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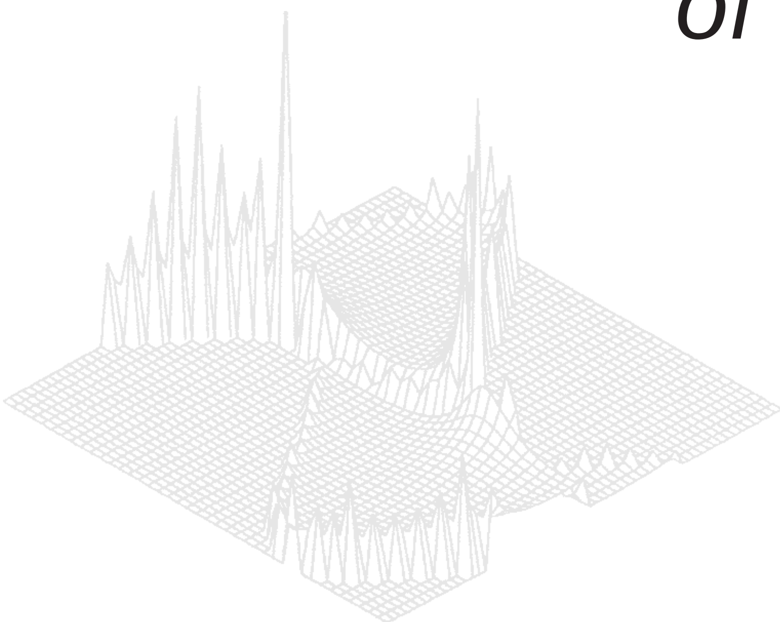

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Preparation of Motion Entangled Coherent States of Two Cavity Mirrors

Shi-Biao Zheng

Department of Electronic Science and Applied Physics,
Fuzhou University, Fuzhou 350 002, P. R. China.
email: sbzheng@pub5.fz.fj.cn

Abstract

A scheme is proposed for the generation of entangled coherent states of two spatially separated cavity mirrors. In the scheme, a two-level atom is sent through two cavities, each having a movable mirror, to produce an entangled photon state for the cavity fields. Then the optomechanical effects further entangle the mirror motions with the cavity fields. A second two-level atom, passing through the cavities, is state-selectively measured, which reduces the mirror motions to an entangled coherent state. We also show how to distinguish such an entangled state from a classical mixture.

1. Introduction

Over the past few years, much effort has been directed to the so-called Schrödinger cat states (Schrödinger 1935), i.e. superpositions of macroscopically distinguishable quantum states. In quantum optics these states are usually given as superpositions of two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, which are separated in phase by π. Though formed by quantum states closest to the classical ones, such superposition states may exhibit various nonclassical properties, such as squeezing and sub-Poissonian statistics (Janszky *et al.* 1993, 1995; Janszky and Vinogradov 1990; Xia and Guo 1989). Recently, such cat states have been realised for both a cavity field (Brune *et al.* 1996) and the motion of a trapped ion (Monroe *et al.* 1996).

In a recent paper, Mancini *et al.* (1997) have shown that a cavity with a movable mirror can also be used to produce Schrödinger cat states of the cavity field. More recently, Bose *et al.* (1997) have shown that such a system can lead to a large variety of nonclassical states of the cavity field. Moreover, it is shown that the mirror can also be prepared in a Schrödinger cat state with many components by a quadrature measurement of the cavity field after its interaction with the moving mirror. The idea of Bose *et al.* (1997) offers a way to generate nonclassical states for a macroscopic object. Recently, we have proposed a scheme to put the mirror into the even or odd coherent states (Zheng 1998).

On the other hand, there have been multi-mode generalisations of the cat states, which are called entangled coherent states (Sanders 1992*a*, 1992*b*), also referred to as superpositions of two-mode coherent states (Chai 1992; Ansari and Man'ko 1994; Dodonov *et al.* 1995). These superposition states may exhibit various nonclassical properties, such as two-mode squeezing and violation of the Cauchy–Schwarz inequality. It has been shown that, under certain conditions, superpositions of two-mode coherent states can exhibit various nonclassical features such as sub-Poissonian photon number statistics, two-mode squeezing, and violations of the Cauchy–Schwarz inequalities (Chai 1992). The strong correlations between these modes can be responsible for the nonclassical features especially

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for the two-mode case. A number of schemes have been proposed for the generation of entangled coherent states for light fields (Sanders 1992*a*, 1992*b*; Wielinga and Sanders 1993; Davidovich *et al.* 1993; Guo and Zheng 1997) and the motions of a trapped ion (Gerry 1997). In this paper we propose a method for preparing such states for two spatially separated cavity mirrors.

