# AN ESTIMATE OF THE DENSITY AND MOTION OF SOLAR MATERIAL FROM OBSERVED CHARACTERISTICS OF SOLAR RADIO OUTBURSTS

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*Summary*

The theory of radio wave generation by multistream charge interaction (Feinstein and Sen 1951) is extended and applied to the observations made by Australian workers (Wild 1950) of the spectrum of outbursts of solar radio-frequency radiation in the frequency range 70–130 Mc/s. The dispersion equation is derived as a function of the velocity of solar material erupting into a static corona and of the temperatures and densities of the material and the corona. The application of the dispersion equation to the Australian data (loc. cit.) enables an estimate to be made of the velocity ($\simeq$500 km./s.) and the particle density ($\simeq 10^8$ cm.$^{-3}$) of the moving solar material.

## I. Introduction

The high thermal temperature ($10^8$–$10^{13}$ °K.) associated with solar noise bursts and outbursts has led several workers to suggest a non-equilibrium mechanism for their origin (Pawsey 1950). The principal one in this field has been space-charge wave amplification in moving interacting beams as in the electron-wave tube (Haeff 1949*a*). Recently, Australian workers (Wild 1950) have obtained, by a sweep-frequency technique, the spectrum of " outbursts " of solar radio-frequency radiation in the frequency range 70–130 Mc/s. They define an " outburst " as a " burst having a particular type of 'dynamic' spectrum, characterized by a drift of spectral features, with time, towards the lower frequencies at a rate of the order of $\frac{1}{4}$ Mc/s. per second ". They tentatively interpret the spectra in terms of the accelerated motion of particles that finally leave the solar atmosphere and cause terrestrial magnetic storms.

The author in the present paper makes an attempt to interpret the Australian data as a phenomenon of radio wave generation by solar material moving through the ionized plasma of the corona. He believes that such an interpretation provides an independent method through radio observations of estimating the density and the motion of the solar corpuscles *in the solar atmosphere*, that, later on, are supposed to cause terrestrial magnetic storms. Theories of the latter, it is known, give us estimates of the motion of the corpuscles in transit from the Sun to the Earth and of their density near the Earth (Chapman and Bartels 1940; Kiepenheuer 1952). The author finds that his solar estimates are consistent with the terrestrial ones.

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## II. Theory

We shall first derive the appropriate dispersion equation* as a function of the velocity of the moving (ionized but macroscopically neutral) solar material and the temperatures and densities of the material and the corona. This dispersion equation we shall consider as a particular case of a more general one, that is, that of $n$ interacting beams of electrons. We shall neglect the motion of the ions, on account of their large mass,† and assume that there is no static space charge, the charge on the ions balancing that on the electrons.

Let the physical quantities, viz. charge density, velocity, and current density of the $n$th interacting beam be denoted, respectively, by

$$\left.\begin{aligned}\rho_n' &= \rho_n + \bar{\rho}_n, \quad v_n' = v_n + \bar{v}_n, \\ i_n' &= i_n + \bar{i}_n = i_n + \bar{\rho}_n v_n + \rho_n \bar{v}_n,\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where the bars denote the A.C. perturbations. We shall suppose these perturbations to be of the first order of small quantities, and neglect all quantities of order higher than the first.

Assume that all A.C. quantities vary as

$$e^{-\Gamma z + j\omega t}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where the $z$-axis is the direction of propagation, $\Gamma$ the propagation constant, and $\omega$ the angular frequency of the disturbance.

In the absence of a static charge, Poisson's equation gives (in rationalized m.k.s. units)

$$\varepsilon_0 \frac{\partial \bar{E}}{\partial z} = \sum_1^n \bar{\rho}_n, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where $\bar{E}$ is the perturbed electric field and $\varepsilon_0$ the dielectric constant of free space.

