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Recent Developments in Rho–Omega Mixing*

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Abstract

The topic of ρ – ω mixing has received renewed interest in recent years and has been studied using a variety of modern techniques. A brief history of the subject is presented before summarising recent developments in the field. The present status of our understanding is discussed.

No discussion of ρ – ω mixing can be self-contained without a brief mention of the Vector Meson Dominance (VMD) model, within which it is studied [1, 2]. A simple example to introduce VMD is the electromagnetic (EM) pion form factor $F_\pi(q^2)$. This quantity represents the multiplicative deviation from the amplitude for the reaction $e^+e^- \rightarrow \pi^+\pi^-$ for the coupling of photons to purely point-like pions. A resonance peak is observed in the cross section (see Fig. 1 showing data from [3]). While this arises from low-energy, non-perturbative QCD processes [4], it can be modelled by assuming the photon couples to pions via vector mesons, the dominant one here being the rho-meson. VMD assumes the photon interacts with hadrons through vector mesons and mathematically, the enhancement seen in the cross section is provided by the pole in the meson propagator at the ρ mass-point. The broadness of the peak seen in the reaction is explained by attributing a ‘width’ Γ_ρ to the rho-meson corresponding to shifting the pole off the real axis, so that it becomes complex and we have

$$m_\rho^2 = \hat{m}_\rho^2 + i\hat{m}_\rho\Gamma_\rho, \quad (1)$$

where \hat{m}_ρ (the ‘mass’ of the rho) and Γ_ρ are real.

The $\pi^+\pi^-$ final state has isospin 1, so in order that G parity be conserved the ρ meson must have isospin 1. The photon can also couple to the three pion system (as the electromagnetic current has isospin 1 and isospin 0 components). Once again, a similar enhancement is seen in the reaction $e^+e^- \rightarrow \pi^+\pi^0\pi^-$ attributed

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to the isospin zero ω , though with a much narrower peak [5] (and a mass $\hat{m}_\omega > \hat{m}_\rho$).

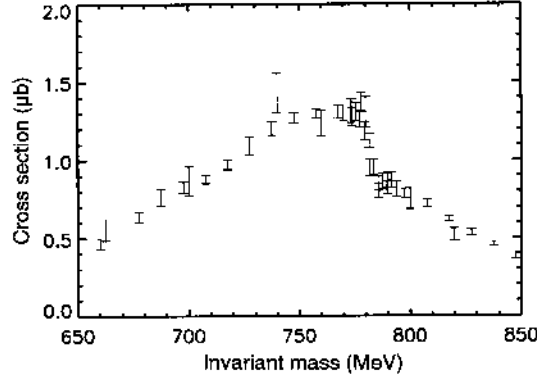


Fig. 1. Cross section for the reaction $e^+e^- \rightarrow \pi^+\pi^-$ from the data of Ref. [3] in the ρ - ω resonance region.

As more data was collected on $e^+e^- \rightarrow \pi^+\pi^-$ and the resolution of the plot of the cross section improved (see Fig. 1), the interference of the ω meson in the reaction was observed [6]. One was now faced with a G symmetry violating interaction, $\omega \rightarrow \pi^+\pi^-$. This could not be explained by the EM process $\omega \rightarrow \gamma \rightarrow \rho \rightarrow 2\pi$ as this is far too small to account for what is seen experimentally. Thus, this interference needs to be incorporated into the VMD picture. This can be done by writing the ρ and ω meson propagators in matrix form and generating the mixing by dressing the bare (isospin pure) matrix elements. We thus have the bare matrix $D_{\mu\nu}^0 = -g_{\mu\nu}D^0$, where

$$D^0 = \begin{pmatrix} (q^2 - m_\rho^2)^{-1} & 0 \\ 0 & (q^2 - m_\omega^2)^{-1} \end{pmatrix}. \quad (2)$$

We then dress this to form the full (or physical) propagator given by $D_{\mu\nu} = D_{\mu\nu}^0 + D_{\mu\alpha}^0 \Pi^{\alpha\beta} D_{\beta\nu}$ where the polarisation function, $\Pi_{\mu\nu} = (g_{\mu\nu} - q_\mu q_\nu / q^2) \Pi$, has off-diagonal elements $\Pi_{\rho\omega}$ which generate the mixing between the isospin pure ρ_I and ω_I .

