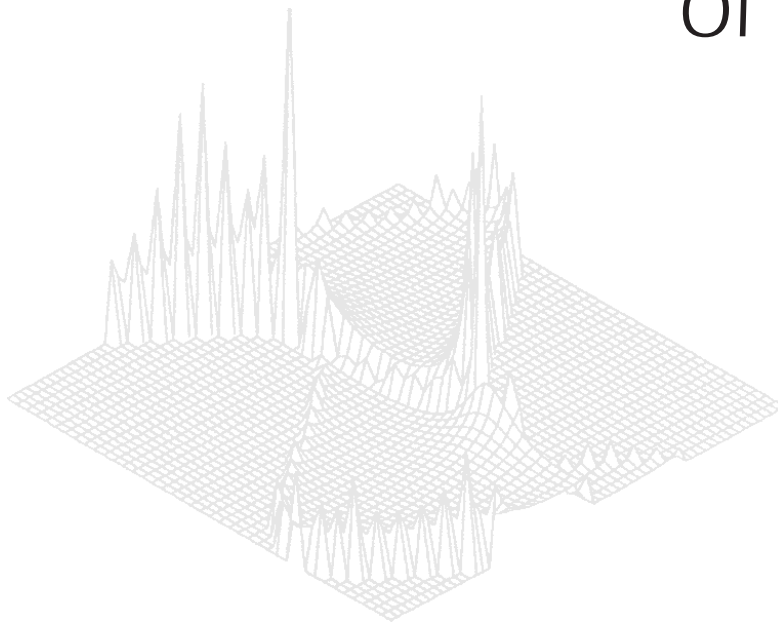

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Electromagnetic and Gravitational Decay of the Higgs Boson

R. Delbourgo and Dongsheng Liu

School of Mathematics and Physics, University of Tasmania,
Hobart, Tas. 7001, Australia.
email: bob.delbourgo@utas.edu.au, d.liu@utas.edu.au

Abstract

The decays of a scalar particle, of either parity, into two photons or into two gravitons are evaluated. The effective interactions are of the form ϕFF , $\phi F\tilde{F}$ or ϕRR , $\phi R\tilde{R}$; in particular, the Higgs meson decay mode into gravitons is tiny and can be neglected.

In two recent papers, Srivastava and Widom (2000a, 2000b) have claimed that the decay width of the Higgs meson into two gravitons is given by $\sqrt{2}G_F m_H^3/16\pi$. In their result, which they say stems from an effective interaction $\phi R/\langle\phi\rangle$, Newton's constant disappears and gets replaced by the Fermi constant, leading to a large magnitude for the process. If their result were true, it would be counter-intuitive to the notion that gravitational interactions are miniscule in particle physics and it would imply that the Higgs mesons disappears *very quickly* into a puff of gravitational plus electromagnetic radiation! In this paper we revisit this interesting problem and derive the correct magnitudes for the decay amplitudes, be the decaying particle scalar or pseudoscalar; our faith in the weakness of induced gravitational (and electromagnetic) effects is happily restored. We consider parity-conserving decays of a 0^+ or 0^- particle into two photons γ or into two gravitons h . A simple helicity amplitude analysis (Delbourgo and Liu 1998) shows that either process is governed by just one reduced helicity amplitude, $\langle k, \lambda; k', \lambda'|S\rangle|0\rangle$, where $\lambda = \lambda' = 1$ or 2 , because of parity conservation. The same conclusion is reached by a covariant amplitude analysis, using the twin principles of gauge and general covariance. If ε represents the external wavefunction of an incoming massless particle (γ or h), we may write down the *unique* couplings:

Scalar $\rightarrow \gamma(k, \lambda)\gamma(k', \lambda')$

$$\mathcal{L}_{\phi\gamma\gamma} = g\varepsilon_{\lambda}^{*\mu}(k)\varepsilon_{\lambda'}^{\rho}(k')T_{\mu\rho}(k, k'); \quad T_{\mu\rho}(k, k') \equiv k'_{\mu}k_{\rho} - \eta_{\mu\rho}k \cdot k', \quad (1)$$

