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#### Influence of Giant Nuclear-spin Polarisation on Resonant Gamma-ray Absorption and Emission\*

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#### Abstract

We propose a new scheme of gamma-quanta amplification without inversion. Laser pumping of electron states creates giant nuclear-spin polarisation via the hyperfine interaction. This results in extreme cooling of the ground-state nuclear spin in a projection which does not absorb both laser pump and gamma-quanta according to selection rules for these transitions. Induced emission from the nuclear excited state is not influenced by the pump. Therefore gamma-quanta travelling inside the pump beam have an opportunity to induce stimulated emission without subsequent quenching by ground state nuclei.

#### 1. Introduction

Lasing becomes increasingly difficult as the wavelength of radiation becomes shorter and reaches the gamma-ray band. The first difficulty comes from small nuclear cross sections relative to atomic ones. This gives rise to a second, more serious obstacle in creating population inversion of nuclear states which demands an enormous energy flux. Many concepts have been proposed to reduce pump requirements. One of them is gain without inversion (GWI) of population of absorbing and emitting nuclear states (Coussement *et al.* 1993; Kocharovskaya 1995). In this paper we consider recoilless gamma-ray transitions as the resonance cross section of Mössbauer nuclei is high. This choice moderates the first difficulty.

It is well recognised that resonant gamma-absorption and gamma-emission of Mössbauer nuclei are reciprocal, as corresponding cross sections are equal to each other. Consequently, gamma-lasing in a Mössbauer sample can be achieved only when the number of excited nuclei predominates over the number of ground-state nuclei. We present a method which breaks this reciprocity and shows the way to achieve gamma-lasing without population inversion. Thus, the second obstacle becomes a solvable problem as well.

Our method takes its origin in the new principle of lasing without inversion formulated for the optical band (Kocharovskaya and Khanin 1988; Scully *et al.* 1989; Harris 1989). We develop a new scheme of induced gamma-emission amplification which differs from that proposed in Coussement *et al.* (1993). There, the scheme contains radio-frequency (RF) excitation of nuclear spin states when

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spin level crossing in an external magnetic field takes place. This RF scheme demands cooling of the sample down to milli-Kelvin temperatures. At room temperature the RF scheme (Coussement *et al.* 1993) does not provide GWI, but one can consider it as supplementary for lasing with population inversion created by cooling.

In our scheme we propose to excite spin sublevels of ground-state nuclei by a resonant laser pump via hyperfine interaction. Following the paper by Coussement et al. we consider Mössbauer nuclei with spin  $I_e = \frac{1}{2}$  in the excited state and spin  $I_g = \frac{3}{2}$  in the ground state. In this crystal the nuclear electric quadrupole interaction splits the ground-state spin  $I_g$  into a doublet:  $|\pm \frac{1}{2}\rangle$  and  $|\pm \frac{3}{2}\rangle$ . We apply a magnetic field **H** parallel to the **c**-axis of a noncubic uniaxial crystal, also as Coussement et al., to obtain nuclear spin sublevel crossing. The Zeeman interaction gives an additional splitting of the doublet sub-states. Interplay between quadrupole and Zeeman interactions may result in crossing of doublet sub-components. We consider the case of  $|-\frac{1}{2}\rangle$  and  $|-\frac{3}{2}\rangle$  sub-components of the magnetic field with c-axis gives a strong mixing of  $-\frac{1}{2}$  and  $-\frac{3}{2}$  spin states at their crossing point. Mixed states at the level crossing split into a smaller doublet g

$$\begin{aligned} |1\rangle &= \cos\psi| - \frac{1}{2}\rangle - \sin\psi| - \frac{3}{2}\rangle \\ |2\rangle &= \sin\psi| - \frac{1}{2}\rangle + \cos\psi| - \frac{3}{2}\rangle \end{aligned} \tag{1}$$

with energy gap  $\hbar\omega_{21}$ , where  $\psi$  defines the degree of states mixing.

