

THE FAILURE OF MATTHIESSEN'S RULE FOR HEAVILY DEFORMED ALLOYS

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Summary

Deviations from Matthiessen's rule, such as were found by Boas and Nicholas (1953) for some heavily deformed alloys, may be explained by assuming that in the strongly disordered regions the thermal vibrations of the atoms contribute much less to the resistance than in perfect regions of the crystal. The magnitude of the calculated effect is compatible with the experimental results.

I. GENERAL ASSUMPTIONS

In the preceding paper (Boas and Nicholas 1953) the occurrence of deviations from Matthiessen's rule for some heavily deformed alloys has been discussed. Whereas these authors ascribe the deviations to a change in the frequency of the atoms vibrating in a crystal containing strains due to plastic deformation, it is here suggested that these deviations are caused by a non-linear additivity of the scattering probabilities of the conduction electrons. It is the purpose of this paper to show that the experimental results are compatible with this explanation.

As in the preceding paper it is assumed that the component of resistivity due to elastic scattering by the deformations is unaffected by temperature changes. It is now further assumed that in the neighbourhood of a static imperfection produced by plastic deformation the displacement of the atoms is large compared with their average thermal displacement. Then the scattering probability of the conduction electrons due to thermal displacements in this region will not be proportional to the mean square thermal displacement of the atoms. There will be a saturation effect and the thermal displacement of an atom will contribute much less to the resistance than it would in a perfect region of the crystal. The thermal component of resistivity would then be

$$\rho_i(T) = \rho_i^\circ(T)(1 - \alpha), \quad \dots\dots\dots (1)$$

where $\rho_i(T)$ is the resistance due to thermal vibrations, $\rho_i^\circ(T)$ is the value of $\rho_i(T)$ in the absence of the imperfections considered, and α is proportional to, and of the order of, the fraction of the volume of the crystal occupied by imperfections. On this basis we can understand the decrease of $\rho_i(T)$ with deformation.

II. PARTICULAR MODELS

In order to check whether the fractional volume α , as found for the observed deviations, is of reasonable order of magnitude it is necessary to assume some

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model for the scattering imperfections. We shall consider two models: scattering by dislocations (disordered regions cylindrical) and scattering by stacking errors (disordered regions laminar).

Consider the following case, typical in regard to order of magnitude:

$$\delta\rho/\rho=30 \text{ per cent.}, \quad \delta(d\rho/dT)/(d\rho/dT)=-2 \text{ per cent.},$$

and ρ is about five times the resistance of copper, so that, from knowledge of the mean free path of electrons in copper, we find that the mean free path l due to scattering by the additional imperfections is 560 Å. The average distance between atoms is 3 Å.

Assuming that scattering is by dislocations, let there be N dislocation lines per cm.²; let r_1 be the radius of the cylindrical regions within which thermal displacements do not contribute to the scattering; and let r_2 be the radius defining the scattering cross section, then

$$\pi N r_1^2 = \alpha = 0.02,$$

$$2N r_2 = 1/l = (5.6 \times 10^{-6})^{-1}. \quad \dots\dots\dots (2)$$

If we now assume $r_1=r_2=r$, which is correct as regards order of magnitude,

$$r = \frac{2}{\pi} \times 0.02 \times 5.6 \times 10^{-6} = 7 \times 10^{-8} \text{ cm.},$$

$$N = 1.3 \times 10^{12} \text{ cm.}^{-2} \quad \dots\dots\dots (3)$$

Both this effective radius and this dislocation density are of reasonable order of magnitude.

Alternatively, if it is assumed that scattering is by stacking errors of the type proposed by Broom (1952), similar agreement is obtained. Let the area of stacking error per unit volume be A , and let p be the probability of scattering for a Fermi electron passing through a stacking error. We assume that the effective volume of a stacking error is two atomic layers thick, i.e. 6 Å. Then

$$6 \times 10^{-8} A = \alpha = 0.02,$$

$$A p = (5.6 \times 10^{-6})^{-1}. \quad \dots\dots\dots (4)$$

Hence $p=0.53$ and $A=3 \times 10^{-5} \text{ cm.}^2$.

The magnitude of the scattering probability thus derived seems reasonable. An elementary wave-fitting calculation, assuming perpendicular incidence and a sudden phase change of $2\pi/3$ gives, after averaging over all phases, a reflection coefficient of 0.5. While the fortuitously close agreement with p is of no particular significance, the present picture is clearly self-consistent in regard to orders of magnitude.

The assumption that the volume occupied by imperfections does not contribute to the thermal scattering of electrons is therefore consistent with observations on the assumption of either of the two models.

III. REFERENCES

- BOAS, W., and NICHOLAS, J. F. (1953).—*Aust. J. Phys.* **6**: 116.
BROOM, T. (1952).—*Proc. Phys. Soc. Lond.* B **65**: 871.