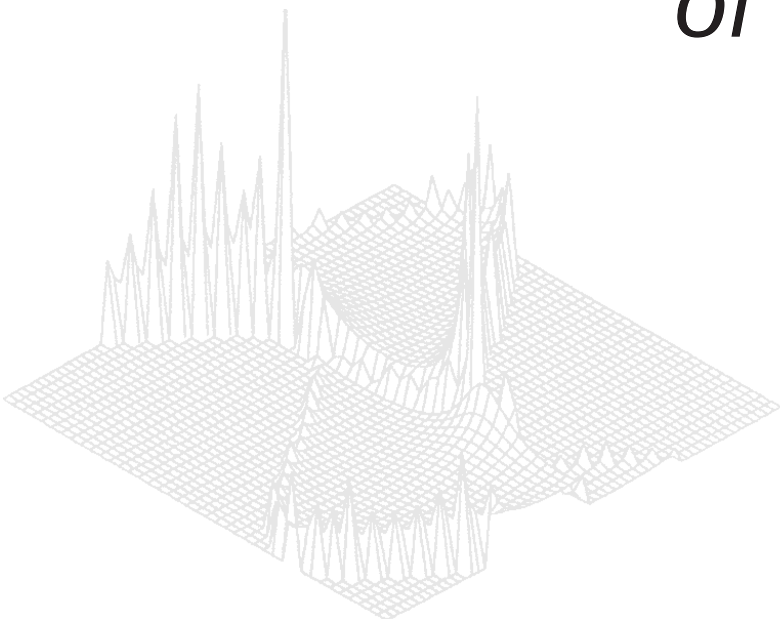
**CSIRO** PUBLISHING

# *Australian Journal of Physics*

Volume 53, 2000

A journal for the publication of
original research in all branches of physics

**www.publish.csiro.au/journals/ajp**

All enquiries and manuscripts should be directed to
*Australian Journal of Physics*
**CSIRO** PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia

Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: ajp@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO and
the Australian Academy of Science

*Aust. J. Phys.*, 2000, **53**, 723–736

# Microwave Absorption in Layered Media: Application to Landmine Detection

*J. A. Hermann*

DSTO Electronics & Surveillance Research Laboratory,
Surveillance Systems Division, PO Box 1500,
Salisbury, SA 5108, Australia.

*Abstract*

The absorption of microwave radiation and subsequent thermal conduction by simple composite media, consisting of parallel layers with disparate thermal properties, is analysed. The solutions for a one-dimensional conduction model are used to investigate the time evolution and distribution of thermal energy within moisture-laden soils containing non-absorbing objects. The application of these results to the detection of landmines is discussed and evaluated.

## 1. Introduction

It is well known that active and passive techniques using infrared radiation may be used for detecting surface-laid and near-surface landmines. The active techniques include beam scanning (e.g. employing IR lasers) to detect specular reflections, while passive techniques include the measurement of characteristic thermal signatures produced when the solar flux interacts with the buried objects. Modelling of the latter technique has been carried out recently by Miller (1997). Experimental and numerical analyses of active infrared imaging of buried objects using heating lamps have been reported by Li *et al.* (1995). Other techniques that have been proposed for landmine detection include various forms of ground-penetrating radar. As it stands, these approaches suffer from the problem of distinguishing between landmines and other features close to, or at, the ground surface (clutter).

It has also been proposed recently by Le Fevre (1995), Carter *et al.* (1996), Hermann and Chant (1999), Fueloep and Hall (1996) and Fueloep and Bird (1996) that the moisture content of the surrounding soil can be utilised for detecting a buried mine, by irradiating the ground with microwaves at the frequency 2.45 GHz. The absorption of microwaves by ground moisture will produce a significant temperature rise near the ground surface, whereas the mine (which is assumed to be essentially non-absorbing) will stay relatively cool. In consequence a characteristic thermal signature will appear at the surface.

The use of microwaves is a relatively attractive approach compared with other thermal methods because it is possible to achieve larger temperature differentials and therefore better resolution (e.g. by an order of magnitude compared with the solar flux method), and also because the input energy can be controlled (e.g. the solar method is only applicable in full sunlight). Furthermore, the strongly penetrating property of MW radiation means that the temporal evolution of the thermal signatures is more rapid than can occur with the solar method, because there is much less dependence on the diffusion of heat from the surface.

 10.1071/PH00001 0004-9506/00/050723$05.00

A hierarchy of increasingly complicated models, involving the absorption of radiation by composite media, may be investigated using the analytical techniques employed in this paper. The simplest type of problem involves the reception of radiant flux at the surface of a semi-infinite absorber. This problem may be augmented and extended in various ways, for example, by the inclusion of additional or alternative boundary conditions, by introducing non-uniform absorption via a continuous variation in the absorptive coefficient, by introducing inhomogeneous absorption via different 'layers' of selectively absorbing and reflecting materials, by including various switching conditions or time-dependent flux, and by including features such as contact resistances between media possessing different thermal and material properties. An essential requirement for an analytic solution within such a model hierarchy is that it should reduce to the solution for a simpler physical model whenever appropriate parametric limits are taken. This requirement provides a useful means for testing the model at each stage in order to discover errors. An alternative error-detecting procedure, which can be used whenever the analytical expressions are complicated or difficult to manipulate, is to program the solutions on a computer with variable parameters and test the limiting parametric behaviour for special cases against the known analytic solutions for those cases.