We consider a gamma-transition from the excited state  $|e\rangle$  to the ground state  $|g\rangle$  with selection rule  $\Delta M = \pm 1$ . When level crossing takes placing, the nucleus decays into a superposition of states  $|1\rangle$  and  $|2\rangle$ , i.e.

$$\left|-\frac{3}{2}\right\rangle = \cos\psi|2\rangle - \sin\psi|1\rangle = |n\rangle \tag{2}$$

if the spectral width of the gamma-quantum  $\gamma$ , defined by the lifetime of the excited state  $\tau_0$  ( $\gamma = 1/\tau_0$ ), is larger than the transition frequency  $\omega_{21}$  between sublevels  $|1\rangle$  and  $|2\rangle$ . Otherwise the nucleus falls into one of these states with a probability defined by the  $\psi$  value.

We also take into account the nuclear spin interaction with the electron shell. In our scheme a laser pump induces a transition between two states of the electron shell (ground and excited states). We choose the value of the magnetic field **H**, which together with the magnetic hyperfine field  $\mathbf{H}_g$  of the ground-state electron, creates a level crossing of the ground-state nucleus. The excited electron state h is assumed to create a different hyperfine field on nuclear spin, so when an electron is excited by the laser pump the nuclear spin sublevels do not cross each other. Then an electron transition from the excited state to ground state is accompanied by a nuclear spin transition from the pure state (for example  $|-\frac{1}{2}\rangle$ ) to the superposition state  $|n\rangle$  (see equation 2) if the selection rule is the same as for gamma-quanta. The energy diagram with transitions between state functions is shown in Fig. 1. We show only the nuclear spin state  $|-\frac{1}{2}\rangle$  in the excited electron state h, as the laser pump transition has a selection rule

 $\Delta M = \pm 1$ . It is supposed that the nucleus has another spin in the excited state e and its interaction with the electron gives the shift of the resonant frequency of the electron transition. Below we show that a Raman pump of the states  $|1\rangle$ ,  $|2\rangle$ ,  $|3\rangle$  (where  $|3\rangle$  is the excited electron state) results in ground-state population trapping and the electron is not excited. Therefore we can confine our consideration to the analysis of the four-level system interaction with the two fields on the adjacent transitions, as shown in Fig. 1.



Fig. 1. Diagram of the energy states: g is the ground state of the nucleus and electron; h is the electron excited state when the nucleus is not excited; and e is the nuclear excited state when the electron is not excited.

Below we show that the Raman pump forces the nucleus to abandon the state  $|-\frac{3}{2}\rangle$  in equation (2) which is involved in the electron transition according to the selection rules. As a result we get a predominant population of the state  $|-\frac{1}{2}\rangle$ , which corresponds to giant nuclear spin polarisation. It differs about a million times from nuclear spin polarisation at room temperature. Electron and gamma-transitions from this state to the excited states are forbidden. Therefore one can reduce, and in principle make zero, the absorption cross section of gamma-quanta by a Raman pump via hyperfine interaction. As the excited nucleus state e is not polarised the emission cross section is not zero. Thus one can get amplification of gamma-quanta without population inversion. We assume the process populating the excited state e results from the parent nucleus decay or the additional pump process (Collins and Carroll 1997). Then the Raman laser pump essentially reduces the pump requirements for the gamma-transition.

We model the initial emission of gamma-quanta by Lorentzian irradiation with randomly shifted phase. Mean dwell time between successive phase shifts is equal to the lifetime of the nucleus in the excited state e. A single gamma-quantum induces Raman excitation of the ground-state doublet g when the frequency of the phase shift  $\gamma$  is higher than the doublet split frequency  $\omega_{21}$ . The laser pump is described by the phase diffusion model. It induces Raman excitation of the nuclear spin doublet g when the decay rate of the electron coherence (i.e. electron polarisation)  $\Gamma$  is larger than the doublet split frequency  $\omega_{21}$ . In the next section we derive kinetic equations for the quantum system interacting with random phase fields.