This paper is organised as follows. In Section 2 we present a scheme for preparing entangled coherent states for two separated mirrors. In Section 3 we show how to distinguish an entangled coherent state from an incoherent mixture. The conclusion appears in Section 4.

2. Generation of Entangled Coherent States

We consider the system composed of a cavity field and a movable mirror. Treating the mirror as a quantum harmonic oscillator we obtain the Hamiltonian for such a system as (Bose *et al.* 1997; Mancini *et al.* 1997)

$$H = \hbar\omega_0 a^+ a + \hbar\omega_m b^+ b - \hbar g a^+ a(b^+ + b), \tag{1}$$

where a^+ and b^+ denote the creation operators for the cavity mode with frequency ω_0 and mirror with frequency ω_m, respectively, and

$$g = \frac{\omega_0}{L}\sqrt{\frac{\hbar}{2m\omega_m}}, \tag{2}$$

with L and m being the length of the cavity and mass of the mirror respectively. In the interaction picture (omitting the free evolution of the field), the relevant time evolution operator is

$$U(t) = e^{ik^2(a^+a)^2[\omega_m t - \sin(\omega_m t)]}\, e^{ka^+a(\eta b^+ - \eta^* b)}\, e^{-ib^+ b\omega_m t}, \tag{3}$$

where $k = g/\omega_m$ and $\eta = 1 - e^{-i\omega_m t}$.

Our purpose is to prepare the entangled coherent states for the mirror motions of two cavities of the above-mentioned type. We assume that the cavity fields are initially in the vacuum state $|0\rangle_1\,|0\rangle_2$, and the movable mirrors in the coherent state $|\alpha\rangle_1\,|\alpha\rangle_2$. Zurek *et al.* (1993) have found the coherent state as the minimum entropy state under a particular condition. By lowering the temperature the mirrors (a harmonic oscillator) may be put in the vacuum state, which can be converted into a coherent state by some kind of kick. In order to generate entangled coherent states for the mirrors we require two-level atoms resonant with the cavity fields. Suppose an atom of such a type initially prepared in the excited state $|e\rangle_1$ is sent through the cavity system. We assume that the atom–field coupling strength Ω is much larger than the mirror frequency ω_m and the field–mirror coupling strength g. In this case during the atom passing through the cavity the field–mirror coupling can be neglected. Upon the passage of the atom through the first cavity, the whole system is in the state

$$\sqrt{\tfrac{1}{2}}\,[\cos(\Omega\tau_1)\,|e\rangle_1\,|0\rangle_1 - i\sin(\Omega\tau_1)\,|g\rangle_1\,|1\rangle_1]\,|0\rangle_2\,|\alpha\rangle_1\,|\alpha\rangle_2, \tag{4}$$

where τ_1 is the interaction time of the atom with the first cavity. After the passage through the second cavity, the state of the whole system is

$$\sqrt{\tfrac{1}{2}}\,[\cos(\Omega\tau_1)\cos(\Omega\tau_2)\,|e\rangle_1\,|0\rangle_1\,|0\rangle_2 - i\cos(\Omega\tau_1)\sin(\Omega\tau_2)\,|g\rangle_1\,|0\rangle_1\,|1\rangle_2 \\ - i\sin(\Omega\tau_1)\,|g\rangle_1\,|1\rangle_1\,|0\rangle_2]\,|\alpha\rangle_1\,|\alpha\rangle_2. \tag{5}$$