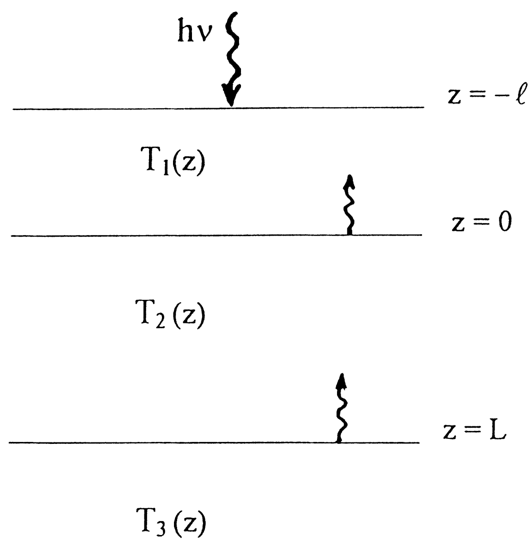



**Fig. 1.** Diagram of a three-layered composite, consisting of a non-absorbing material sandwiched between two absorbing layers, and irradiated by an external microwave source.

Fig. 1 depicts an irradiated three-layer composite, in which a non-absorbing layer is sandwiched between two absorbing layers. Reflection occurs at each interface of the buried object, as well as at the surface of the soil. The flow of heat in this simple model is envisaged as being one-dimensional. Even when the central layer is of finite extent, a one-dimensional model may be entirely appropriate for sufficiently short irradiation times and at positions well removed from the outer edge of the layer. Apart from the application of this methodology to detecting buried objects, including objects of military origin, other possible applications of the analytic solutions of these fairly general absorption–conduction equations are to the study of microwave cooking and the effects of microwave irradiation upon biological tissues, and for this reason the long-time behaviour also may be of interest.

## 2. Microwave Absorption by a Homogeneous Medium

Understanding the manner in which radiation of a given power can heat a homogeneous absorbing medium is an essential prerequisite for modelling the impact of buried objects upon the temperature distribution at the surface. The thermal conduction equation for this situation is sufficiently simple to allow precise analytic solutions to be found, even if one includes a heat flux or elevated temperature at the surface (which might be associated with either continuous or discontinuous sunlight, for example). The form and properties of the heat conduction equation appropriate to this task are discussed in the well-known book on heat conduction by Carslaw and Jaeger (1990).

It was demonstrated in the previous modelling of thermal techniques by Miller (1997) and by Hermann and Chant (1999) that the one-dimensional thermal model provides a reasonably accurate approximation to the observed temperature anomaly for situations in which it is only required to measure the maximum temperature differential, and providing that the object is positioned sufficiently close to the surface and the irradiation time is sufficiently short. A major advantage in using the one-dimensional model is that it provides a simply programmed, fast algorithm which will be useful for rapid assessment purposes.

In the simplest absorptive model there is no significant heat flow at the surface, and the rate of dissipation of radiant energy as heat per unit volume per unit time $A(z)$ follows negative exponential behaviour with respect to the propagation distance $z > 0$ (the Beer–Lambert law). In more general models the rate of heat production will be a function of the spatial and temporal variables, as well as temperature. It is also assumed that the radiant source is switched on at time $t = 0$, and that the initial temperature is $T_0$ at all points within the active medium.

The differential equation in time and space describing the temperature distribution $T(z,t)$ is

$$\frac{\partial^2 T}{\partial z^2} = \kappa^{-1}\frac{\partial T}{\partial t} - \frac{A(z)}{K} \quad , \tag{1a}$$

subject to the initial and boundary conditions

$$T(z,0) = T_0\,, \qquad T(\infty,t) = T_0\ , \qquad \left.\frac{\partial T}{\partial z}\right|_{z=0} = 0 \tag{1b}$$

in which we have assumed for the driving term the attenuation law $A(z) = A_0 e^{-\alpha z}$, and where $\alpha$ is the radiation extinction coefficient, $\kappa$ is the thermal diffusivity and $K$ is the thermal conductivity. It should be noted that the power dissipation per unit volume, $A$, is given by

$$A = -\,\mathrm{d}I/\mathrm{d}z = \alpha I, \qquad \alpha = k_0 \varepsilon_r^{1/2} \tan\delta$$

where $I(z)$ is the irradiance (power density) of the microwave beam, $k_0$ is the radiation wave-number for free space, $\varepsilon_r = n_0^2$ is the relative dielectric constant, and $\delta$ is the loss tangent.

As a matter of mathematical convenience one may employ the transformed coordinates $x = \alpha(\kappa t)^{1/2}$ and $y = \alpha z$. An exact solution of equation (1) (see Carslaw and Jaeger 1990, p. 80) then has the form

$$T = T_0 + \frac{A_0}{K\alpha^2}\left\{2x\,\mathrm{ierfc}\left(\frac{1}{2}\frac{y}{x}\right) - \mathrm{e}^{-y} + \tfrac{1}{2}\mathrm{e}^{x^2}\left[\mathrm{e}^{y}\mathrm{erfc}\left(x + \frac{1}{2}\frac{y}{x}\right) + \mathrm{e}^{-y}\mathrm{erfc}\left(x - \frac{1}{2}\frac{y}{x}\right)\right]\right\} \tag{2}$$

where the notations erfc($u$) and ierfc($u$) refer to the error function and the first integral of the error function respectively (these functions are fully defined and discussed in Appendix 2 of Carslaw and Jaeger). The temperature evolution at the surface (being relevant to any thermal scanning) is most generally given by

$$T_s = T_0 + \frac{A_0}{K\alpha^2}\left\{-1 + \frac{2x}{\pi^{1/2}} + \mathrm{e}^{x^2}\mathrm{erfc}\,(x)\right\} \quad . \tag{3}$$

In the small-time limit specified by $t << d^2/\kappa$, where $d = \alpha^{-1}$ is the so-called 'penetration depth', equation (3) reduces to

$$T_s = T_0 + \frac{A_0 \kappa t}{K} \tag{4a}$$

and in the large-time limit specified by $t >> d^2/\kappa$ equation (3) reduces to

$$T_s = T_0 + \frac{2A_0}{K\alpha}\left(\frac{\kappa t}{\pi}\right)^{1/2} \quad . \tag{4b}$$