The charge distribution of the $n$th beam must satisfy the equation of continuity :

$$\frac{\partial \rho_n'}{\partial t} + \frac{\partial i_n'}{\partial z} = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

From (1), (2), and (4), we derive

$$(j\omega - \Gamma v_n)\bar{\rho}_n - \Gamma \bar{v}_n \rho_n = 0. \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

We assume the plasma to be of sufficiently low density so that we can neglect the frictional forces. Then we have the equation of motion for the $n$th beam :

$$\frac{\partial v_n'}{\partial t} + v_n' \frac{\partial v_n'}{\partial z} + \frac{1}{\rho_n'} \frac{\partial}{\partial z}\left(\frac{kT_n}{m} \rho_n'\right) = \frac{e}{m} E', \quad \ldots\ldots \quad (6)‡$$

* The physical theory of space-charge wave amplification in moving interacting charges has been well discussed in the literature. The author is fully aware of the differences of opinion in the field, particularly with reference to the application of laboratory data to solar atmospheres. Discussion of such niceties is beyond the scope of the present paper.

† The ions contribute terms that are $(\omega_i/\omega_e)^2$ times the electronic terms in equation (10) of this paper, where $\omega_i$ is the ionic and $\omega_e$ the electronic plasma frequency given by equation (11).

‡ Equation (6) results on multiplying the Boltzmann equation :

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e}{m} \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

by $\mathbf{v}$ and integrating with respect to $\mathbf{v}$.

where $T_n$ is the temperature of the $n$th beam, $e$* the charge and $m$ the mass of the electron, and $k$ is the Boltzmann constant.

Substitute (1) in (6) and obtain

$$\frac{\partial \bar{v}_n}{\partial t}+v_n\frac{\partial \bar{v}_n}{\partial z}+\frac{1}{\rho_n}\frac{\partial}{\partial z}\left(\frac{kT_n}{m}\bar{\rho}_n\right)=\frac{e}{m}\bar{E}, \qquad (7)$$

as the static space charge is assumed to be zero.

For the variation (2) of the A.C. quantities, equation (7) reduces to

$$(\mathrm{j}\omega-\Gamma v_n)\bar{v}_n-\frac{\Gamma}{\rho_n}\frac{kT_n}{m}\bar{\rho}_n=\frac{e}{m}\bar{E}. \qquad (8)$$

Eliminate $\bar{v}_n$ between (5) and (8) and obtain for the A.C. charge density:

$$\bar{\rho}_n=\frac{(e/m)\bar{E}\rho_n}{(\mathrm{j}\omega-\Gamma v_n)^2/\Gamma-\Gamma(kT_n/m)}. \qquad (9)$$

From (3) and (9), we have the dispersion equation:

$$\sum_1^n\frac{\omega_n^2}{(\omega+\mathrm{j}\Gamma v_n)^2+(kT_n/m)\Gamma^2}=1, \qquad (10)$$

where $\omega_n$ is the plasma angular frequency, given by

$$\omega_n^2=\frac{e}{m\varepsilon_0}\rho_n. \qquad (11)$$

For solar material of density $\rho_2$ and temperature $T_2$, erupting with velocity $v$ into a static corona of density $\rho_1$ and temperature $T_1$, the dispersion equation (10) reduces to

$$\frac{\omega_1^2}{\omega^2+(kT_1/m)\Gamma^2}+\frac{\omega_2^2}{(\omega+\mathrm{j}\Gamma v)^2+(kT_2/m)\Gamma^2}=1. \qquad (12)$$

Introduce in (12) the following dimensionless quantities:

$$\left.\begin{aligned}\gamma&=\frac{\mathrm{j}\Gamma v}{\omega}, & \alpha_n&=\frac{kT_n}{mv^2},\\ \beta_n&=\frac{\omega^2}{\omega_n^2}, & x&=\frac{\beta_2}{\beta_1}.\end{aligned}\right\} \qquad (13)$$

The equation (12) then reduces to a quartic in $\gamma$:

$$\alpha_1(\alpha_2-1)\beta_1x\gamma^4-2\alpha_1\beta_1x\gamma^3-\{\alpha_1(\beta_1x-1)+(\alpha_2-1)(\beta_1-1)x\}\gamma^2$$
$$+2(\beta_1-1)x\gamma+(\beta_1-1)x-1=0. \qquad (14)$$

Equation (14) will give the range of $\beta_1$ for which $\gamma$ is complex for a fixed set of values of the parameters $\alpha_1$, $\alpha_2$, and $x$.