We select the interaction times appropriately so that $\Omega\tau_1 = \pi/4$, $\Omega\tau_2 = \pi/2$. Then we obtain

$$\frac{-i}{\sqrt{2}}[|0\rangle_1\,|1\rangle_2 + |1\rangle_1\,|0\rangle_2]|\alpha\rangle_1|\alpha\rangle_2, \tag{6}$$

with the atom left in the ground state $|g\rangle_1$. Here we assume the transit time of the atom through each cavity is $\pi/2\Omega$. The interaction time of the atom with the first cavity field can be shortened by using a static electric field to Stark shift the atom out of resonance for a proper time during the passage through this cavity (Davidovich *et al.* 1994). After an interaction time t the field–mirror coupling leads to the state

$$\begin{aligned}\frac{-i}{\sqrt{2}}e^{ik^2[\omega_m t-\sin(\omega_m t)]}[&|1\rangle_1\,|0\rangle_2\,|\alpha e^{-i\omega_m t} + k(1-e^{-i\omega_m t})\rangle_1\,|\alpha e^{-i\omega_m t}\rangle_2\\ &+ |0\rangle_1\,|1\rangle_2\,|\alpha e^{-i\omega_m t}\rangle_1\,|\alpha e^{-i\omega_m t} + k(1-e^{-i\omega_m t})\rangle_2],\end{aligned} \tag{7}$$

where we have discarded the atomic state. We choose the interaction time t appropriately so that $\omega_m t = \pi$. Thus we have

$$\frac{-i}{\sqrt{2}}e^{ik^2\pi}[|1\rangle_1\,|0\rangle_2\,|-\alpha+2k\rangle_1\,|-\alpha\rangle_2 + |0\rangle_1\,|1\rangle_2\,|-\alpha\rangle_1\,|-\alpha+2k\rangle_2]. \tag{8}$$

We now send a second resonant two-level atom, initially in the ground state $|g\rangle_2$ through the cavity system. After the first cavity, the system is in the state

$$\begin{aligned}\frac{-i}{\sqrt{2}}e^{ik^2\pi}\{&[\cos(\Omega\tau_1')\,|g\rangle_2\,|1\rangle_1\,|0\rangle_2 - i\sin(\Omega\tau_1')\,|e\rangle_2\,|0\rangle_1\,|0\rangle_2]|-\alpha+2k\rangle_1\,|-\alpha\rangle_2\\ &+ |g\rangle_2\,|0\rangle_1\,|1\rangle_2\,|-\alpha\rangle_1\,|-\alpha+2k\rangle_2\}.\end{aligned} \tag{9}$$

When the atom emerges from the second cavity the system evolves to

$$\begin{aligned}\frac{-i}{\sqrt{2}}e^{ik^2\pi}\{&[\cos(\Omega\tau_1')\,|g\rangle_2\,|1\rangle_1\,|0\rangle_2 - i\sin(\Omega\tau_1')\cos(\Omega\tau_2')\,|e\rangle_2\,|0\rangle_1\,|0\rangle_2\\ &- \sin(\Omega\tau_1')\sin(\Omega\tau_2')\,|g\rangle_2\,|0\rangle_1\,|1\rangle_2]|-\alpha+2k\rangle_1\,|-\alpha\rangle_2\\ &+ [\cos(\Omega\tau_2')\,|g\rangle_2\,|0\rangle_1\,|1\rangle_2 - i\sin(\Omega\tau_2')\,|e\rangle_2\,|0\rangle_1\,|0\rangle_2]|-\alpha\rangle_1\,|-\alpha+2k\rangle_2\}.\end{aligned} \tag{10}$$

We choose the interaction times τ_1' and τ_2' appropriately so that $\Omega\tau_1' = \pi/2$, $\Omega\tau_2' = \pi/4$. Then we have

$$\begin{aligned}\frac{-i}{2}e^{ik^2\pi}\{&[-i|e\rangle_2\,|0\rangle_1\,|0\rangle_2 - |g\rangle_2\,|0\rangle_1\,|1\rangle_2]|-\alpha+2k\rangle_1\,|-\alpha\rangle_2\\ &+ [|g\rangle_2\,|0\rangle_1\,|1\rangle_2 - i|e\rangle_2\,|0\rangle_1\,|0\rangle_2]|-\alpha\rangle_1\,|-\alpha+2k\rangle_2\}.\end{aligned} \tag{11}$$

It should be noted that in a realistic experiment the two atoms will have the same velocity. Thus, during the passage of the second atom through the second cavity we should use a Stark field to shorten the corresponding atom–field interaction time as we do during the passage of the first atom through the first cavity. The field–mirror interaction time is essentially the time interval between the two atoms.