Using these results, one may carry out simple estimates of the temperature rise to be expected at different depths and at the surface of an absorbing medium for a particular microwave system and a given irradiation time. Following the previous work on landmine detection reported by Le Fevre (1995), and referred to in a conference paper by Carter *et al.* (1996), we can assume that a simple workable microwave system will operate at a frequency of 2.45 GHz, utilising a horn antenna positioned immediately above the soil surface, and producing a surface radiant power level of at least 1 kW. Let us also assume that the horn's aperture area is $15 \times 15$ cm$^2$. For moist sandy soil (moisture content 4%) the penetration depth is roughly $d = \alpha^{-1} = 20$ cm. Other parameters assumed for this exercise are $\kappa = 0.2 \times 10^{-6}$ m$^2$ s$^{-1}$ and $K = 0.442$ W °C$^{-1}$ m$^{-1}$. With these representative values, and with the additional value of $5 \times 10^4$ deg. for the temperature parameter $A_0/K\alpha^2$ (corresponding to a modest radiant power of 1 kW), one finds that the surface of the irradiated ground would be expected to experience a temperature increase of 20 °C within the first 60 s.

## 3. Composite Solid with a Surface Absorbing Layer

In our first generalisation of the simple homogeneous model of the previous section, a radiation-absorbing layer of thickness $l$ is interposed between a non-absorbing layer, initially assumed to be infinitely thick, and the atmosphere. The representation of the non-absorbing buried object as having infinite depth is often a useful approximation for landmines, particularly where the radiation-absorbing layer of soil above them is relatively thin. The overall temperature distribution $T(z, t)$ has two components, $T_1(z, t)$ and $T_2(z, t)$, which in the simplest model correspond to absorbing and non-absorbing regions respectively. The appropriate differential equations describing the thermal processes are:

$$\frac{\partial^2 T_1}{\partial z^2} = \kappa_1^{-1}\frac{\partial T_1}{\partial t} - \frac{A_0}{K_1}\left\{ e^{-\alpha(z+l)} + r e^{\alpha(z-l)} \right\} \quad , \qquad -l \le z \le 0 \, , \tag{5a}$$

$$\frac{\partial^2 T_2}{\partial z^2} = \kappa_2^{-1}\frac{\partial T_2}{\partial t} \quad , \qquad 0 \le z \le \infty \, , \tag{5b}$$

where $r$ is the reflectivity of the upper surface of the buried object ($0 \le r \le 1$), while the initial conditions are

$$T_1(z,0) = T_2(z,0) = T_0 \tag{6a}$$

and the boundary conditions at the two interfaces and at infinity are

$$\frac{\partial T_1}{\partial z} = 0 \quad , \qquad z = -l \, , \tag{6b}$$

$$T_1 = T_2 ; \qquad K_1 \frac{\partial T_1}{\partial z} = K_2 \frac{\partial T_2}{\partial z} \quad , \qquad z = 0 \, , \tag{6c}$$

$$T_2 = T_0 \quad , \qquad z = \infty \, . \tag{6d}$$

It should be noted that equation (5a) actually represents two conduction equations, one for the temperature rise associated with the absorption of the forward propagating field and the other associated with the absorption of the reflected field. The rate of heat generation from the forward and backward fields is therefore represented by the first term and second terms respectively on the right-hand side of equation (5a).

Analytical methods for solving simultaneous differential equations of this type include the use of Laplace transforms. A number of valuable techniques may be found in Carslaw and Jaeger (1990) and in Spiegel (1965). Applying ordinary Laplace transforms with respect to the time variable, the above equations become

$$\frac{d^2 \overline{T}_1}{dz^2} = q_1^2 \overline{T}_1 - \kappa_1^{-1} T_0 - \frac{A_0}{K_1 p}\left\{ e^{-\alpha(z+l)} + r e^{\alpha(z-l)} \right\} \qquad -l \le z \le 0 \, , \tag{7a}$$

$$\frac{d^2 \overline{T}_2}{dz^2} = q_2^2 \overline{T}_2 - \kappa_2^{-1} T_0 \quad , \qquad 0 \le z \le \infty \, , \tag{7b}$$

where $p$ is the Laplace transform variable and the parameters $q_i(p)$ have been defined as

$$q_i(p) = (p/\kappa_i)^{1/2} \tag{8}$$

together with the transformed boundary conditions

$$\frac{d T_1}{dz} = 0 \quad , \qquad z = -l \, , \tag{9a}$$

$$\overline{T}_1 = \overline{T}_2 ; \qquad K_1 \frac{\mathrm{d}\overline{T}_1}{\mathrm{d}z} = K_2 \frac{\mathrm{d}\overline{T}_2}{\mathrm{d}z} \ , \qquad z = 0 \, , \tag{9b}$$

$$\overline{T}_2 = \overline{T}_0 \ , \qquad z = \infty \, . \tag{9c}$$

By considering the properties of the above set of equations (7) and (9) it is possible to infer suitable algebraic expressions for the Laplace transforms $\overline{T}_i(z, p)$, i.e.

$$\overline{T}_1(z\,,\,p) = A\cosh q_1(p)(z+l) + B\sinh q_1(p)(z+l) + D\mathrm{e}^{-\alpha(z+l)} + G\mathrm{e}^{\alpha(z-l)} + E\,, \tag{10a}$$

$$\overline{T}_2(z,p) = C\mathrm{e}^{-q_2(p)z} + F \ , \tag{10b}$$

where the coefficients $A$, $B$, $C$, $D$, $E$ and $F$ are generally functions of the transform variable $p$.