#### 2. Random Phase Field Interaction with the Quantum System

We reduce our consideration of the gamma-quantum interaction with nuclei, polarised by the Raman pump via hyperfine interaction, to the double  $\Lambda$  scheme (Kocharovskaya 1995) (see Fig. 1). In this scheme the laser pump

$$E_R(t) = E_R \exp\left[i\omega t + i\Phi(t) - ikz\right] \tag{3}$$

excites the resonant transitions  $|1\rangle \rightarrow |3\rangle$ ;  $|2\rangle \rightarrow |3\rangle$ ; and the probe field

$$E_P(t) = E_P \exp\left[i\Omega t + i\alpha(t) - iKz\right] \tag{4}$$

is applied to the transitions  $|1\rangle \rightarrow |4\rangle$  and  $|2\rangle \rightarrow |4\rangle$ . Here  $\omega, \Omega; k, K$ ; and  $\Phi(t), \alpha(t)$ are the frequencies; wave numbers; and phases of these fields, respectively. The probe field arises due to nuclei decaying in an external source. We consider this field as a sequence of monochromatic fractions (quanta) with uncorrelated phases (Lorentzian irradiation). Mean dwell time between successive phase shifts is equal to the lifetime of the nucleus in the excited state  $\tau_0$ . Time intervals between successive phase shifts (i.e. the duration  $\tau$  of the field fraction with constant phase) obey the Poisson distribution

$$dW(t) = \exp(-\tau/\tau_0)\frac{d\tau}{\tau_0}.$$
(5)

The correlation function of this field is described by the expression

$$\langle E_P(t)E_P^*(t_0)\rangle = E_P^2 K(t-t_0) \exp[i\Omega(t-t_0)],$$
 (6)

where

$$K(t - t_0) = \langle \exp\left[i\alpha(t) - i\alpha_0(t_0)\right] \rangle = \int \int e^{i(\alpha - \alpha_0)} \varphi(\alpha_0) \varphi(\alpha_0, t_0 | \alpha, t) d\alpha d\alpha_0$$
$$\varphi(\alpha_0) = \lim_{t = t' \to \infty} \varphi(\alpha', t' | \alpha_0, t_0),$$

and where  $\varphi(\alpha_0, t_0 | \alpha, t)$  is the density of the conditional probability that the phase, being equal to  $\alpha_0$  at time  $t_0$ , changes its value to  $\alpha$  at time t;  $\varphi(\alpha_0)$ is the probability to find the phase  $\alpha_0$  within any cross section of the random process. We consider the stationary, discontinuous Markovian process. The density of conditional probability of this process obeys the forward Kolmogorov–Feller equation (Burshtein 1968; Feller 1966; Wodkiewicz and Eberly 1985)

$$\frac{\partial}{\partial t}\varphi(\alpha_0, t_0|\alpha, t) = -\frac{1}{\tau_0}\varphi(\alpha_0, t_0|\alpha, t) + \frac{1}{\tau_0}\int\varphi(\alpha_0, t_0|\beta, t)\,f(\beta|\alpha)d\beta \tag{7}$$

with the initial condition of  $\varphi(\alpha_0, t_0 | \alpha, t_0) = \delta(\alpha - \alpha_0)$ . The function  $f(\beta | \alpha)$  defines the probability for the phase  $\beta$  to change to the value  $\alpha$  at the time of the phase jump. When the random process is uncorrelated, then  $f(\beta | \alpha)$  coincides

with the stationary distribution  $\varphi(\alpha)$ . One can obtain from (7) the following equation for the irradiation correlation function:

$$\frac{\partial}{\partial t}K(t-t_0) = -\frac{1}{\tau_0}K(t-t_0).$$
(8)

Its solution gives the Lorentzian power spectrum of the field

$$P(\Omega') = Re \frac{1}{\pi} \int_0^\infty \langle E_P(t) E_P^*(0) \rangle e^{-i\Omega' t} dt = \frac{|E_P|^2}{\pi} \frac{\tau_0}{1 + (\Omega - \Omega')^2 \tau_0^2}$$
(9)

with a half-width  $1/\tau_0$ .

The laser pump has correlated phase jumps. This process is described by the conditional probability

$$f(\alpha_0|\alpha) = f(\alpha - \alpha_0) = f(\alpha_0 - \alpha), \qquad (10)$$

which is an even function as the probability of finding a new phase after the jump is symmetrically distributed relative to the phase value prior to the jump. The phase correlation is defined by the dispersion of the function  $f(\vartheta)$ , where  $\vartheta = \alpha - \alpha_0$ , i.e. by the mean value of the phase jump. For example, in the limit of infinitely small jumps, when the mean jump-value is zero, this function takes the form  $f(\vartheta) = \delta(\vartheta)$  and we have no phase change. When the mean value of the phase jump is finite and small, the individual phase changes after each jump are also small but with time the cumulative phase variation grows. This random walk of the phase corresponds to the phase diffusion process. With an increase of the  $f(\vartheta)$ -function dispersion the process modifies, and, finally, when  $f(\vartheta)$  coincides with the stationary distribution function  $\varphi(\alpha)$ , the process changes qualitatively. It becomes almost the same as an uncorrelated process.