Pseudoscalar $\rightarrow \gamma(k, \lambda)\gamma(k', \lambda')$

$$\mathcal{L}_{\phi_5\gamma\gamma} = g_5\varepsilon_{\lambda}^{*\mu}(k)\varepsilon_{\lambda'}^{\rho}(k')k^{\alpha}k'^{\beta}\epsilon_{\mu\rho\alpha\beta}, \quad (2)$$

Scalar $\rightarrow h(k, \lambda)h(k', \lambda')$

$$\mathcal{L}_{\phi hh} = G\varepsilon_{\lambda}^{*\mu\nu}(k)\varepsilon_{\lambda'}^{\rho\sigma}(k')[T_{\mu\rho}(k, k')T_{\nu\sigma}(k, k') + T_{\nu\rho}(k, k')T_{\mu\sigma}(k, k')]/2; \quad (3)$$

Pseudoscalar $\rightarrow h(k,\lambda)h(k',\lambda')$

$$\mathcal{L}_{\phi_5 hh} = G_5 \varepsilon_{\lambda}^{*\mu\nu}(k) \varepsilon_{\lambda'}^{*\rho\sigma}(k') [\varepsilon_{\mu\rho\alpha\beta} k^\alpha k'^\beta T_{\nu\sigma}(k, k') + (3 \text{ other perms})]/4. \quad (4)$$

In these equations η stands for the Minkowski metric, about which we have expanded the gravitational metric field ($g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}$) and with respect to which Lorentz scalar contractions are made.

There are four significant points about equations (1)–(4). Firstly, we note that insistence on current and stress tensor conservation means that each photon is accompanied by one power of momentum and each graviton comes with two powers of momentum, entirely in keeping with low-energy theorems (Weinberg 1964, 1965). Secondly, we can encapsulate the gauge invariances by regarding the effective couplings above as corresponding to effective gauge interactions,

$$\begin{aligned} g\phi F^{\mu\nu} F_{\mu\nu}/4, \quad g_5\phi_5 \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}/8, \\ G\phi R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}/2\kappa^2, \quad G_5\phi_5 \varepsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma\alpha\beta}/4\kappa^2, \end{aligned} \quad (5)$$

where $\kappa^2 = 8\pi G_{\text{Newton}}$. Thirdly, the mass dimensions of g , g_5 are M^{-1} and of G , G_5 are M^{-3} . Fourthly and most importantly, the curvature tensor R has only two powers of derivatives, but of course contains all powers of the graviton field h , as befits a nonlinear theory. Thus it is quite impossible for an interaction like ϕR to lead to two-graviton decay, as it is essential to conjure up *four* powers of momentum in the decay amplitude, as we find in the gravitational contribution to the chiral anomaly (Kimura 1969; Delbourgo and Salam 1972; Eguchi and Freund 1976).^{*} In this respect we find the Srivastava–Widom claims unacceptable, notwithstanding that one is dealing with a spontaneously broken gauge symmetry and a vacuum expectation value which is closely tied to the Higgs boson field.

It is straightforward to compute the decay rates ensuing from the interactions (1) to (4). Because gravitons and photons are massless the results are really quite simple in the end; if m denotes the mass of the decaying scalar, one finds the partial widths,

$$\begin{aligned} \Gamma_{\phi\gamma\gamma} &= |g|^2 m^3/64\pi, \quad \Gamma_{\phi_5\gamma\gamma} = |g_5|^2 m^3/64\pi, \\ \Gamma_{\phi hh} &= |G|^2 m^7/256\pi, \quad \Gamma_{\phi_5 hh} = |G_5|^2 m^7/512\pi. \end{aligned} \quad (6)$$

The first and third of these are relevant to the Higgs boson. The only remaining question is the magnitude of the various coupling constants, g to G_5 .

