THE SURFACE TEMPERATURE OF THE MOON

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Summary

Results of numerical calculations are given for (i) the fall in surface temperature during a lunar eclipse, (ii) the variation in surface temperature at the equator during a lunation, and (iii) the variation in microwave temperature at the equator during a lunation. These calculations are made on the assumption that heat is lost from the surface by radiation according to the fourth power law. Two models are considered, firstly, that in which the surface material of the Moon is homogeneous, and secondly, that in which it consists of a thin skin of poor conductor on a better conducting substratum.

The experimental results are discussed in the light of these calculations and it is found that none of the proposed models fits them all adequately, and, though there is a slight preference for the thin skin model over the homogeneous solid, it is not possible to discriminate between the two on the information at present available.

I. Introduction

Observations of the surface temperature of the Moon have been available for many years and attempts have been made to deduce information about the nature of the surface from them. The theoretical background necessary for such attempts is the theory of conduction of heat in the semi-infinite solid with prescribed supply of heat at its surface from the Sun, and with loss of heat from its surface by radiation according to the fourth power law. The accurate fourth power law must be used because of the large range of temperatures involved, and, since this makes the problem non-linear, calculations are difficult and must be carried out numerically. The object of this paper is to give numerical information about the solution of this problem for a fairly wide variety of cases, and also some discussion of the experimental results.

In considering the radiation from the Moon as a whole, each element of its surface may be regarded as a semi-infinite solid with its own thermal properties and angle of incidence of the Sun's rays. To get a strict comparison between theory and experiment it would be necessary to integrate the emission from these over the relevant portion of the disk, but as a first approximation all that can be done is to assume that the restricted portion of the disk observed in the optical experiments will behave as a sort of average semi-infinite solid, and to determine the properties of this material.

With regard to the nature of the surface material, three possibilities are to be recognized. Writing K, ρ, c, and $\kappa=K/\rho c$ for the thermal conductivity, density, specific heat, and diffusivity of the material, and using c.g.s. units,

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calorie, and °K., these are: (i) normal igneous rock for which $(K\rho c)^{-\frac{1}{2}}$ has a value of about 20, (ii) a granular or cellular substance such as pumice or lava gravel for which $(K\rho c)^{-\frac{1}{2}}$ is of the order of 100, or (iii) fine dust for which Wesselink (1948) has shown that, under the conditions of low atmospheric pressure existing on the Moon, $(K\rho c)^{-\frac{1}{2}}$ will be of the order of 1000. It has also been suggested by various authors that the surface layers may not be homogeneous in depth, for example, that a thin layer of dust might lie on the surface of a better conductor such as gravel or solid rock. It will be assumed throughout that the thermal properties of the material are independent of temperature; in view of our ignorance of the nature of the material it seems hardly worth while introducing the additional complication of variation with temperature: this matter is referred to again in Section II.

Three types of experimental information are available, (i) infra-red observations of the fall in temperature during a lunar eclipse, (ii) infra-red observations of the variations in surface temperature during a lunation, and (iii) observations of the variation in radio-microwave temperature during a lunation. Any proposal about the nature of the surface must be consistent with all three.

II. Eclipse Observations

Pettit and Nicholson (1930) in 1927 and Pettit (1940) in 1939 made observations of the fall in temperature of the surface during a lunar eclipse. Epstein (1929) attempted to calculate the thermal properties of the surface from the results for the 1927 eclipse and obtained a value of 120 for $(K\rho c)^{-\frac{1}{2}}$ from which he concluded that the lunar surface is covered with some substance such as pumice. He used a linear theory (in effect assuming that the surface loses heat at a rate proportional to the fourth power of its initial temperature instead of to the fourth power of its actual temperature), and it was pointed out by both Wesselink (1948) and Jaeger and Harper (1950) that, when the accurate fourth power law of radiation is used, a much higher value is obtained: both these authors give single curves calculated for values of $(K\rho c)^{-\frac{1}{2}}$ of the order of 1000 and compare them with the experimental results. A family of such curves for various values of $(K\rho c)^{-\frac{1}{2}}$ is given in Figure 1 which makes the comparison with experiment clearer.