The equations for the correlation function and for the spectrum of the field with correlated phase jumps coincide with equation (8) and equation (9), respectively. Only the parameter  $1/\tau_0$  is different. It is to be replaced by

$$\frac{1}{\tau_1} = \frac{1}{\tau_0} (1 - \langle \cos \vartheta \rangle); \qquad \langle \cos \vartheta \rangle = \int \cos \vartheta \ f(\vartheta) \, d\vartheta \,. \tag{11}$$

Therefore the spectrum of correlated irradiation is narrower that the frequency of the phase jump  $1/\tau_0$ .

Let us consider the evolution of the quantum system excited by the field with random phase. Its partial density matrix  $\stackrel{\wedge}{\rho}(\alpha)$  obeys the Burshtein (1968) equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{\mathcal{H}}(\alpha), \hat{\rho}(\alpha)] - \frac{1}{\tau_0} \hat{\rho}(\alpha) + \frac{1}{\tau_0} \int d\beta f(\beta|a) \hat{\rho}(\beta) + \hat{R}(\hat{\rho}(\alpha)), \quad (12)$$

where  $\stackrel{\wedge}{\mathcal{H}}(\alpha)$  is the Hamiltonian of the quantum system excited by the field fraction with constant phase  $\alpha$ , and  $\stackrel{\wedge}{R}(\stackrel{\wedge}{\rho}(\alpha))$  is the operator describing relaxation

of the diagonal and non-diagonal components of the density matrix induced by internal interactions in the sample. We consider the phase difference between the field and induced polarisation for each field fraction, as just this value contains the information about field absorption. This phase difference is defined by the variable

$$\sigma_{ge}(\alpha) = \rho_{ge}(\alpha) \exp\left(-i\Omega t - i\alpha + iKz\right), \qquad (13)$$

where g (e) denotes the ground (excited) states (Shoemaker 1978). Mean values of this variable and population difference

$$\langle \sigma_{ge} \rangle = \int \sigma_{ge}(\alpha) \, d\alpha; \qquad w = \int \left[ \rho_{ee}(\alpha) - \rho_{gg}(\alpha) \right] d\alpha$$
(14)

satisfy the equations

$$\dot{w} = 2i \left( A_{eg} \langle \sigma_{ge} \rangle - A_{ge} \langle \sigma_{eg} \rangle \right) - \frac{1}{T_1} (w - w_0) ,$$
$$\langle \dot{\sigma}_{ge} \rangle = \left( i\Delta - \frac{1}{T_2} - \frac{1}{\tau_1} \right) \langle \sigma_{ge} \rangle + iA_{ge} w , \qquad (15)$$

$$\langle \sigma_{eg} \rangle = \langle \sigma_{ge} \rangle^* \,,$$

where  $A_{ge} = d_{ge}E_P/\hbar$ ;  $d_{ge}$  is the matrix element of the dipole transition from the state g to the state e;  $T_1$  and  $T_2$  are relaxation times of the population difference and polarisation;  $w_0$  is the equilibrium population difference; and  $\Delta = \omega_{eg} - \Omega$  is a tuning parameter. We note that for uncorrelated irradiation the relation  $\tau_1 = \tau_0$  is valid.

Solution of equation (15) gives the mean value of polarisation in the instantaneous reference frame linked rigidly to the field phase. Therefore its analysis yields reliable information about absorption of the field  $E_P(t)$ . The dephasing rate in equation (15) has an extra component  $1/\tau_1$  equal to the half-width of the field spectrum. In the next paragraph we consider the solution of a similar equation for the quantum system excited by the Raman pump  $E_R(t)$  and the probe  $E_P(t)$  fields. As their phase noises have a different origin one can make a phase average for both fields independently. Then Lorentzian irradiation of gamma-quanta contributes to the dephasing rate  $1/T_2$  of the gamma-transition, whereas the laser pump gives broadening of its own resonant transition equal to its spectrum width.