The important result for the temperature transform at the surface is

$$\overline{T}_1(-l\,,p) = A + D + G\mathrm{e}^{-2\alpha l} + E\,,, \tag{11}$$

which readily translates as

$$\overline{T}_1(-l,p) = p^{-1}T_0 + \frac{A_0}{K_1(q_1^2-\alpha^2)p}\left\{1 + r\,\mathrm{e}^{-2\alpha l} + \Omega(q_1,\alpha,\sigma,l)\sum_{n=0}^{\infty}(-\beta)^n\,\mathrm{e}^{-2nq_1 l}\right\}\,, \tag{12}$$

$$\Omega(q_1,\alpha,\sigma,l) =$$

$$\frac{\alpha}{q_1}\left(\frac{2\,(1-r)}{1+\sigma}\,\mathrm{e}^{-\alpha l - q_1 l} + \left(\beta\,\mathrm{e}^{-2q_1 l} - 1\right)\left(1 - r\mathrm{e}^{-2\alpha l}\right)\right) - \frac{2\sigma(1+r)}{1+\sigma}\,\mathrm{e}^{-\alpha l - q_1 l}\,, \tag{13a}$$

$$\beta = \frac{\sigma-1}{\sigma+1}; \qquad \sigma = \frac{K_2}{K_1}\left(\frac{\kappa_1}{\kappa_2}\right)^{1/2} \, . \tag{13b}$$

The parameter $\sigma$ is the ratio of the thermal inertias of the absorbing and non-absorbing media (the relative thermal inertia). The necessary Laplace inversion may be accomplished by utilising contour integration techniques or by making repeated use of standard identities which involve the transform parameters $p$ and $q_1(p)$. The result obtained by the author for the surface temperature is as follows:

$$T_1(-l\,,t) = T_0 +$$

$$\frac{A_0}{\alpha^2 K_1}\left\{\begin{array}{l}\left(\mathrm{e}^{\alpha^2\kappa t} - 1\right)\left(1 + r\mathrm{e}^{-2\alpha l}\right) + \\ \sum\limits_{n=0}^{\infty}(-\beta)^n\left[\frac{\mathrm{e}^{-\alpha l}(1-\mathrm{r})}{1+\sigma}\Psi_{2n+1} + \frac{1}{2}\left(1 - r\mathrm{e}^{-2\alpha l}\right)(\beta\Psi_{2n+2} - \Psi_{2n}) - \frac{\sigma(1+r)\,\mathrm{e}^{-\alpha l}}{1+\sigma}\Phi_{2n+1}\right]\end{array}\right\} \tag{14}$$

$$\Psi_m(l,t) = -4\alpha(\kappa t)^{1/2}\,\mathrm{ierfc}\left(\frac{ml}{2(\kappa t)^{1/2}}\right) + $$
$$+\, e^{\alpha^2\kappa t}\left\{ e^{-m\alpha l}\,\mathrm{erfc}\left(\frac{ml}{2(\kappa t)^{1/2}} - \alpha(\kappa t)^{1/2}\right) - e^{m\alpha l}\,\mathrm{erfc}\left(\frac{ml}{2(\kappa t)^{1/2}} + \alpha(\kappa t)^{1/2}\right)\right\} \tag{15a}$$

$$\Phi_m(l,t) = -2\,\mathrm{erf}c\left(\frac{ml}{2(\kappa t)^{1/2}}\right) + $$
$$+\, e^{\alpha^2\kappa t}\left\{ e^{-m\alpha l}\,\mathrm{erfc}\left(\frac{ml}{2(\kappa t)^{1/2}} - \alpha(\kappa t)^{1/2}\right) + e^{m\alpha l}\,\mathrm{erfc}\left(\frac{ml}{2(\kappa t)^{1/2}} + \alpha(\kappa t)^{1/2}\right)\right\} \tag{15b}$$

where $\kappa_1$ has been abbreviated to $\kappa$. A variety of surface temperature curves may be plotted using equation (14). A case of special interest occurs when $\sigma = 1$ ($\beta = 0$), and for this case the limiting expressions for the short- and long-time regimes may be derived by simple analysis. The results are found to be

Short-time regime: $$T_1(-l,t) = T_0 + \frac{A_0\kappa t}{K_1}\left(1 + r e^{-2\alpha l}\right) \ . \tag{16a}$$

Long-time regime: $$T_1(-l,t) = T_0 + \frac{2A_0}{\alpha K_1}\left(\frac{\kappa t}{\pi}\right)^{1/2}\left(1 - e^{-\alpha l}\right)\left(1 + r e^{-\alpha l}\right) \ . \tag{16b}$$

These expressions reduce to equations (4a) and (4b) as $l \to \infty$, i.e. the problem tends to the simpler homogeneous case described in Section 2. The short-time solution can also be obtained easily from the original differential equation (5a) by setting the second-order spatial derivative to zero at the surface position and performing the time integration. The same result is also obtained by considering the behaviour of the Laplace transform solution, equation (12), as the transform variable $p$ increases indefinitely. In this limit the thermal behaviour becomes independent of the parameter $\beta$. Physically, this means that insufficient time has elapsed for the surface to be influenced by the buried non-absorbing layer by thermal conduction processes alone, and that the reflectivity of the interface is the only factor influencing the surface temperature. This solution is therefore a good approximation for the parameters used in the example calculated at the end of Section 2 for landmine depths greater than 1 cm and for irradiation times shorter than 60 s. For smaller depths, equation (12) shows that the Laplace transform can be expanded as a series of negative exponential functions in multiples of the parameter $q_1 l$. In many situations it is sufficient to take only a few of these terms in order to obtain accurate short-time behaviour.