Thus we introduce the decay mode $\gamma \rightarrow \omega_I \rightarrow \rho_I \rightarrow 2\pi$ to model what is seen in experiment. However, we should also consider the intrinsic decay $\omega_I \rightarrow 2\pi$. An argument by Renard, though, claimed this effect will be suppressed and hence can be ignored [2, 9] (this will be discussed further below). The dressed propagator $D(q^2)$ will then have off-diagonal elements, but the propagator could be diagonalised by transforming to the physical basis, $\rho = \rho_I - \epsilon \omega_I$ and $\omega = \omega_I + \epsilon \rho_I$. The isospin violating mixing angle, ϵ is given by [2]

$$\epsilon = \frac{\Pi_{\rho\omega}}{m_\omega^2 - m_\rho^2}, \quad (3)$$

where we use the complex masses of Eq. (1). A model for the pion form factor could now be written down as

$$F_\pi(q^2) = \frac{\hat{m}_\rho^2}{g_\rho(q^2 - m_\rho^2)} + \epsilon \frac{\hat{m}_\omega^2}{g_\omega(q^2 - m_\omega^2)} \quad (4)$$

which produced a remarkably good fit to the experimental data from only a few parameters. In the absence, though, of any theoretical model, $\Pi_{\rho\omega}$ has to be fitted to the data [10].

The mixing of elementary particles due to symmetry breaking had been considered by Coleman and Schnitzer for the vector case [11]. They discussed two kinds of mixing, mass (or particle) mixing, which was constant and current (or vector) mixing which was momentum dependent. The conclusion reached was that although mass mixing is perfectly adequate for spinless particles, current mixing is better for spin one particles, because it does not violate the conservation of electric charge (and hence $\Pi_{\rho\omega}$ should be proportional to q^2). Although the specific example they addressed was ω – ϕ mixing, they mentioned that off-diagonal (see above) current mixing was suitable for the study of ρ – ω mixing which, at the time, had been examined by Glashow [12]. It was a number of years before this suggestion was followed up [8], prompted by the direct experimental evidence for ρ – ω mixing in the pion form factor [6].

Coon [13] studied ρ – ω mixing in the one boson exchange model of the short-distance nuclear force as a contributing mechanism for the generation of the charge symmetry violation (CSV) seen experimentally [14]. The resulting potential is proportional to $\Pi_{\rho\omega}$ and, as it turns out, the value of $\Pi_{\rho\omega}$ extracted in the measurement of the pion form factor (Eq. (4)) has the right sign and magnitude to produce a reasonable fit to the data.

Although Coon *et al.* realised that the mixing function would in general be momentum dependent, it was claimed that its value at the ρ or ω mass point was all that was needed. Nevertheless, its extraction in the pion form factor is for time-like q^2 , while the vector mesons in the boson exchange model of the NN force have space-like momentum. Therefore, any momentum dependence could have significant implications for the standard treatment of CSV using ρ – ω mixing. This was first realised by Goldman *et al.* [15] who constructed a simple model in which $\Pi_{\rho\omega}$ is generated by a quark loop. The amplitude for the mixing is given by the difference between the $\bar{u}u$ loop and the $\bar{d}d$ loop. In the limit of isospin invariance ($m_u = m_d$) the mixing would vanish. The prediction of a *significant* momentum dependence for ρ – ω mixing forced Goldman *et al.* to conclude that it would strongly reduce the standard class III and IV CSV NN potential. So can ρ – ω mixing be simply assumed to be independent of momentum, and if not, what does this say about nuclear models?