The problem in conduction of heat proposed by the eclipse experiments may be stated as follows. The homogeneous semi-infinite solid $x>0$ is initially at constant* temperature v_0 (°K.) and in equilibrium with the solar radiation $E\sigma v_0^4$ absorbed by it, where E is its emissivity and σ is the Stefan-Boltzmann constant. During the penumbral stage of the eclipse, the radiation absorbed from the Sun may be written $E\sigma v_0^4 f(t)$, where $f(t)$ can be calculated from the circumstances of the eclipse and vanishes at the beginning of the umbral phase. During both umbra and penumbra the surface loses heat at the rate $E\sigma v_s^4$,

* The initial temperature is in fact that due to the monthly periodic variation of surface temperature; if this is used instead of the constant value the change is very small.

where v_s is its temperature. Then if t_0 is the duration of penumbra, and v is the temperature in the solid, the equations to be solved are :

$$\frac{\partial^2 v}{\partial x^2}-\frac{1}{\varkappa}\frac{\partial v}{\partial t}=0, \qquad x>0,\ t>0, \qquad \text{(1)}$$

$$v=v_0, \qquad x>0,\ t=0, \qquad \text{(2)}$$

with the boundary condition at $x=0$

$$K\frac{\partial v}{\partial x}=E\sigma v^4-E\sigma v_0^4 f(t), \qquad 0<t<t_0, \qquad \text{(3)}$$

$$=E\sigma v^4, \qquad t>t_0. \qquad \text{(4)}$$

Because of the occurrence of the fourth power in (3) and (4) the problem is non-linear and a simple explicit solution for v cannot be obtained. Wesselink (1948) and Jaeger and Harper (1950) have made numerical calculations using Schmidt's method which is very well adapted to problems of this type (cf. Jaeger 1950).

In Figure 1 a family of curves showing the ratio of the surface temperature v_s to the initial temperature v_0 as a function of t/t_0 is given : the curves are specified by the single parameter

$$P=\frac{\sigma E v_0^3 t_0^{\frac{1}{2}}}{(10K\rho c)^{\frac{1}{2}}}, \qquad \text{(5)}$$

and the chosen values of P are 2, 1·5, 1, 0·2, and 0·03, corresponding, for $t_0=74$ min., $v_0=370$ °K., and $E=1$, to values 1370, 1026, 685, 137, and 20 of $(K\rho c)^{-\frac{1}{2}}$. The time interval used in the Schmidt process was $t_0/20$ or 3·7 min. For $f(t)$ a smooth curve drawn through some figures given by Pettit and Nicholson (1930) has been used ; this is the curve marked " Insolation " in Figure 1 ; the temperature is not very sensitive to the form of $f(t)$ and in fact a linear fall $f(t)=(t_0-t)/t_0$ gives curves very little different from those of Figure 1.

The results of Pettit's 1939 eclipse observations are shown by the crosses in Figure 1 and reasonable agreement is obtained if P lies between 1·5 and 2, that is $(K\rho c)^{-\frac{1}{2}}$ between 1370 and 1030. The values $P=0·2$ and 0·03 correspond to pumice and bare rock respectively, and it appears that the possibility of any large proportion of the surface being covered with these is quite ruled out. As remarked above, Wesselink (1948) has shown that values of $(K\rho c)^{-\frac{1}{2}}$ of the order of 1000 are likely for dust under lunar conditions, so it seems probable that the surface, or a very considerable fraction of it, is composed of dust.

The most notable discrepancy between the calculated curves of Figure 1 and the experimental values is that the latter fall more slowly in the umbral phase. Both Jaeger and Harper (1950) and Lettau (1951) remark that this may be due to an increase of thermal conductivity, either with depth or with temperature.

Variation of conductivity with temperature is an attractive explanation but a difficult one to discuss. Some calculations have been made with conductivity varying as the cube of the absolute temperature but these yield curves which still fall too rapidly in umbra.

The other explanation, an increase in conductivity with depth, which could easily be provided by a layer of dust on a substratum of rock or pumice, can easily be made to give an adequately slow fall of temperature during umbra.

where A is the amplitude of the insolation. In all the calculations below we shall take $E=1$; this is justifiable since the emissivity of the lunar surface is known to be high, and reducing the value of E to $0 \cdot 9$ is found to make a change of at most one or two degrees in temperature. The value $0 \cdot 0258$ cal. cm.$^{-2}$ sec.$^{-1}$ will be used for A; this quantity again is not well known and varies with position in the orbit, but a 10 per cent. change in A gives a change of only one or two degrees in temperature at lunar midnight.

