66$ fm
nucleon–core mass	1390 MeV	~ 1300 MeV
constituent quark rms size	$0 \cdot 59$ fm	—
chiral quark constituent mass	270 MeV	—
u/d quark constituent mass	300 MeV	~ 340 MeV
s quark constituent mass	525 MeV	~ 510 MeV
0^+ diquark rms size	$0 \cdot 78$ fm	—
0^+ diquark constituent mass	692 MeV	>400 MeV
1^+ diquark constituent mass	1022 MeV	—
0^- diquark constituent mass	1079 MeV	—
1^- diquark constituent mass	1369 MeV	—
MIT core bag–constant	$(154 \text{ MeV})^4$	$(146 \text{ MeV})^4$
MIT nucleon–core mass (no cm corr.)	1500 MeV	~ 1300 MeV

GCM hadronisation [1]. They are particularly effective in reducing the numerical complexity when solving the baryon Faddeev integral equations. Note that a nucleon–core mass of 1390 MeV is consistent with the first Faddeev computation of the nucleon-core mass in [18]. This is also consistent with the idea that dressing of the nucleon–core by NG bosons causes a decrease of some 300 MeV. Incorporation of the spin 1^+ diquark state into the nucleon–core computation is also expected to decrease the nucleon mass. Other compuations of the nucleon–core state [19, 20, 21, 22] in the context of the NJL limit of the GCM have always adjusted the NJL parameters so that the nucleon–core mass actually fitted the experimental nucleon mass, leaving out the significant NG dressing effect.

Also shown is the GCM predicted value for the MIT bag constant, using expression (21). This is somewhat larger then the usual MIT value and gives a MIT nucleon-core mass of 1500 MeV without cm corrections. With cm corrections we might expect this to reduce to near the covariant Faddeev value of 1390 MeV, i.e the GCM MIT bag constant value appears to be consistent with the expected nucleon–core mass. The further reduction in the GCM $\mathcal{B}$ value when chiral symmetry is partially restored due to meson dressing of isolated nucleons, and a further enhanced reduction for nucleons inside nuclei, was discussed in Section 5.

8. A New NG Mass Formula

The usual NG mass formula is

$$M_\pi^2 = \frac{(m_u + m_d)\rho}{f_\pi^2}\,,\tag{30}$$

where

$$\rho = \langle \overline{q}q \rangle = N_c tr[G(x=0)] = 12 \int \frac{d^4q}{(2\pi)^4}\sigma_s(q^2)\,.\tag{31}$$

This expression for ρ is divergent in QCD. The values of m and $\langle \overline{q}q \rangle$ are then usually quoted as being relative to some cutoff momentum, often 1 GeV.

However, analysis of the GCM [10] gives rise to a different mass formula in which

$$\rho = 24 \int \frac{d^4q}{(2\pi)^4}\epsilon_s(q^2)c(q^2)\sigma_s(q^2)\,,\tag{32}$$

with the factor $c(s) = B(s)^2/(sA(s)^2 + B(s)^2)$, and the expansion of the m dependence of $B(q)$ from (12)

$$B(q)\mid_{m\neq0} +m = B(q)\mid_{m=0} +\epsilon_s(q)m + O(m^2)\tag{33}$$

defines the current mass enhancement factor $\epsilon_s(q)$. Despite the apparent difference between (31) and (32), Langfeld and Kettner [23] have shown that they are indeed equal.

9. Conclusions

We have summarised here some of the facets of the GCM and its relationship to the fundamental theory (QCD) and to various phenomenological models which have been profitably employed in modelling QCD. Significantly the GCM permits not only the derivation of these phenomenologies, but also particular expressions for the numerous parameters that arise in these models, such as the MIT bag constant, its modification within nuclear matter, and the ChPT parameters. Fundamentally the GCM is non-local and so the predictions do not involve divergences and the consequent renormalisations, once the gluon propagator is given its scale dependence.

References

[1] R.T. Cahill, Aust. J. Phys. **42** (1989) 171; Proc. Workshop on Diquarks, Turin, (Eds M. Anselmino and E. Predazzi), p. 201 (World Scientific, Singapore, 1989); Nucl. Phys. **A543** (1992) 63c.

[2] N. Brown and M.R. Pennington, Phys. Rev. **D39** (1989) 2723; K. Büttner and M.R. Pennington, Phys. Rev. **D52** (1995) 5220.

[3] P. Marenzoni, G. Martinelli, N. Stella and M. Testa, Phys. Lett. **B318** (1993) 511.

[4] P. Marenzoni, G. Martinelli and N. Stella, hep-lat/9410011.

[5] H. Reinhardt, Phys. Lett. **B244** (1990) 316.

[6] J. Gasser and H. Leutwyler, Ann. Phys. (NY) **158** (1984) 142; Nucl. Phys. **B250** (1985) 465.

[7] C.D. Roberts, R.T. Cahill, M.E. Sevior and N. Iannella, Phys. Rev. **D49** (1994) 125.

[8] R.T. Cahill and S.M. Gunner, Phys. Lett. **B359** (1995) 281.

[9] M.R. Frank and T. Meissner, Phys. Rev. **C53** (1996) 2410.

[10] R.T. Cahill and S.M. Gunner, Mod. Phys. Lett. **A10** (1995) 3051; The pion mass formula, hep-ph/9602240.

[11] R.T. Cahill and C.D. Roberts, Phys. Rev. **D32** (1985) 2419.

[12] P.A.M. Guichon, Phys. Lett. **B200** (1988) 235.

[13] M.R. Frank and P C. Tandy, Phys. Rev. **B46** (1992) 338.

[14] K. Saito and A.W. Thomas, Phys. Lett. **B327** (1994) 9; Phys. Rev. **C52** (1995) 2789.

[15] X. Jin and B.K. Jennings, Phys. Lett. **B374** (1996) 13.

[16] S.J. Stainsby and R.T. Cahill, Mod. Phys. Lett. **A9** (1994) 3551.

[17] R.T. Cahill, S.M. Gunner and N. Stella, Using the lattice gluon propagator in the global colour model for QCD, in preparation.

[18] C.J. Burden, R.T. Cahill and J. Praschifka, Aust. J. Phys. **42** (1989) 147.

[19] A. Buck, R. Alkofer and H. Reinhardt, Phys. Lett. **286** (1992) 29.

[20] N. Ishii, W. Bentz and K. Yazaki, Phys. Lett. **B301** (1993) 165; Phys. Lett. **B318** (1993) 26.

[21] S. Huang and J. Tjon, Phys. Rev. **C49** (1994) 1702.

[22] H. Meyer, Phys. Lett. **B337** (1994) 37.

[23] K. Langfeld and C. Kettner, private communication.