A lot of model calculations were performed following this [16] which all produced similar results. At a more formal level, it was shown that for any model in which the vector mesons coupled to a conserved current, the mixing must vanish at $q^2 = 0$ [7], precisely the constraint on vector mixing expected by Coleman and Schnitzer [11]. In light of this, alternative mechanisms for nuclear CSV were proposed, involving isospin violation at the meson-nucleon vertex [17], rather than in the propagator. As both the vertex and propagator parts of the

NN interaction are off-shell, they are dependent on the choice of interpolating field for the vector mesons. It was thus argued that one could find fields so that the sum of vertex and propagator contributions is equivalent to a configuration in which all CSV occurs through a momentum independent mixed propagator [18]. This argument, though, has been disputed on the grounds of unitarity and analyticity [19].

However, the success of NN models in which the CSV is generated by a fixed-valued $\rho-\omega$ mixing provided considerable incentive to argue against momentum dependence. Miller [21] considered the mixing of the photon and the rho. Traditional VMD has a fixed coupling between the photon and the rho, but if this coupling were also generated by the kind of momentum dependent loop processes used for $\rho-\omega$ mixing [16] then the photon-rho coupling would be strongly momentum dependent, hence destroying the successful VMD phenomenology. However, an equivalent *momentum dependent* version of VMD exists (which we shall refer to as VMD1; the traditional version we shall call VMD2 [2]), which was described by Sakurai thirty years ago [1]. VMD1 differs from VMD2 by having a linear-in- q^2 photon-rho coupling *and* a direct coupling of the photon to the hadronic field, unlike in VMD2 where photon-hadron interactions take place exclusively through a vector meson. As an example, the pion form factor was plotted using VMD1 [10] and the results are indistinguishable from the usual VMD2. Thus Miller's worry could be addressed by comparing the loop models [16] not to VMD2, but to VMD1 [22]. This makes sense not only because the photon- ρ mixings will be momentum dependent, but also because if the photon is now allowed to couple to quarks (say) to form the loop, then we would expect it to be able to couple to the quarks in hadrons, hence introducing a direct photon-hadron coupling not found in VMD2, but appearing in VMD1.

We might like, though, to make some model-independent statement about $\rho-\omega$. This is difficult because the underlying theory, QCD, is presently inaccessible at the relevant energies. In the past 20 years we have developed some model independent treatments of low energy strong interactions, and two of these have been used to look at $\rho-\omega$ mixing. The first is the technique of QCD sum-rules (QCDSR) [23]. The basic idea is that one examines two-point functions of various hadronic currents, expanding them out in powers of $1/q^2$. At high q^2 QCD can be treated perturbatively due to asymptotic freedom, but cannot be handled in this manner for low q^2 (for example, around the ρ mass). So to work with the current correlators at low q^2 we have to appeal to phenomenology (importantly the resonances which are related to the vacuum structure). In this sense QCD sum rules are a bit of an art, because there is no set method for using them.

Interestingly, one of the first examples of the use of QCDSR by the original authors was $\rho-\omega$ mixing. The problem is set up by considering the two-point function

$$C_{\rho\omega}^{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T(J_\rho^\mu(x) J_\omega^\nu(0)) | 0 \rangle, \quad (5)$$

where $J_\rho^\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2$ and $J_\omega^\mu = (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)/6$. This *current* correlator (Eq. (5)) was then used by Hatsuda *et al.* [24] to examine the momentum

dependence of ρ – ω mixing by equating it with the mixed propagator (after extracting the transverse tensor $g_{\mu\nu} - q_\mu q_\nu / q^2$)

$$D_{\rho\omega}(q^2) = \frac{\Pi_{\rho\omega}(q^2)}{(q^2 - \hat{m}_\rho^2)(q^2 - \hat{m}_\omega^2)}. \quad (6)$$