## 4. Composite Solid with a Buried Non-absorbing Layer

A more useful model than that described in Section 3 consists of three regions: (i) a surface absorber of thickness $l$, in contact with (ii) a non-absorbing layer of thickness $L$, which in turn contacts (iii) a deeper layer of absorbing material. In this model, the second (deeper) absorbing region is infinitely thick and is assumed to be composed of the same material as the upper absorbing region. The temperature distributions within the upper and lower absorbing regions are designated $T_1(z, t)$ and $T_3(z, t)$ respectively, and the temperature distribution within the non-absorbing middle region is designated $T_2(z, t)$. The heat conduction equations together with the initial and boundary conditions describing this situation are

$$\frac{\partial^2 T_1}{\partial z^2} = \kappa_1^{-1}\frac{\partial T_1}{\partial t} - \frac{A_0}{K_1}\left\{e^{-\alpha(z+l)} + r_{\mathrm{eff}}\, e^{-\alpha(-z+l)}\right\} \quad ,-l \le z \le 0 \,, \tag{17a}$$

$$\frac{\partial^2 T_2}{\partial z^2} = \kappa_2^{-1}\frac{\partial T_2}{\partial t} \quad , \qquad 0 \le z \le L \,, \tag{17b}$$

$$\frac{\partial^2 T_3}{\partial z^2} = \kappa_1^{-1}\frac{\partial T_3}{\partial t} - \frac{A_0 t_{\mathrm{eff}}}{K_1} e^{-\alpha(z+l-L)} \quad , \qquad L \le z \le \infty \,, \tag{17c}$$

$$T_1(z,0) = T_2(z,0) = T_3(z,0) = T_0 \tag{18a}$$

$$\frac{\partial T_1}{\partial z} = 0 \quad , \qquad z = -l \,, \tag{18b}$$

$$T_1 = T_2; \;\; K_1\frac{\partial T_1}{\partial z} = K_2\frac{\partial T_2}{\partial z} \quad , \qquad z = 0 \,, \tag{18c}$$

$$\overline{T}_2 = \overline{T}_3; \;\; K_2\frac{\partial T_2}{\partial z} = K_1\frac{\partial T_3}{\partial z} \quad , \qquad z = L \,, \tag{18d}$$

$$T_3 = T_0 \quad , \qquad z = \infty \,. \tag{18e}$$

In equations (17a) and (17c) the effective reflection and transmission parameters $r_{\mathrm{eff}}$ and $t_{\mathrm{eff}}$ can be expressed as composites of the reflectivities, $r_1$ and $r_2$, of the upper and lower surfaces of the buried object respectively. By considering the forward and backward distributions of microwave energy, and allowing for transmission and reflection from each surface, one easily obtains the following exact results at vertical incidence:

$$r_{\mathrm{eff}} = \frac{r_1 + r_2 - 2r_1 r_2}{1 - r_1 r_2} \quad , \tag{18f}$$

$$t_{\mathrm{eff}} = \frac{(1-r_1)(1-r_2)}{1 - r_1 r_2} \quad . \tag{18g}$$

As required, the sum of $r_{\mathrm{eff}}$ and $t_{\mathrm{eff}}$ is unity. The corresponding transforms and boundary conditions in the transform space become

$$\frac{d^2 \overline{T_1}}{dz^2} = q_1^2\, \overline{T_1} - \kappa_1^{-1} T_0 - \frac{A_0}{K_1 p}\{e^{-\alpha(z+l)} + r_{\mathrm{eff}}\, e^{\alpha(z-l)}\} \quad , \qquad -l \le z \le 0 \,, \tag{19a}$$

$$\frac{d^2 \overline{T_2}}{dz^2} = q_2^2\, \overline{T_2} - \kappa_2^{-1} T_0 \quad , \qquad 0 \le z \le L \,, \tag{19b}$$

$$\frac{d^2\overline{T}_3}{dz^2} = q_1^2\overline{T}_3 - \kappa_1^{-1}T_0 - \frac{A_0 t_{\mathrm{eff}}}{K_1 p}\, e^{-\alpha(z+l-L)} \quad , \qquad L \le z \le \infty \, , \tag{19c}$$

$$\frac{d\overline{T}_1}{dz} = 0 \quad , \qquad z = -l \, , \tag{20a}$$

$$\overline{T}_1 = \overline{T}_2 ; \;\; K_1 \frac{d\overline{T}_1}{dz} = K_2 \frac{d\overline{T}_2}{dz} \quad , \qquad z = 0 \, , \tag{20b}$$

$$\overline{T}_2 = \overline{T}_3 ; \;\; K_2 \frac{d\overline{T}_2}{dz} = K_1 \frac{d\overline{T}_3}{dz} \quad , \qquad z = L \, , \tag{20c}$$

$$\overline{T}_3 = T_0 \quad , \qquad z = \infty \, , \tag{20d}$$

Once again, the properties of the above set of equations enable one to assume the following forms for the Laplace transforms of the temperature distributions:

$$\overline{T}_1 = A\cosh q_1(z+l) + B\sinh q_1(z+l) + D\mathrm{e}^{-\alpha(z+l)} + H\mathrm{e}^{\alpha(z-l)} + E \quad , \tag{21a}$$

$$\overline{T}_2 = C\cosh q_2 z + F\sinh q_2 z + G \quad , \tag{21b}$$

$$\overline{T}_3 = M\mathrm{e}^{-q_1(z-L)} + P\mathrm{e}^{-\alpha(z+l-L)} + N \quad , \tag{21c}$$

where, as previously, $q_i(p)^2 = p/\kappa_i$ . For this problem the variables $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $M$, $N$ and $P$ have to be determined. Since effectively eleven equations are at hand to describe the flow of heat, the problem is fully determined. Substituting the assumed expressions into the differential equations and equating like terms, one obtains