#### 3. Double $\Lambda$ Scheme in Gamma-Optics

Before calculating the integral absorption of gamma-quanta by nuclei with ground-state spin polarised by laser pump, we consider the four-level system shown in Fig. 1. We assume that it is excited by two fractions of the fields (equations 3 and 4) with constant phases  $\Phi(t) = \text{const}$  and  $\alpha(t) = \text{const}$ . Equations for the

density matrix of this system are

$$\dot{\rho}_{11} = i \left( \mathcal{A}_{14} + \mathcal{B}_{13} \right) - w_{12}\rho_{11} + \sum_{n \neq 1} w_{n1}\rho_{nn};$$

$$\dot{\rho}_{22} = i \left( \mathcal{A}_{24} + \mathcal{B}_{23} \right) - w_{21}\rho_{22} + \sum_{n \neq 2} w_{n2}\rho_{nn};$$

$$\dot{\rho}_{33} = -i \left( \mathcal{B}_{13} + \mathcal{B}_{23} \right) - \left( w_{31} + w_{32} \right)\rho_{33};$$

$$\dot{\rho}_{44} = -i \left( \mathcal{A}_{14} + \mathcal{A}_{24} \right) - \left( w_{41} + w_{42} \right)\rho_{44};$$

$$\dot{\sigma}_{g4} = \left( i\Delta_g - \Gamma_\gamma \right)\sigma_{g4} + iA_{g4} \left( \rho_{44} - \rho_{gg} \right) - iA_{g'4}\rho_{gg'}, \qquad (16)$$

$$\dot{\sigma}_{g3} = \left( i\delta_g - \Gamma \right)\sigma_{g3} + iB_{g3} \left( \rho_{33} - \rho_{gg} \right) - iB_{g'3}\rho_{gg'},$$

$$\rho_{12} = (i\omega_{21} - \Gamma_M) \,\rho_{12} + i \left(A_{14}\sigma_{42} - A_{42}\sigma_{14}\right) + i \left(B_{13}\sigma_{32} - B_{32}\sigma_{13}\right) \,,$$

where  $\mathcal{A}_{g4} = A_{g4}\sigma_{4g} - A_{4g}\sigma_{g4}$ ;  $\mathcal{B}_{g3} = B_{g3}\sigma_{3g} - B_{3g}\sigma_{g3}$ ;  $A_{g4} = S(g)d_{g4}E_P/\hbar$ ;  $B_{g3} = S(g)d_{g3}E_R/\hbar$ ;  $S(1) = -\sin\psi$ ;  $S(2) = \cos\psi$ ;  $d_{g4} = d_{4g} = d_P$ ;  $d_{g3} = d_P$ ;  $d_{g4} = d_{g4}$ ;  $d_{g4}$ ;  $d_{g4} = d_{g4}$ ;  $d_{g4}$  $d_{3g} = d_R; \ \Delta_g = \omega_{4g} - \Omega; \ \delta_g = \omega_{3g} - \omega; \ \sigma_{g4} = \rho_{g4} \exp\left(-i\Omega t - i\alpha + iKz\right); \ \sigma_{g3} = \rho_{g3} \exp\left(-i\omega t - i\Phi + ikz\right); \ g = 1,2; \ \text{and} \ w_{nm} \ \text{is the probability of a}$ relaxation transition from the state n to the state m. The states 4 and 3 decay spontaneously to the couple of the ground states g(1, 2). Transitions between states 1 and 2 are induced by the spin-lattice interaction. As  $\hbar\omega_{21} \ll kT$ (where T is the temperature of the sample), transition probabilities  $w_{21}$  and  $w_{12}$ are equal [since they satisfy the relation  $w_{21}/w_{12} = \exp(\hbar\omega_{21}/kT)$ ]. The main contribution to the dephasing rate of the gamma-transition  $\Gamma_{\gamma}$  is given by the value  $1/4\tau_0$ . Here we take into account that the nucleus decays from state 4 to the states 1 and 2 simultaneously. Therefore, the probability of the transition  $4 \rightarrow q$  equates to  $1/2\tau_0$ . Elastic scattering of the phonons on the nucleus gives also the contribution to the zero-phonon line of the gamma-transition, whereas it gives the main contribution to the electron transition dephasing rate  $\Gamma$  at room temperature. The nuclear-spin dephasing rate  $\Gamma_M$  is caused by magnetic interactions with neighbouring nuclei, with electron spin, etc. All these relaxation rates are related as follows:

$$\sum_{g} w_{4g} = 4\Gamma_{\gamma} = \frac{1}{\tau_0} \gg w_{21} = w_{12}; \qquad \Gamma \gg w_{3g} \gg w_{21}.$$
(17)