## III. Results and Conclusions

We assume $T_1$ (electron temperature of corona)$\simeq 10^6$ °K.,† and $T_2$ (temperature of moving material)$\simeq 10^4$ °K. To fit the Australian data we have to determine our disposable parameters $v$ and $x$, which will give an estimate of the motion and the density of the solar corpuscles. Table 1 gives a summary of the results obtained.

* With the proper sign (— in this case).

† We have neglected the small variation in coronal electron temperature with height.

The second column of Table 1 gives the electron density, $n_1$, of the corona as computed from the Allen-Baumbach formula (Allen 1947):

$$n_1 = 10^8(1\cdot55\rho^{-6}+2\cdot99\rho^{-16}) \text{ electrons cm.}^{-3}, \quad \ldots\ldots \ (\mathbf{15})$$

where $\rho$ is the distance from the centre of the Sun in units of the solar radius. The fourth column gives the electron concentration, $n_2$, of the moving material at the different heights. The density was fixed at an arbitrary height over the solar surface ($1\cdot68\times10^8$ cm.$^{-3}$ at $11\cdot6\times10^4$ km.), and the densities at the remaining heights were calculated according to the inverse square law dilution from the solar centre. The fixing of the density was done by trial and error so as to get amplification within the observed range of frequencies. The sixth column gives the velocities, $v$, of the moving material at the different heights

TABLE 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Height above Sun's Surface $h$ (km.) | Electron Density of Corona $n_1$ (cm.$^{-3}$) | Plasma Frequency $f_1$ (Mc/s.) | Electron Density of Moving Material $n_2$ (cm.$^{-3}$) | $x=\frac{n_1}{n_2}$ | Velocity of Moving Material $v$ (km./s.) | Velocity of Ca$^+$ (Milne's Formula) (km./s.) | Frequency Bandwidth of Amplification $f$ (Mc/s.) |
| $\times10^4$ | $\times10^8$ | | $\times10^8$ | | | | |
| 8·0 | 1·34 | 104·0 | 1·83 | 0·73 | 305 | | 93·3–159·7 |
| 10·0 | 1·03 | 91·0 | 1·74 | 0·59 | 388 | 86 | 86·2–147·7 |
| 11·6 | 0·84 | 82·5 | 1·68 | 0·50 | 500 | 244 | 80·5–141·3 |
| 14·0 | 0·67 | 73·0 | 1·58 | 0·42 | 600 | 366 | 74·6–131·8 |
| 16·0 | 0·55 | 66·5 | 1·51 | 0·36 | 740 | 439 | 69·3–125·4 |

that conform with the drift of the cut-off frequency towards lower frequencies at the rate of 0·22 Mc/s. per second (Wild 1950, p. 402). The cut-off frequency is supposed to be the plasma frequency, $f_1$, at the level concerned (third column).* For comparison, the velocities have been given in the seventh column as calculated from Milne's (1926) theory of the expulsion of calcium ions by radiation pressure. The eighth column gives the frequency range, $f$ $(=\omega/2\pi)$, within which amplification is possible.

Figure 1 gives the amplification *v.* frequency curves for the different heights and velocities listed in Table 1. The ordinate is the amplification factor $A=(\omega/v)\gamma_2$, where $\gamma_2$ is the imaginary part of $\gamma$ in (**14**). With increasing velocity, the curves shift toward the lower frequencies. This is in conformity with the observed drift of spectral features, with time, towards the lower frequencies, and the tendency of the spectrum to show signs of subsiding at the higher frequencies (Wild 1950). Also, observational evidence on prominence motion and the theory of magnetic storms indicate an outward acceleration of solar material.

* $f_1=\omega_1/2\pi$, where $\omega_1$ is given by (**11**).