$$E = G = N = \frac{T_0}{p} \quad , \tag{22a}$$

$$D = \frac{A_0}{K_1 p(q_1^2 - \alpha^2)} \quad , \tag{22b}$$

$$B = \frac{\alpha}{q_1} D\left(1 - r_{\mathrm{eff}}\, \mathrm{e}^{-2\alpha l}\right) \quad , \tag{22c}$$

$$H = r_{\mathrm{eff}} D \quad , \tag{22d}$$

$$P = t_{\mathrm{eff}} D \quad , \tag{22e}$$

together with a set of four linear equations linking the parameters $A$, $C$, $F$ and $M$. The desired temperature transform at the surface is

$$\overline{T}_1(-l, p) = A + D + H\mathrm{e}^{-2\alpha l} + E \tag{23}$$

so it only remains to solve for the parameter $A$. This can be done by applying a standard inversion technique to the linear matrix equation for the four unknown quantities:

$$\begin{bmatrix} \sinh q_1 l & 0 & -\sigma & 0 \\ \cosh q_1 l & -1 & 0 & 0 \\ 0 & \sigma \sinh\ q_2 L & \sigma \cosh\ q_2 L & 1 \\ 0 & \cosh\ q_2 L & \sinh\ q_2 L & -1 \end{bmatrix} \begin{bmatrix} A \\ C \\ F \\ M \end{bmatrix} = \begin{bmatrix} A_2 \\ -A_1 \\ -\dfrac{\alpha}{q_1} t_{\mathrm{eff}} \\ \mathrm{e}^{-\alpha l} t_{\mathrm{eff}} \end{bmatrix} D \quad , \tag{24}$$

leading to the general result

$$\bar{T}_1(-l, p) = p^{-1} T_0 + D \left\{ 1 + r_{\mathrm{eff}}\, \mathrm{e}^{-2\alpha l} + \frac{\sigma\ [(1 - \ \alpha / q_1)\ \mathrm{e}^{-\alpha l} t_{\mathrm{eff}} - A_1 X\,] + A_2 Y}{\Delta} \right\} \ , \tag{25}$$

where

$$A_1(p) = (1 + r_{\mathrm{eff}})\mathrm{e}^{-\alpha l} + (1 - r_{\mathrm{eff}}\mathrm{e}^{-2\alpha l})\, \frac{\alpha}{q_1}\ \sinh q_1 l \quad , \tag{26a}$$

$$A_2(p) = \frac{\alpha}{q_1}\ \{(1 - r_{\mathrm{eff}})\mathrm{e}^{-\alpha l} - (1 - r_{\mathrm{eff}}\, \mathrm{e}^{-2\alpha l}) \cosh q_1 l\} \quad , \tag{26b}$$

$$X(p) = \cosh q_2 L + \sigma \sinh q_2 L \quad , \tag{27a}$$

$$Y(p) = \sinh q_2 L + \sigma \cosh q_2 L \quad , \tag{27b}$$

$$\Delta(p) = Y(p) \sinh q_1 l + X(p) \sigma \cosh q_1 l \ \cdot \tag{28}$$

Using the inversion theorem, one obtains for the surface temperature the formal solution:

$$T_1(-l\,, t) = T_0 + \frac{A_0}{\alpha^2 K_1}\ \mathrm{e}^{-\alpha(l\ +\ z)} \left( \mathrm{e}^{\alpha^2 \kappa t} - 1 \right) \left( 1 + r_{\mathrm{eff}}\, \mathrm{e}^{-2\alpha l} \right) + \frac{A_0 \kappa_1}{2\pi i K_1} \int_{\gamma\, -\, \mathrm{i}\infty}^{\gamma\, +\, \mathrm{i}\infty} \mathrm{e}^{\lambda t}\, \frac{[1 - \alpha\,(\kappa/\lambda)^{1/2}]\, \sigma\, \mathrm{e}^{-\alpha l} t_{\mathrm{eff}} + A_2(\lambda)\, Y(\lambda) - A_1(\lambda) \sigma X(\lambda)}{\lambda(\lambda - \kappa\alpha^2) \Delta(\lambda)}\, \mathrm{d}\lambda\, . \tag{29}$$

In the contour integration procedure $\lambda$ is treated as a complex variable, and the integrand has a series of poles along the imaginary axis. Because of the branch point at $\lambda = 0$, it is necessary to make a branch cut along the real axis.

Notwithstanding the general complexity, some cases may be evaluated with relative ease, for example the situation in which one may assume the value

$$\sigma = \frac{K_2}{K_1} \left( \frac{\kappa_1}{\kappa_2} \right)^{1/2} = 1 \tag{30}$$

represents an important and commonly occurring special case. The recent work of Hermann and Chant (1999) indicates that some larger landmines produce $\sigma$ values which cluster around unity over a wide range of soil types. For this case equation (25) reduces to the surface temperature transform

$$\bar{T}_1(-l,p) = p^{-1}\bar{T}_0 + D\begin{Bmatrix}\left(1-\dfrac{\alpha}{q_1}\right)\left(1+t_{\mathrm{eff}}\,\mathrm{e}^{-\alpha l-q_1 l-q_2 L}-\mathrm{e}^{-\alpha l-q_1 l}\right)\\ +r_{\mathrm{eff}}\left(1+\dfrac{\alpha}{q_1}\right)\mathrm{e}^{-\alpha l}\left(\mathrm{e}^{-\alpha l}-\mathrm{e}^{-q_1 l}\right)\end{Bmatrix} \tag{31}$$