Let us turn back to equation (12) which describes the interaction of the random-phase field with the quantum system and generalise it for the case of interaction with two random fields. Phase average (see equation 14) gives equations similar to (16). As in equation (15) the additional contributions to the decay rates  $\Gamma$  and  $\Gamma_{\gamma}$  appear:

$$\Gamma'_{\gamma} = \Gamma_{\gamma} + \frac{1}{\tau_0}; \qquad \Gamma' = \Gamma + \frac{1}{\tau'_1}, \qquad (18)$$

where  $1/\tau'_1$  obeys equation (11). It should be noted that the durations of the gamma-quantum  $\tau_0$  and laser quantum  $\tau'_0$  are different as they have different origins.

All equations for the population  $\rho_{nn}$  (n = 1, 2, 3, 4) and equations for nondiagonal elements of the density matrix of the ground state spin  $\rho_{12}$  do not change as the random phase does not influence the relaxation of these variables.

Raman excitation of the four-level system by two fields  $E_P(t)$  and  $E_R(t)$  may be useful for amplification without inversion only when the resonant frequency of the spin states  $\omega_{21}$  is smaller than the effective half-widths of gamma and electron transitions  $\Gamma'$  and  $\Gamma'_{\gamma}$ . Below we omit the primes in the notation keeping in mind that coherence decay rates of relevant transitions depend on the parameters of the driving fields.

We consider gamma-quanta of coherence length longer than the length of the sample. Moreover, the sample is supposed to be optically 'thin'. Then the probe field amplitude  $E_P(t)$  at the output edge of the sample satisfies the equation

$$E_P(L) = E_P(0) + \Delta E_P; \qquad \Delta E_P = 4\pi KNL Im \left( d_{41} \langle \sigma_{14} \rangle + d_{42} \langle \sigma_{24} \rangle \right), \quad (19)$$

where  $E_P(0)$  is the amplitude of the gamma-quantum; N is the concentration of the nuclei in the sample; L is the sample length and  $\langle \sigma_{g4} \rangle$  is the stationary value of the nondiagonal components of the density matrix averaged over all realisations of the random process. Equation (19) describes the average variation of the probe field amplitude  $E_P(0)$  in the time interval which is much longer than  $\tau_0$ . When  $\Delta E_P$  has the same sign as  $E_P(0)$ , then the probe field is amplified. Otherwise the field  $E_P(t)$  is absorbed. When  $\Delta E_P/E_P = q$  and |q| < 1, then one has to make 1/|q| measurements to detect this effect.

Below we show that even when the sample is an optically 'thick' absorber, Raman excitation makes it transparent and one can use equation (19) to describe gamma-quanta propagation in the sample along the laser beam. Equation (19) is violated only when the concentration of excited state nuclei is large enough to produce the avalanche of gamma-quanta. To describe this process one has to solve Maxwell–Bloch equations for the field and polarisation of the sample. When the rate of the gamma-quantum avalanche growth is small enough, one can simply calculate the gain coefficient of the field  $E_P(t)$  per unit length

$$G_P = \frac{\Delta E_P}{LE_P} \,. \tag{20}$$

As the Lorentzian field is small, we calculate the stationary response of the sample in the linear approximation. Solution of equation (16), averaged over all possible realisations of the random process, gives the following equation for the probe field gain:

$$G_P = \frac{4\pi KN |d_P|^2}{\hbar\Gamma_{\gamma}} \left\{ \rho_{44} - \left(\rho_{22}\cos^2\psi + \rho_{11}\sin^2\psi\right) + \sin\psi\cos\psi\left(\rho_{12} + \rho_{21}\right) \right\}, \quad (21)$$

where  $\rho_{mm}$  and  $\rho_{gg'}$  are stationary values of the density matrix. Equation (21) is valid under the condition  $\Gamma_{\gamma} \gg \Delta_1, \Delta_2$ .