As pointed out by Maltman [25] though, the association of the correlator with the off-diagonal propagator is only relevant if one uses interpolating fields for the ρ and ω mesons proportional to the currents J_ρ^μ and J_ω^μ , otherwise the correlator cannot be used to provide information about the off-shell behaviour of the mixing element of the vector meson propagator. Hatsuda *et al.* concentrated on the effect of the ρ and ω in this correlator, which as the most nearby resonances might be expected to play the dominant role. However, Maltman found that the ρ and ω contributions actually partially cancel. Because of this, the ϕ , although quite far away and hence contributing with a much lesser strength than the individual ρ and ω , becomes important for the isospin-breaking correlator (an effect not considered in the previous two analyses).

In all analyses the sum-rule result is ultimately compared to the data for the G-parity violation seen in $e^+e^- \rightarrow \pi^+\pi^-$. The correlator, though, is only relevant to the contribution from the mixing of the isospin pure states, ρ_I – ω_I , to the isospin breaking seen in the process. The competing process, $\omega_I \rightarrow \pi^+\pi^-$, is overlooked (as mentioned earlier), but Maltman found the ρ_I – ω_I contribution (as determined by the current correlator in QDCSR) under-estimates the isospin-violation seen experimentally. We shall discuss the matter of intrinsic decay further below.

Leinweber *et al.* in two recent paper [26] examined the effects of including the widths of the ρ and ω mesons in the QCDSR calculation performed by Hatsuda *et al.* They replaced the real parts of the mass in Eq. (6) by the complex pole positions given in Eq. (1). Following Maltman [25], they included the ϕ mesons, but found that its contribution was negligible. Perhaps of most interest, following the nuclear CSV debate, was their claim that for certain values of λ , $\Pi_{\rho\omega}$ has the same sign and similar magnitude in the space-like region to the on-shell value.

Because the reaction $e^+e^- \rightarrow \pi^+\pi^-$ is the only place ρ – ω is actually seen, it was decided that a new and general analysis should be performed [20]. The two effects normally ignored, momentum dependence of $\Pi_{\rho\omega}$ and the intrinsic decay $\omega_I \rightarrow \pi^+\pi^-$, were included. As two recent fits to data had been performed [30] all that needed to be done was to construct a precise theoretical expression for the form factor, which could be compared to the numbers extracted from these analyses for the expression

$$F_\pi(q^2) \propto P_\rho + Ae^{i\phi}P_\omega, \quad (7)$$

where $P_{\rho,\omega}$ are the poles of the ρ and ω propagators. The starting point was the mixed matrix formalism. In the data one sees two resonance peaks—a broad one associated with the physical (as opposed to isospin-pure) rho and a narrow one associated with the physical ω . Thus, the mixed propagator should only contain two poles in the physical basis. We choose this basis to be $\rho = \rho_I - \epsilon_1\omega_I$ and $\omega = \omega_I + \epsilon_2\rho_I$ and write the propagator in the new, physical basis. This allows us to fix $\epsilon_{1,2}$ by demanding that there are no poles in the off-diagonal pieces, i.e. that all resonant behaviour is associated with the physical mesons. This gives