Ordinary Laplace inversion in this case is straightforward, and produces the result

$$T_1(-l,t) = T_0 + \frac{A_0}{2\alpha^2 K_1}\begin{Bmatrix}(1+r_{\mathrm{eff}}\,\mathrm{e}^{-2\alpha l})\Phi_0(t)-(1-r_{\mathrm{eff}}\mathrm{e}^{-2\alpha l})\Psi_0(t)+\\ t_{\mathrm{eff}}\,\mathrm{e}^{-\alpha l}[(\Phi_1(\Lambda,t)-\Psi_1(\Lambda,t)]+(1-r_{\mathrm{eff}})\Psi_1(l,t)-(1+r_{\mathrm{eff}})\Phi_1(l,t)]\end{Bmatrix} \tag{32}$$

where

$$\Lambda = l+\mu L; \qquad \mu = (\kappa_1/\kappa_2)^{1/2}. \tag{33}$$

**Table 1. Soil, radiation and landmine parameters used in the sample calculation**

| | |
|---|---|
| Soil type/moisture level: | Sandy 4% |
| Loss tangent: | 0.06 |
| Relative dielectric constant: | 6.0 |
| Absorption coefficient ($m^{-1}$): | 7.54 |
| Thermal conductivity ($W\ m^{-1}\ K^{-1}$): | 0.45 |
| Thermal diffusivity ($m^2\ s^{-1}$): | $0.30 \times 10^{-6}$ |
| Landmine thickness (cm): | 12.0 |
| Relative thermal inertia σ: | 1.0 |
| Mine-surface reflectivities: | $r_1 = r_2 = 0.1$ |
| Power density ($kW\ m^{-2}$): | 8.36 |
| Power dissipation per volume ($J\ m^{-3}\ s^{-1}$): | $2.88 \times 10^5$ |

Note that the functions $\Psi_m(l, t)$ and $\Phi_m(l, t)$ reduce to a degenerate case when $m = 0$ (i.e. they no longer depend upon the thickness of the upper layer), hence the altered notation for the lowest order functions within equation (32). The differential change in surface temperature (from the irradiated free soil value) has been plotted as a function of the irradiation time for the $\sigma = 1$ case using (32). The representative soil, radiation and landmine parameter values used in plotting the temperature curves are listed in Table 1. These values have all been obtained from data provided in the paper by Hermann and Chant (1999). The results are presented in Fig. 2. The full curves correspond to landmine upper-face depths of (lower to upper) 1, 2 and 5 mm, while the three positive dashed curves correspond to the depths of (upper to lower) 1, 2 and 3 cm. The lower dotted line shows the increasing temperature deficit with irradiation time for a surface-level mine. It should be noted that the radiant power density employed in this example is quite modest, and one may obtain larger temperature-time gradients if required simply by raising the power level. Ascertaining the most appropriate power level in practice depends in turn

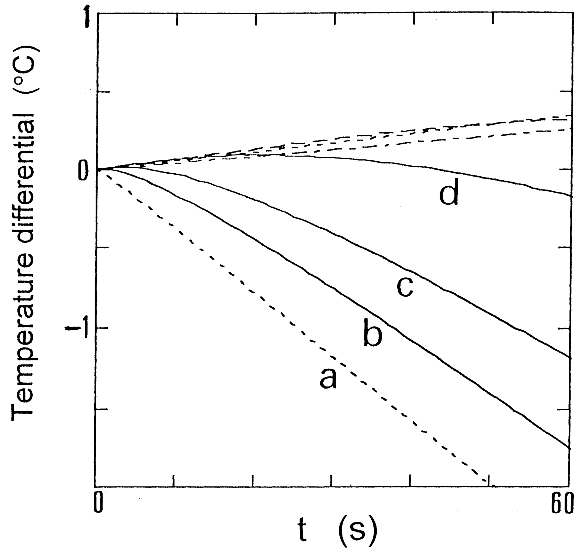



**Fig. 2.** Temperature differential from the free-soil value plotted as a function of the irradiation time in seconds. The mine has thickness $L$ = 12 cm, with a reflectivity value of 0.1 at both its upper and lower faces. The labelled curves represent upper-face depths of (*a*) zero (surface level), (*b*) 1 mm, (*c*) 2 mm and (*d*) 5 mm; the dashed curves for the positive anomalies correspond to the depths 1, 2 and 3 cm (upper to lower) respectively.

upon detector sensitivity. In this context it should be noted that many modern IR detectors have sensitivities of 0.1 °C or better.

The results in Fig. 2 clearly show that temperature profiles can exhibit either 'hot spots' (elevated local temperature) or 'cold spots' (depressed local temperature), depending upon the mine depth and to some extent the irradiation time. The hot spots appear when sufficient radiant reflection occurs at the upper and lower mine faces, and when the subsequent enhanced absorption within the upper layer is able to overcome the temperature-lowering effect of the sub-surface (non-absorbing) region. It is noteworthy that hot spots and cold spots were both observed by Fueloep and Bird (1996) in their microwave experiments with different landmine types, and that sample calculations reported by Hermann and Chant (1999) indicate temperature anomalies in approximate agreement with those measurements. Evidently there is a 'break-even' depth for which the surface temperature remains unchanged.

One can easily derive the short-time behaviour of the three-layer case by allowing the transform variable $p$ (i.e. the $q_i$ parameters) to become large in equation (31). The inverse Laplace transform in this limit is

$$T_1(-l, t) = T_0 + \frac{A_0 \kappa t}{K_1}(1 + r_{\text{eff}} e^{-2\alpha l}) \tag{34}$$

and is a generalisation of equation (16a), again supposing that $t << l^2 / \kappa_1$ . It may be noted that the upper dashed curves in Fig. 2 (corresponding to $l$ = 1, 2 and 3 cm) tend to this limiting linear case. The short-time behaviour of a shallower, less-absorbing layer may be obtained by expanding the Laplace transform of $T_1$ as a series of negative exponentials.