We now perform a state-selective measurement on the atom. If we detect the atom in the excited state $|e\rangle_2$, the two cavity mirrors are projected to the entangled coherent state

$$N_+(|-\alpha+2k\rangle_1\,|-\alpha\rangle_2 + |-\alpha\rangle_1\,|-\alpha+2k\rangle_2), \tag{12}$$

and the cavity fields are left in the vacuum state $|0\rangle_1\,|0\rangle_2$. Here N_+ is a normalisation factor. On the other hand, the detection of the ground state $|g\rangle_2$ leads to the entangled state

$$N_-(|-\alpha+2k\rangle_1\,|-\alpha\rangle_2 - |-\alpha\rangle_1\,|-\alpha+2k\rangle_2), \tag{13}$$

and the cavity fields are left in the state $|0\rangle_1\,|1\rangle_2$. In this case the motion of the mirror of the second cavity will be further modified by the photon in the second cavity. Thus, it is difficult to observe this entangled state. Choosing the value of α appropriately we can obtain entangled coherent states of special interest. For the case where $\alpha = 0$ we obtain the following entangled state (Sanders 1992*a*, 1992*b*):

$$N_+(|2k\rangle_1\,|0\rangle_2 + |0\rangle_1\,|2k\rangle_2). \tag{14}$$

When $\alpha = k$ we obtain a two-mode cat state of another type (Chai 1992; Ansari and Man'ko 1994; Dodonov *et al.* 1995):

$$N_+(|k\rangle_1\,|-k\rangle_2 + |-k\rangle_1\,|k\rangle_2). \tag{15}$$

3. An Entangled Coherent State versus an Incoherent Mixture

In this section we suggest a method to distinguish the superposition state of equation (12) from a classical mixture of the form

$$\tfrac{1}{2}[|-\alpha+2k\rangle_1\,|-\alpha\rangle_{22}\,\langle-\alpha|_1\,\langle-\alpha+2k| + |-\alpha\rangle_1\,|-\alpha+2k\rangle_{22}\,\langle-\alpha+2k|_1\,\langle-\alpha|]. \tag{16}$$

In order to do so we send an atom initially in the excited state $|e\rangle_3$ through the cavity system. We choose the interaction times of the atom with the two cavities in the same way as those of the first atom. After the cavity fields interact with the mirror for a time π/ω_m, another atom initially in the ground state $|g\rangle_4$ is sent through the cavity system. We choose the atom–field interaction times in the same way as those of the second atom. If the mirror state is given by equation (12) the system finally evolves to

$$\begin{aligned}&\frac{-iN_+}{2}e^{ik^2\pi}\{|g\rangle_4\,|0\rangle_1\,|1\rangle_2[|\alpha-2k\rangle_1\,|\alpha+2k\rangle_2 - |\alpha+2k\rangle_1\,|\alpha-2k\rangle_2]\\ &\quad - i|e\rangle_4\,|0\rangle_1\,|0\rangle_2[2|\alpha\rangle_1\,|\alpha\rangle_2 + |\alpha-2k\rangle_1\,|\alpha+2k\rangle_2 + |\alpha+2k\rangle_1\,|\alpha-2k\rangle_2]\}.\end{aligned} \tag{17}$$

Now the probability of finding the atom in the excited state $|e\rangle_4$ is given by

$$P_e = \frac{1}{4\left(1+e^{-4k^2}\right)}\left(3+e^{-16k^2}+4e^{-4k^2}\right). \tag{18}$$

For simplicity we here have assumed that α is real. On the other hand, if the mirror is initially in a classical mixture of equation (16) the probability of finding the atom in the state $|e\rangle_4$ is

$$P'_e = \tfrac{1}{2}\left(1+e^{-4k^2}\right). \tag{19}$$

Therefore, by measuring the probability of the atom in the excited state we can determine whether the two cavity mirrors are in a superposition or a incoherent mixture of two two-mode coherent states.