Because the decays are induced by quantum effects, they first arise at one-loop level. We may determine the couplings by applying the standard Feynman rules to the graphs depicted in Fig. 1. In quoting the answers below, for a source field ψ of mass m_ψ circulating round the loop which is either scalar or spinor and of unit charge e , we should point out that certain graphs of Fig. 1 are absent or give vanishing contributions; for example, there is no Fig. 1d for photons, but it certainly arises for gravitons; similarly, Fig. 1c does

^{*} In this connection, observe that the pseudoscalar interactions are the same as the axial current anomalies when the pseudoscalar field is divided out, both for electromagnetism and for gravity. It is also worth mentioning that in regard to the scalar gravitational interaction, the Gauss–Bonnet invariant sum is of little utility because of the inclusion of the ϕ field.

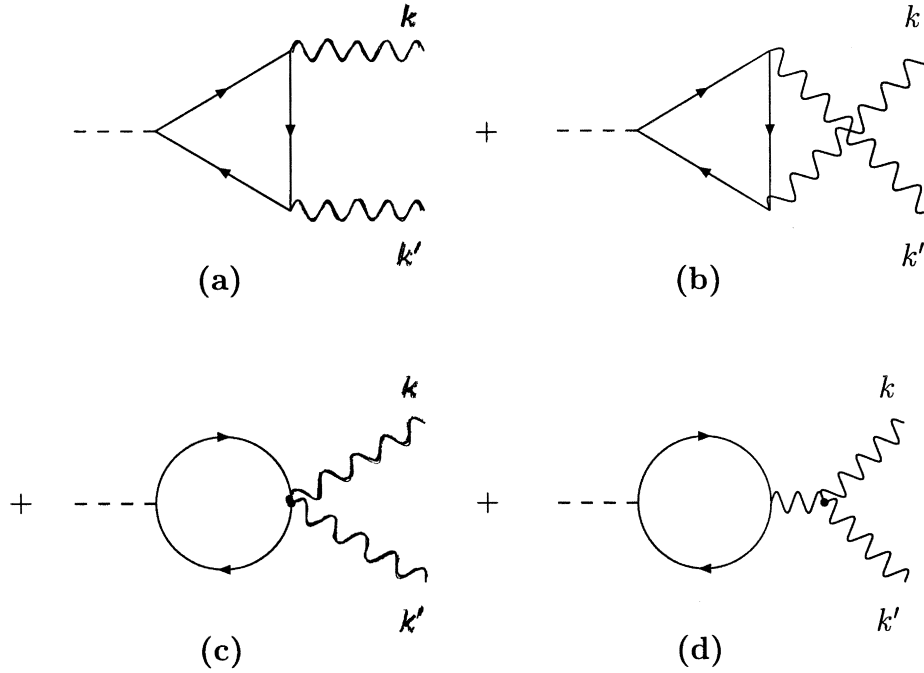


Fig. 1. One-loop contributions to Higgs decay into two massless gauge particles, where the circulating loop particle can be scalar, spinor or vector.

not occur for fermions undergoing electromagnetic decay but is present for gravitons and always exists for boson sources. As a purely technical point, we have applied dimensional regularisation when calculating these diagrams, in order to respect gauge invariance.

$$g = \frac{e^2 g_{\phi\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) (4xy-1) / (m_\psi^2 - m^2 xy); \quad \text{fermion loop} \quad (7a)$$

$$g = \frac{e^2 g_{\phi\psi\psi}}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) xy / (m_\psi^2 - m^2 xy); \quad \text{boson loop} \quad (7b)$$

$$g_5 = \frac{e^2 g_{\phi_5\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) / (m_\psi^2 - m^2 xy); \quad \text{fermion loop only} \quad (8)$$

$$G = \frac{\kappa^2 g_{\phi\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) xy(1-4xy) / (m_\psi^2 - m^2 xy); \quad \text{fermion loop} \quad (9a)$$

$$G = -\frac{\kappa^2 g_{\phi\psi\psi}}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) x^2 y^2 / (m_\psi^2 - m^2 xy); \quad \text{boson loop} \quad (9b)$$

$$G_5 = -\frac{\kappa^2 g_{\phi_5\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1-x-y) xy / (m_\psi^2 - m^2 xy); \quad \text{fermion loop only.} \quad (10)$$

Note that $g_{\phi\psi\psi}$ is dimensionless when ψ is a spinor, but acquires dimensions of mass when ψ is a scalar field. Anyhow, we see that all the integrals can be written as combinations of the basic integral

$$I_n(\mu) \equiv \int_0^1 \int_0^1 dx dy \theta(1-x-y) (xy)^n / (1-\mu xy); \quad \mu = m^2/m_\psi^2, \quad (11)$$

which can be expressed as sums of dilog functions* and the like. As there is little to be gained by looking up tables of Spence functions, it turns out to be much easier to evaluate the I_n numerically for input values of μ .