One of the clear advantages of using the more general three-layer model, rather than the two-layer model, is that one can satisfactorily account for reflection from both the upper and lower surfaces of the buried object. The two-layer model will often provide an approximate and satisfactory description of the thermal effects when both of the reflection coefficients are small ($r_1, r_2 << 1$), and for this case the effective reflection coefficient from the upper face may be taken to be $r_1 + r_2$ as a first approximation. For larger reflection coefficients one has no choice and the three-layer model must be used. The three-layer model will also obviously provide a much better description with some geometries, for example when the buried object is relatively thin.

## 5. Discussion

The layered-medium model of radiant absorption explored within this paper possesses a number of limitations, some associated with the simplifications required to either remove inessential features or to make the model more tractable. One such simplification is the assumption that the different absorbing regions possess identical thermal parameters. However, this restriction can be easily relaxed if required without unduly increasing the complexity of the analysis. The generalisation would thus be a relatively small refinement. Further assumptions made in the present modelling are (a) that the contact resistances (at the surfaces between the absorbing layer and the non-absorbing layer) are zero, and (b) that no heat is lost from the soil–atmosphere interface. These omissions have been justified because, within the short-time regime at least, they may often be treated as second-order effects. Nevertheless, it is recognised that a more comprehensive analysis would aim to include these physical processes as part of a more realistic treatment of the heat fluxes across the boundary surfaces. The main aim of this paper has been to explore the overall usefulness of the methodology.

The time-dependent analysis has been kept fairly general, since it is envisaged that the results could be useful in a variety of contexts, including applications which involve both short-time and long-time radiant exposures. Within the domain of microwave absorption several potential applications of this type of heating model come to mind, including microwave cooking and the irradiation of biological materials, perhaps over longer time scales. Also, notwithstanding that the application considered in this paper utilises microwaves, the analytical results may be relevant and applicable to radiant absorption at other wavelengths.

Some obvious physical requirements would also hinder the applicability of the proposed moisture-based technique to landmine detection, for example the technique will work optimally only within a certain soil moisture range, and the moisture level requirement might well create a problem if landmine clearances cannot be scheduled to coincide with those times of the year for which the surface moisture levels are close to optimal. Other possible problem areas include the likely necessity to employ a portable power generator, also operator safety aspects associated with the generation of thousands of watts of microwave energy (somewhat greater than the power densities created by microwave-oven magnetrons), and also ground reflection problems.

If very much longer periods of irradiation are involved, and the lateral dimensions of the buried object are small, it might be necessary to retain transverse terms within the model conduction equation and to allow for the transverse flow of heat. The validity and usefulness of the model developed, with or without the transverse features, will require adequate laboratory and field testing under realistic conditions.

In order to reduce the false alarm rate, it also would be useful to image the area of interest during the irradiation process at two quite different times. When coupled with the magnitude and shape of the temperature anomaly, this technique would, in principle, be able to provide some target classification characteristics.

## 6. Conclusions

The physical processes underpinning a proposed technique for detecting and recognising non-absorbing sub-surface bodies, based on the microwave irradiation of moisture-laden layers assumed to surround the object, have been analysed mathematically. A one-dimensional model has been found to be useful and adequate for describing short-time exposures (defined as $t << d^2 / \kappa$, where $d$ is the penetration depth and $\kappa$ is the diffusivity of the absorbing medium). The one-dimensional approximation may also be valid in other circumstances, however its principal value lies in providing a relatively simple approach to estimating the temperature disturbance and in identifying the important physical process and parameters.

The analytical results have been used to assess the impact of using microwave radiation to enhance the thermal signatures of certain types of landmines (usually the plastic-cased varieties, but not the metallic-cased types) embedded within moist soils. The results may also be used to explore the possibility of distinguishing between different types of landmines on the basis of the magnitude, sign and time evolution of the accompanying surface temperature anomalies. Each irradiated area of concern may be rapidly scanned in the infrared and its thermal signature analysed. Apart from its application to landmine detection, the analysis should be useful within other contexts where radiation might be used to effect sub-surface heating.

## Acknowledgments

The author is grateful to DSTO colleagues Ian Chant and Karl Fueloep for helpful discussions and the provision of useful information relating to microwave absorption within soils, as well as to Garry Newsam who reviewed the manuscript and suggested some improvements. One of the referees corrected some errors and provided helpful insights which have improved the quality of the presentation.

## References

Carslaw, H. S., and Jaeger, J. C. (1990). 'Conduction of Heat in Solids', 2$^{nd}$ edn (Clarendon Press: Oxford).

Carter, L. J., Bryant, G. H., Le Fevre, M., and Wong, W. C. (1996). *In* 'Detection of Abandoned Land Mines', IEE Conference Publication no. 431.

Fueloep, K., and Bird, R. (1996). DSTO (Australia) Report No. DSTO-TR-0402.

Fueloep, K., and Hall, J. (1996). DSTO (Australia) Report No. DSTO-TR-0295.

Hermann, J. A., and Chant, I. J. (1999). SPIE Proceedings 3710 (AeroSense 1999, Detection and Remediation Technologies for Mines and Mine-like Targets IV), p. 14.

Le Fevre, M. (1995). University of Auckland (New Zealand) Report EE95/67, Dept of Elect. & Electron. Eng., University of Auckland.

Li, P., Maad, A., Moshary, F., Arend, M. F., and Ahmed, S. (1995). *Appl. Optics* **34**, 5809.

Miller, A. (1997). CSIRO (Australia) Report No. DMS-C95/95, CSIRO Div. of Math. & Stat.

Spiegel, M. R. (1965). 'Laplace Transforms', Ch. 7 (Schaum Publishing: New York).

Manuscript received 4 January, accepted 22 May 2000