The velocities (305–740 km./s.) and the electron density ($\simeq 10^8$ cm.$^{-3}$) of the moving material *in the solar atmosphere* should be compared with the velocity (1000–1500 km./s.) and density (100 cm.$^{-3}$) of solar corpuscles near the Earth, as estimated from the theory of magnetic storms (Chapman and Bartels 1940 ; Kiepenheuer 1952). Whipple and Gossner (1949) obtain, from the scattering of sunlight by electrons, the upper limit of electron density near the Earth $\simeq 10^3$ cm.$^{-3}$, which is in close accord with our value.

No particular significance need be attached to the values of the amplification factors. A first-order theory as developed above can only indicate the qualitative trend. We shall need a non-linear theory to achieve quantitative accuracy. Nevertheless, we can estimate if it is possible to receive the observed radio flux on the Earth, on the assumption that the source of the available energy is the initial difference of energy between the interacting streams (Haeff 1949*b*). Let

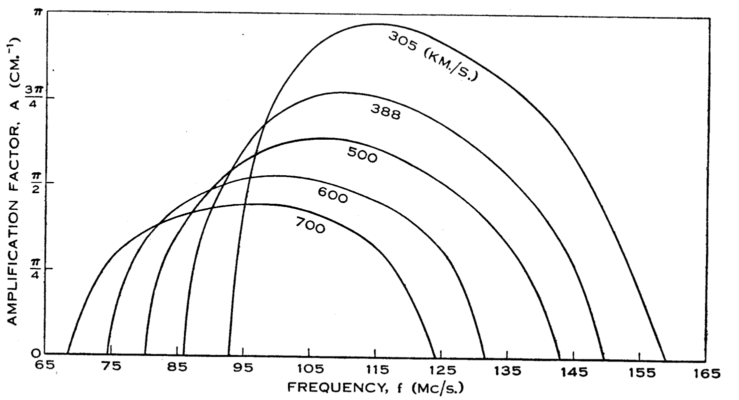


Fig. 1.—Curves of amplification $A$ *v.* frequency $f$, for different velocities of solar material ; wave growth$=\exp[Az]$.

us assume an efficiency factor $\alpha$ for this conversion. Then the power, $P$, generated per unit bandwidth at solar surface by solar material erupting into a static corona, is given by

$$P = \tfrac{1}{2}\alpha N m v^3 S/\Delta\nu, \qquad (16)$$

where $N$ is the density, $m$ the mass, and $v$ the velocity of the electrons, $\Delta\nu$ is the frequency bandwidth within which the power is radiated, and $S$ is the area of the active surface ejecting the corpuscles.

Hence, if $R$ is the Earth-Sun distance, the power flux at the Earth is

$$F = \frac{P}{4\pi R^2} = \tfrac{1}{2}\alpha N m v^3 \frac{S}{4\pi R^2 \Delta\nu}. \qquad (17)$$

Setting $N = 10^8$ cm.$^{-3}$, $m = 9\cdot 12\times 10^{-28}$ gm., $v = 500$ km./s., $S$ = one hundred-millionth of solar surface $= \dfrac{4\pi(0\cdot 695\times 10^{11})^2}{10^4}$ cm.$^2$, $R = 149\cdot 5\times 10^6$ km., and $\Delta\nu = 60$ Mc/s. in equation (**17**), we have

$$F = 2\times 10^{-16}\,\alpha \text{ W.m.}^{-2}\,(\text{c/s.})^{-1}. \qquad (18)$$

From the Australian data:

$$F \simeq 10^{-19} \text{ to } 10^{-20} \text{ W.m.}^{-2} \text{ (c/s.)}^{-1}. \quad \ldots\ldots\ldots\ldots \quad (19)$$

Hence the range of $\alpha$ is

$$\alpha \simeq 5 \times 10^{-4} \text{ to } 5 \times 10^{-5}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

We see that enough kinetic energy is available in the moving beam to account for the observed radio flux at the Earth.

We may finally remark that during an outburst particles may be thrown out from the active area in spurts having different concentrations and velocities. This may result in several "drifting" peaks as observed by the Australians. Further, in the course of the computations, it was found that, for a fixed velocity, the frequency spectrum shifted towards higher frequencies for higher particle concentrations. This indicates the possibility of outbursts even in the centimetric range for very high particle concentrations.

## IV. Acknowledgments

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