The state  $|4\rangle$  is populated by nuclear pump or due to the spontaneous decay of another higher energy nucleus. We suppose that the rate of this process is much smaller than the decay rate  $1/\tau_0$ . The Raman pump does not influence the population of the state  $|4\rangle$  as it induces the transitions  $|g\rangle \rightarrow |3\rangle$  only. When  $\Gamma \gg \delta_1, \delta_2$ , its absorption coefficient is

$$G_R = \frac{4\pi k N |d_R|^2}{\hbar \Gamma} \left\{ \rho_{33} - \left( \rho_{22} \cos^2 \psi + \rho_{11} \sin^2 \psi \right) + \sin \psi \cos \psi \left( \rho_{12} + \rho_{21} \right) \right\}.$$
 (22)

The stationary solution of the averaged equation (16) gives the following values of the density matrix components in (22):

$$\rho_{11} = \rho_{22} = \rho_{12} = \rho_{21} \approx \eta_{12}/2; \qquad \rho_{33} \approx 0, \tag{23}$$

where  $\eta_{12}$  is the sum of the populations of the states 1, 2 and 3. Equation (23) is valid provided that (i) the spin states  $-\frac{1}{2}$  and  $-\frac{3}{2}$  (see Fig. 1) are mixed in equal proportion, i.e.  $\psi = \pi/4$  (see equation 1), and (ii) the pump rate obeys the condition

$$|B_{g3}|^2 \gg \Gamma \Gamma_M; \quad \Gamma \omega_{21}$$

Substitution of the density matrix components, equation (23), in equation (22) gives zero value for the absorption coefficient of Raman pump  $E_R(t)$ . Propagating in the sample, this field creates a giant nondiagonal component of the spin-sublevel density matrix which corresponds to the spin polarisation in the state  $|-\frac{1}{2}\rangle$ . This giant polarisation is impossible to create by radio frequency (RF) excitation at room temperature, as the RF field excites transverse magnetisation which is proportional to the thermal population difference. The population difference is equal to the ratio of the spin energy gap and temperature. The optical pump 'transfers' the population difference of the optical transition (which is equal to one) to the nuclear spin coherence. It results in cooling down to a nearly zero temperature of the ground-state nuclei. Substitution of the density matrix components, equation (23), in equation (21) gives a similar result, showing that nuclear spin polarisation neutralises gamma-quanta absorption as well. The gain coefficient  $G_P$  becomes positive for any population of the ground state, if there is a small number of excited state nuclei  $N_4$  satisfying the relation

$$N_4/N > w_{12}\Gamma/|B_{13}|^2. (24)$$

If the Raman pump has a selection rule  $\Delta M = 0$ , or any other one assuming the transitions to the ground spin-state with the projection  $|-\frac{1}{2}\rangle$ , then it polarises the nuclear spin in the state  $|-\frac{3}{2}\rangle$  and nondiagonal elements of the spin density matrix become negative, i.e.  $\rho_{12} = \rho_{21} = -\eta_{12}/2$ . In turn, equation (22) is also changed, as the plus sign before the nondiagonal components changes to minus. Both changes compensate each other and as a result the absorption coefficient  $G_R$ remains zero. The sample becomes transparent for the Raman pump again; but equation (21) is not changed. Therefore the negative nondiagonal components of the spin-density matrix result in enhancing the probe field absorption instead of its suppression. Thus, our scheme of GWI demands the same selection rules for the probe and pump fields. Moreover, it demands applying a constant magnetic field at the value where level crossing takes place and spin mixing is perfect  $(\psi = \pi/4)$ . When the magnetic field does not fit this value, the parameter  $\psi$ differs from  $\pi/4$  and absorption elimination becomes incomplete.

#### 4. Conclusion

We have presented an example of laser induced transparency for gamma-quanta. A laser pump makes the sample transparent for the field with a frequency much greater than the frequency of excitation. We calculated the gain coefficient for gamma-irradiation. As this method is general and has no limitations, it can be applied to GWI on optical transitions from highly-excited electron states. Moreover, instead of nuclear spin states crossing in a magnetic field, one can use the other method to obtain the optical branching. For example, a circularly polarised laser beam induces transition to the superposition of the spin states in a transverse magnetic field, resulting in transverse magnetisation of the sample (Kohmoto *et al.* 1983).

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