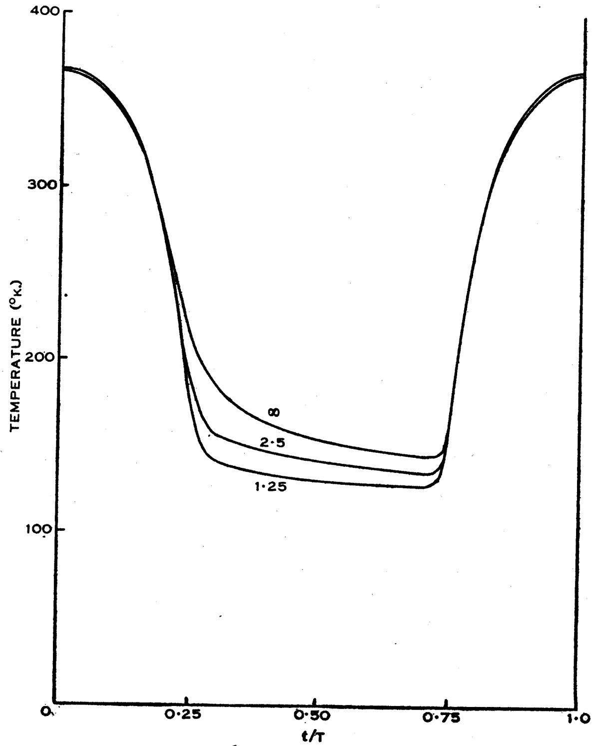

Fig. 3.—Surface temperature of a solid for which $(K\rho c)^{-\frac{1}{2}}=125$, covered by a skin of poor conductor. The numbers on the curves are the values of D specifying the skin.

Figure 2 shows the surface temperature of homogeneous semi-infinite solids with the boundary conditions (6) and (7) and with the values 1000, 500, 250, 125, and 20 of $(K\rho c)^{-\frac{1}{2}}$. Taking 120 °K. as the temperature at lunar midnight, and having regard to the large uncertainty in this value, it appears that values of $(K\rho c)^{-\frac{1}{2}}$ between 200 and 1000 are possible and in particular, as remarked by Wesselink, that dust with $(K\rho c)^{-\frac{1}{2}}$ of the order of 1000 will satisfy both the eclipse observations and the present ones reasonably well.

Next, in view of the suggestion from the eclipse and microwave observations that a two layer model consisting of a thin skin of poor conductor on a better-conducting substratum may fit the results better, it is desirable to make calculations of the surface temperature with this model. Since the skin is thin its thermal capacity may be neglected* for these long period processes, and the model becomes that of a skin of thermal resistance R on a substratum with thermal properties K, ρ, c. The situation may be described in terms of two parameters, $(K\rho c)^{-\frac{1}{2}}$, and

$$D=T^{\frac{1}{2}}(K\rho c)^{-\frac{1}{2}}/R. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Since the results are most interesting if the substratum is a much better conductor than the skin (which we shall assume in the calculations to be dust), only the values 20 and 125 for $(K\rho c)^{-\frac{1}{2}}$ will be considered. Also, since Piddington and Minnett (1949) have deduced the value $D=\sqrt{2\pi}$ from their microwave observations, the results have been calculated for this value and simple fractions of it. The results are shown in Figures 3 and 4 for $(K\rho c)^{-\frac{1}{2}}=125$ and 20, respectively. The curves for $D=\infty$ are those of Figure 2 for the homogeneous solid with no surface skin. It appears that the effect of decreasing D is to lower the night-time curves and to flatten them, an effect similar to that remarked in the eclipse calculation. It would be most interesting to have complete experimental curves during the lunar night for comparison.

It appears that " midnight " temperature as low as 120 °K. can be attained with suitable values of D. For solids with $(K\rho c)^{-\frac{1}{2}}$ of 20, 125, and 250 the necessary values of D would be of the orders of 0·1, 0·6, and 2 respectively. To see the orders of magnitude involved, suppose the skin has conductivity K' and thickness d, so that $R=d/K'$. If we assume the skin to be dust with the reasonable values $K'=2\cdot8\times10^{-6}$, $\rho'=1\cdot8$, $c'=0\cdot2$, $(K'\rho'c')^{-\frac{1}{2}}\doteqdot1000$, the values of its thickness d for various values of R and $(K\rho c)^{-\frac{1}{2}}$ are shown in Table 1.