expressions for $\epsilon_{1,2}$ similar to Eq. (3) but with arguments for $\Pi_{\rho\omega}$ at m_ω^2 and m_ρ^2 respectively, as noted by Harte and Sachs [31]. Thus, in general the bases are not related by a simple rotation, and the pure and real bases are not equivalent (as the transformation between them is not orthogonal). For the case that $\Pi_{\rho\omega}$ is either fixed or linear in q^2 , the off-diagonal terms can be made to disappear completely, but in general they survive and contribute to the non-resonant (i.e. no singular piece) background (although this is only a minor effect). The second part of the analysis centres on the decay $\omega_I \rightarrow \pi^+\pi^-$. A closer examination of the Renard argument shows that the cancellation is not exact, and a reasonable fraction of the intrinsic decay survives, which adds to the total interaction and this turns out to be *crucial* to the analysis. In a world of exact experimental precision the pre-factor $Ae^{i\phi}$ of the ω pole in the expression for the form factor (Eq. (7)) would enable us to pin down the values of the two unknowns, $\Pi_{\rho\omega}$ and the strength of $\omega_I \rightarrow \pi^+\pi^-$. Unfortunately this is not the case, and the considerable uncertainty in the Orsay phase ϕ and the lesser uncertainty in A allows a whole spread of values for the two unknowns; $\Pi_{\rho\omega}$ can take values in the set $(-840, -6240)$ MeV². Naturally if there were no contribution from $\omega_I \rightarrow \pi^+\pi^-$ we would recover the usual analysis and obtain $\Pi_{\rho\omega} = -3960$ MeV² (cf. the value -4520 ± 600 MeV² [32]). In light of this Maltman's QCDSR analysis is quite interesting, as it seems to provide theoretical evidence for a non-zero contribution from intrinsic decay, leading us to re-think the present status of the traditional extraction of $\Pi_{\rho\omega}$. It also brings into question the value of $\Pi_{\rho\omega}$ used in nuclear models.

Another model-independent method for treating the strong interaction at low energies is chiral perturbation theory (ChPT). It is the subject of many recent reviews [27], and essentially it sets up an effective model involving the pseudo-scalar octet and admitting all terms allowed by the symmetry of the original QCD Lagrangian, organised as a perturbative series in q^2 . However, the symmetries of QCD in question (chiral symmetry, isospin symmetry) are not exact; the main feature of ChPT is that it breaks these symmetries, for a meson theory, exactly as QCD breaks them. The various free parameters of the theory are then fixed by comparison to experiment.

As a perturbative series in q^2 , ChPT is only reliable in the low momentum region. The relatively heavy vector mesons, therefore, do not fit naturally into it. As resonances of QCD processes (which is really what they represent), they play a very important role in strong physics, but unfortunately ChPT breaks down well before the q^2 of the poles we associate with vector mesons. Thus, it is usually with an assumption such as VMD that the vectors mesons are fitted into ChPT, to create an effective model incorporating ChPT. One such model, in which the vector mesons appear in the Lagrangian as antisymmetric tensors has been used by Urech to study ρ - ω mixing [28].

The hadronic currents, as appearing in the correlator, have no such difficulty and appear quite naturally in ChPT. Maltman [25] thus used ChPT as a complement to his QCDSR calculation of the mixed correlator. He examined the correlator to one-loop order $O(q^4)$ in ChPT obtaining a result dependent on both q^2 and the mass difference of the neutral and charged kaon (vanishing when these masses are equal, which in ChPT occurs when $m_u = m_d$). The result was much smaller than the corresponding QCDSR result, indicating that the next

order contribution [two-loops, $O(q^6)$] needed to be included as well. The two-loop calculation [29] seems to show that the chiral series is not convergent enough to allow one to truncate even at $O(q^6)$ (which is the present-day limit of ChPT).

The study of ρ – ω mixing has pushed VMD and available data to its limit. So far we have established, within the matrix approach to VMD, that we expect the mixing of the pure isospin states to satisfy $\Pi_{\rho\omega}(0) = 0$ (for models in which the mesons couple to conserved currents), and that, due to experimental uncertainty in the pion form-factor, we cannot distinguish between $\rho_I - \omega_I$ mixing and the intrinsic decay $\omega_I \rightarrow \pi^+\pi^-$. As we have seen, QCD sum rules and ChPT are providing new insights to this old problem, but their results are open to interpretation. Physical processes in total, rather than their individual contributions are what we actually observe and so our studies need to reflect this. It remains for a completely consistent field theory based description of isospin violation in both the time-like (ρ – ω mixing) and space-like (nuclear CSV) to be constructed. We look forward to future developments in this field.

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