4. Conclusion

It is necessary to give a brief discussion on the experimental feasibility of the proposed scheme. We set the following parameters (Bose *et al.* 1997; Mancini *et al.* 1997): $\omega_0 \approx 10^{16}\,\mathrm{s}^{-1}$, $\omega_m \approx 10^3\,\mathrm{s}^{-1}$, $L \approx 1\,\mathrm{cm}$, $m \approx 10\,\mathrm{mg}$, $T_c \approx 10^{-2}\,\mathrm{s}$, with T_c being the cavity lifetime. In this case we obtain $g \approx 0.7 \times 10^3\,\mathrm{s}^{-1}$. The required field–mirror interaction time is about $\pi \times 10^{-3}$ s. The atomic radiative time T_r is of the order of 10^{-2} s, and the atom–field coupling strength can be set to $\Omega = 2\pi \times 24\,\mathrm{kHz}$ (Brune *et al.* 1996), much larger than g. The corresponding atom–field interaction time is $\pi/2\Omega \sim 10^{-5}$ s. Thus, both the atom–field interaction time and the field–mirror coupling time are shorter than T_c and T_r. At present, we are not able to make an estimate of the decoherence time-scale T_m of the mirror motion. However, T_m depends on ω_m as $T_m \sim (\omega_m)^3$ (Bose *et al.* 1997). Thus, in principle we can control T_m by choosing ω_m appropriately. For the above-mentioned values of g and ω_m, we get $k = g/\omega_m = 0.7$. Then we have $P_e \simeq 0.78$ and $P'_e \simeq 0.57$. The difference between these two probabilities should be large enough for us to distinguish an entangled state from a classical mixture.

In summary, we have proposed a scheme for the generation and detection of entangled coherent states of the motions of two cavity mirrors. Our scheme provides a way to produce and measure entanglement between two spatially separated macroscopic objects.

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References

Ansari, N. A., and Man'ko, V. I. (1994). *Phys. Rev.* A **50**, 1942.
Bose, S., Jacobs, K., and Knight, P. L. (1997). *Phys. Rev.* A **56**, 4175.
Brune, M., Hagley, E., Dreyer, J., Maitre, X., Maali, A., Wunderlich, C., Raimond, J. M., and Haroche, S. (1996). *Phys. Rev. Lett.* **77**, 4887.
Chai, C. (1992). *Phys. Rev.* A **46**, 7187.
Davidovich, L., Maali, A., Brune, M., Raimond, J. M., and Haroche, S. (1993). *Phys. Rev. Lett.* **71**, 2360.
Davidovich, L., Zagury, N., Brune, M., Raimond, J. M., and Haroche, S. (1994). *Phys. Rev.* A **50**, R895.
Dodonov, V. V., Man'ko, V. I., and Nikonov, D. E. (1995). *Phys. Rev.* A **51**, 3328.
Gerry, C. C. (1997). *Phys. Rev.* A **55**, 2478.
Guo, G. C., and Zheng, S. B. (1997). *Opt. Commun.* **133**, 142.
Janszky, J., and Vinogradov, A. V. (1990). *Phys. Rev. Lett.* **64**, 2771.
Janszky, J., Domokos, P., and Adam, P. (1993). *Phys. Rev.* A **48**, 2213.
Janszky, J., Domokos, P., Szabo, S., and Adam, P. (1995). *Phys. Rev.* A **51**, 4191.
Mancini, S., Man'ko, V. I., and Tombesi, P. (1997). *Phys. Rev.* A **55**, 3042.
Monroe, C., Meekhof, D. M., King, B. E., and Wineland, D. J. (1996). *Science* **272**, 1131.
Sanders, B. C. (1992*a*). *Phys. Rev.* A **45**, 6811.

Sanders, B. C. (1992*b*). *Phys. Rev.* A **46**, 2966.
Schrödinger, E. (1935). *Naturwissenschaften* **23**, 807; 823; 844.
Wielinga, B., and Sanders, B. C. (1993). *J. Mod. Opt.* **40**, 1923.
Xia, Y., and Guo, G. (1989). *Phys. Lett.* A **136**, 281.
Zheng, S. B. (1998). *Quant. Semiclass. Opt.* **10**, 657.
Zurek, W. H., Habib, S., and Paz, J. P. (1993). *Phys. Rev. Lett.* **70**, 1187.

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