TABLE 1

THICKNESS OF DUST CORRESPONDING TO VARIOUS VALUES OF D AND $(K\rho c)^{-\frac{1}{2}}$

D \ $K\rho c)^{-\frac{1}{2}}$	250	125	20
2·5	0·45	0·22	0·04
1·25	0·89	0·45	0·07
0·625		0·89	0·14
0·25			0·36
0·1			0·89

It appears that in all cases skins of thickness less than 1 cm. are involved.

It may be concluded that, as for the eclipse observations, the results are reasonably well fitted either by a homogeneous solid for which $(K\rho c)^{-\frac{1}{2}}=1000$ or by a layer of such a solid some millimetres thick on a better-conducting substratum.

* It should perhaps be remarked that the thermal capacity of the skin is not neglected in the eclipse calculations; in fact much of the heat extracted in the penumbral phase comes from the skin.

IV. Radio-Microwave Observations

Rock material is partially transparent to radio microwaves whose wavelengths are of the order of 1 cm. Thus the radiation observed from a solid on these wavelengths is not determined merely by the surface temperature but is the total effect of emission from a region near the surface. If v is the temperature at depth x below the surface of the semi-infinite solid, we shall call

$$v_m = \frac{1}{\alpha}\int_0^\infty e^{-\alpha x} v \, dx \quad \dots\dots\dots\dots\dots\dots \quad (9)$$

the microwave temperature at normal incidence. Here α is an attenuation coefficient characteristic of the material and depending on its electrical con-

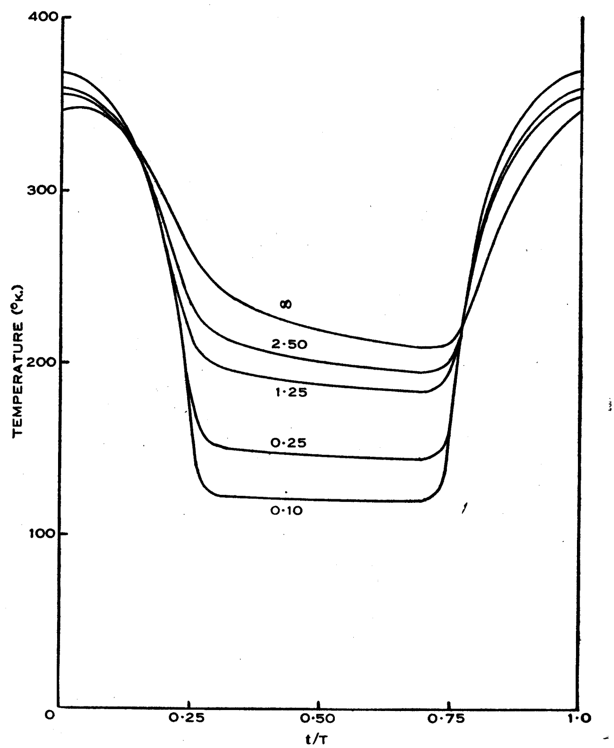

Fig. 4.—Surface temperature of a solid for which $(K\rho c)^{-\frac{1}{2}}=20$, covered by a skin of poor conductor. The numbers on the curves are the values of D specifying the skin.

ductivity. If the surface is seen from a direction included at θ to the normal, α is to be replaced by $\alpha \sec\theta$. The whole question is discussed in detail by Piddington and Minnett (1949).

Formulae for the calculation of microwave temperatures are given by Jaeger (1953). They involve two parameters, $(K\rho c)^{-\frac{1}{2}}$ and

$$C = \alpha(\varkappa T)^{\frac{1}{2}}. \quad \dots\dots\dots\dots\dots\dots \quad (10)$$

Microwave temperatures at normal incidence, for the homogeneous semi-infinite solids for which $(K\rho c)^{-\frac{1}{2}}$ has the values 1000 and 125, and with the boundary conditions (6) and (7), that is, for the Moon at its equator, are shown in Figures 5 and 6 respectively. It appears that decreasing C reduces the amplitude of the oscillation and increases the phase lag of the maximum. $C=\infty$ gives the surface temperature as in Figure 2.