Focussing on the Higgs boson now, and in respect of spin 0 and spin 1/2 contributions to the decay widths $\Gamma_{H\gamma\gamma}$ and Γ_{Hgg} , we should remind ourselves that $g_{H\psi\psi} = -2m_\psi^2/v$ for scalar isosinglets and $g_{H\psi\psi} = -m_\psi/v$ for quarks, where $v = 246$ GeV is the Higgs vacuum expectation value. Therefore the couplings entering the partial widths (6) equal $g = -e^2 I_1(\mu)/\pi^2 v$ and $G = -\kappa^2 I_2(\mu)/\pi^2 v$ from a boson loop as well as $g = e^2 [I_0(\mu) - 4I_1(\mu)]/\pi^2 v$ and $G = -\kappa^2 [I_1(\mu) - 4I_2(\mu)]/\pi^2 v$ from a fermion loop; in particular, a massless loop particle ($\mu \rightarrow \infty$) reassuringly gives zero! Since the Higgs boson in the standard model couples just to the quarks/leptons and gauge bosons, apart from its self-interactions, we get the fermionic contributions

$$g = \frac{e^2}{2\pi^2 v} \left[\sum_{q=u,c,t} \frac{4}{3} (I_0(m_q) - 4I_1(m_q)) + \sum_{q=d,s,b} \frac{1}{3} (I_0(m_q) - 4I_1(m_q)) + \sum_{l=e,\mu,\tau} (I_0(m_l) - 4I_1(m_l)) \right], \quad (12)$$

$$G = \frac{-\kappa^2}{2\pi^2 v} \left[\sum_{q=u,c,t} \frac{4}{3} (I_1(m_q) - 4I_2(m_q)) + \sum_{q=d,s,b} \frac{1}{3} (I_1(m_q) - 4I_2(m_q)) + \sum_{l=e,\mu,\tau} (I_1(m_l) - 4I_2(m_l)) \right]. \quad (13)$$

To complete the calculation for electroweak theory one should add the contributions from the W and Z gauge bosons, including ghost field contributions as required by the gauge-fixing scheme in use. These have been computed previously (Kniehl 1994) and are a factor of 21/4 greater than the fermionic terms. These corrections have even been computed to two-loop order in electroweak theory by Djouadi *et al.* (1998), but in any event we see that the gravitational decay rate is largely determined by the top quark (and the heavy gauge bosons), with a magnitude $G \sim G_{\text{Newton}} m_t^2 / v m_H^2$ that is fixed by the Newtonian constant. This enters equation (6) and duly gives a miniscule result for the gravitational decay rate, of the order of

* The analytic results read $I_0 = [\text{dilog}z_+ + \text{dilog}z_-]/z$, $I_1 = [\text{dilog}z_+ + \text{dilog}z_- - z/2]/z^2$, $I_2 = [\text{dilog}z_+ + \text{dilog}z_- - z/2 - z^2/24]/z^3$, where $z^\pm \equiv 2/[1 \pm \sqrt{1-4\mu}]$. Also worth noting are the limits $I_n(\mu = 0) = (n!)^2/(2n+2)!$, corresponding to a superheavy massive loop particle; and for $\mu \gg 1$, $I_0(\mu) \approx \log^2(\mu)/2\mu$, $I_1(\mu) \approx -1/2\mu$, $I_2(\mu) \approx -1/6\mu$, corresponding to a light fermion loop.

$$\Gamma_{Hhh} \sim m_H (G_{\text{Newton}} m_H^3 / v)^2 / 500 \sim m_H \times 10^{-75}.$$

This is as it should be: the disappearance of the Higgs boson into gravitational radiation will never be observed.

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