Microwave temperatures may also be calculated for the case of a homogeneous solid covered with a thin skin of poor conductor of negligible heat capacity and from which the microwave emission may be neglected. In this case three parameters are involved, viz. $(K\rho c)^{-\frac{1}{2}}$, D, and C, so that it is

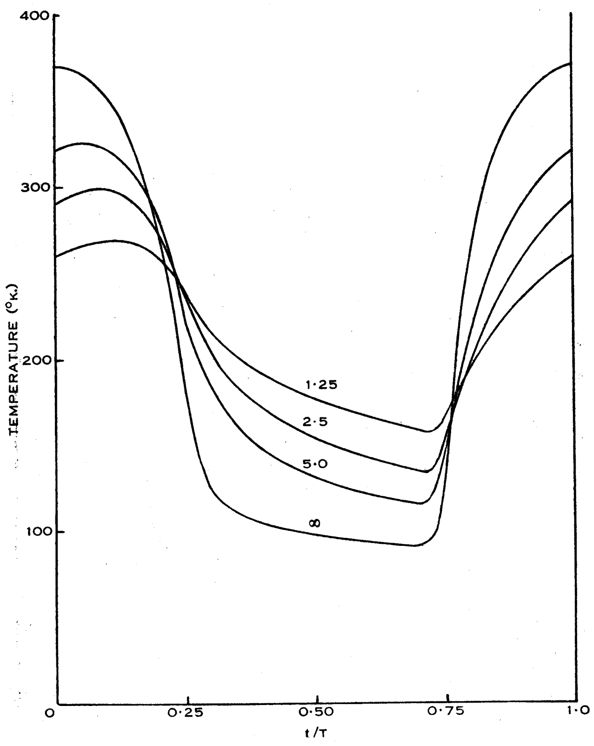

Fig. 5.—Microwave temperatures for a homogeneous solid for which $(K\rho c)^{-\frac{1}{2}}=1000$. The numbers on the curves are the values of $C=\alpha(\varkappa T)^{\frac{1}{2}}$.

impossible to give complete results, but the values of the maximum, mean, and minimum temperatures for a number of cases are shown in Table 2, while curves for the case

$$D=C=\sqrt{2\pi}, \qquad (11)$$

which is of special interest in connection with the work of Piddington and Minnett, are given in Figure 7.

The comparison of the experimental microwave results with theory is extremely difficult. It is possible only to observe the whole disk, and not a relatively small area of it as in the optical case: further, the sensitivity of the equipment varies with position in the aerial beam. Thus the observed result is an average over all latitudes and longitudes, allowing for the angle of emergence of the radiation and position in the aerial beam. Piddington and Minnett

TABLE 2

MAXIMUM, MEAN, AND MINIMUM MICROWAVE TEMPERATURES FOR VARIOUS VALUES OF THE PARAMETERS C AND D

		$(K\rho c)^{-\frac{1}{2}}=1000$			$(K\rho c)^{-\frac{1}{2}}=125$			$(K\rho c)^{-\frac{1}{2}}=20$		
D	C	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.
∞	∞	370	211	89	366	237	144	347	264	207
	5·0	325	211	114	329	237	164	322	264	221
	2·5	298	211	134	306	237	179	308	264	229
	1·25	269	211	157	283	237	197	292	264	240
2·5	∞				304	231	170	308	257	218
	5·0				284	231	185	294	257	227
	2·5				271	231	195	284	257	233
	1·25				257	231	206	275	257	241
1·25	∞				278	226	181	289	251	221
	5·0				263	226	193	278	251	229
	2·5				254	226	200	272	251	233
	1·25				244	226	208	265	251	239
0·25	∞							249	232	215
	5·0							244	232	220
	2·5							241	232	223
	1·25							238	232	226
0·1	∞							227	219	210
	5·0							225	219	213
	2·5							223	219	214
	1·25							222	219	216

represent their results for the average temperature over the disk by the sinusoid

$$239+40\cdot 3\cos\left(\frac{2\pi t}{T}-\frac{\pi}{4}\right) \quad\ldots\ldots\ldots\ldots\ldots \quad (12)$$

From this they deduce

$$249+52\cdot 0\cos\left(\frac{2\pi t}{T}-\frac{\pi}{4}\right) \quad\ldots\ldots\ldots\ldots\ldots \quad (13)$$

for the microwave temperature at normal incidence at the equator (that is, the quantity calculated above). Clearly, the derivation of (**13**) from (**12**) involves a great deal of approximation. Finally, they conclude from the phase lag of 45° in (**13**) that the surface layer of the Moon cannot be regarded as homogeneous but can be represented fairly well by the thin skin model mentioned above and subject to the conditions (**11**).

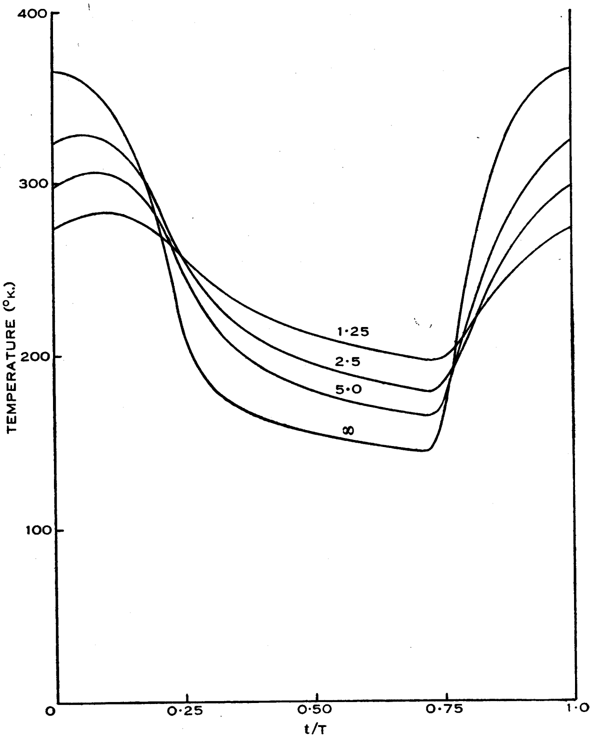

Fig. 6.—Microwave temperatures for a homogeneous solid for which $(K\rho c)^{-\frac{1}{2}}=125$. The numbers on the curves are the values of $C=\alpha(\kappa T)^{\frac{1}{2}}$.

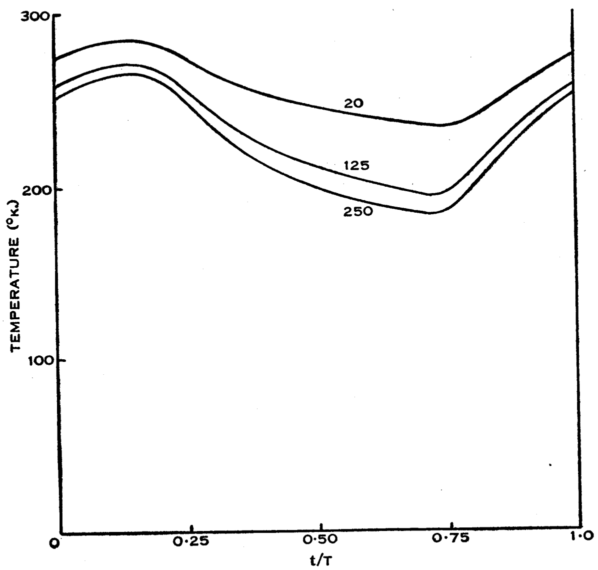

Fig. 7.—Microwave temperatures for a solid with a surface skin for which $D=C=\sqrt{2\pi}$. The numbers on the curves are the values of $(K\rho c)^{-\frac{1}{2}}$.

Their discussion has been criticized by Bracewell (personal communication) who has shown by a more refined analysis of the experimental results (involving both phase and amplitude) that, while a best fit is obtained with values near to (**11**), a wide range of variation in these parameters is consistent with the experimental results and in particular that the homogeneous solid is not altogether ruled out.

Comparing (**13**) with the results of Table 2 and Figure 7, it appears that it is difficult to fit both the optical and microwave results with any model. The discrepancy is essentially between the rather high mean temperature demanded by the microwave results and the rather low temperature during the lunar night which the models of Section III were designed to fit. If either of these is relaxed somewhat, reasonable agreement can be obtained with either a homogeneous solid with $(K\rho c)^{-\frac{1}{2}}$ in the range 500 to 1000 or with a thin layer of such a solid on a substratum with $(K\rho c)^{-\frac{1}{2}}$ of the order of 100 ; the third possibility, that of a thin layer on a substratum of rock with $(K\rho c)^{-\frac{1}{2}}$ of the order of 20, seems less likely because it leads to low values of the amplitude of the oscillation of the microwave temperature.

V. Acknowledgments

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