# RECENT STUDIES OF CHROMOSPHERIC SPICULES

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#### Summary

Five spicule film sequences taken on different dates in H $\alpha$  were studied. Maximum heights of spicules ranged downward from 20,000 km. Lifetimes ranged from the film limit of 0.3 min to about 30 min. Distributions of maximum heights and lifetimes indicated the occurrence of numerous small, short-lived spicules that were not resolved in these films. The mean upward velocity of the larger spicules was 31 km/sec, with a broad distribution. The absence of evidence of gravitational deceleration, together with a correlation between individual velocities and maximum heights above the chromosphere, suggests a modified interpretation of the spicule process. The data yield a tentative upper limit to the number of spicules on the entire Sun of  $2 \cdot 2 \times 10^4$ .

## I. INTRODUCTION

During the past 10 years, chromospheric spicules have gained steadily in interest and significance. They are involved in theories of the structure and conditions in the chromosphere, and of the transport of material and kinetic energy into the corona; they are probably related to the granular turbulence structure of the photosphere; and they even suggest a connexion with the origins of M-region storms.

Observers of eclipses have noted the profusion of spikes and filaments that emanate from the upper chromosphere. On the basis of such observations, Menzel (1931) suggested that the chromosphere may be composed entirely of such structures. Roberts's discovery in 1943 that the spicules could be observed with the Lyot-type coronagraph attracted renewed interest to the subject. Roberts's earlier papers (Roberts 1945; Roberts *et al.* 1949) on spicules have reported the results of preliminary studies.

Hedeman (1949) has reported a study of very small prominences within the size range of spicules. Mohler (1951) obtained heights and numbers of spicules from eclipse observations. Dizer (1952) has reported an investigation of equatorial spicules; he finds systematic differences in the behaviour of spicules associated with quiet or agitated regions of the chromosphere.

Thomas (1948) called attention to the significance of spicules to the theoretical interpretation of the chromosphere. His theory sought to explain the broadened line profiles and the high temperatures found for the chromosphere by Redman (1942), which agreed with the density gradients of Wildt (1947), in terms of *mechanical* transport of energy from the photosphere to the

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atmosphere of the Sun. He developed a theoretical mechanism of such transport in the form of supersonic jets, which he assumed correspond to the spicules. Recently doubts were raised by Woolley and Allen (1950) as to the existence of a high chromospheric temperature. Redman (1953), however, found from 1952 eclipse data a new value of 17,000 °K. The evidence for a high temperature has been reinforced by the recent findings by Athay *et al.* (1953), Dimock *et al.* (1953), and others from High Altitude Observatory spectra of the 1952 eclipse. From studies of free-bound emission in the Balmer continuum, they have obtained a strong height gradient of temperature, leading to values of the order of 25,000 °K or more in the upper chromosphere.

The identification of the photospheric granules with turbulent cells in the convection layer inevitably suggests the possibility that the spicules originate in this turbulence and may even be projections through the chromosphere of individual granule cells. We believe that the research reported here strengthens the hypothesis of such a direct relation between spicules and granules, though it by no means disposes of all the difficulties. The bases for this statement will be developed in detail in our conclusions.

Allen (1944) suggested that the white-light coronal streamers, rotating with the Sun, are the agents of the periodic M-region storms. The authors and others (Roberts, Grenchik, and Billings 1953; Rush and Roberts 1953), following up this suggestion, have proposed the hypothesis that such coronal streamers are neutral beams of ionized material ejected in the spicules and focused by the interactions of the magnetic fields of sunspot groups with each other, or with a general solar field.

We undertook the present research to develop as detailed as possible an understanding of the behaviour of spicules from available observations, and particularly to evaluate more accurately their numbers, dimensions, and velocities—the parameters that bear most significantly on the questions of material and energy transport. Our results generally confirm those already reported. Certain inconsistencies with other work are noted.

# II. OBSERVATIONAL MATERIAL

Five 35-mm film sequences exhibiting spicule activity were studied. Table 1 lists the pertinent data for these films. All were exposed on Eastman 103-H $\alpha$  film in the 5-in. coronagraph at Climax, through a birefringent filter having a half-width of approximately 4 Å, centred on H $\alpha$ . About 90° of the limb was included in the picture frame. Exposure times were approximately 2 sec.

These five films are the best that were obtained before the end of the detailed reduction program. Preliminary studies of films Nos. 1 and 3 have been reported by Roberts *et al.* (1949). Nos. 4 and 5 are distinctly superior to the first three, because of their higher time-resolution and much richer spicule activity. Whether the latter effect is due to better seeing or actually greater spicule abundance is uncertain.

Reductions of these films were carried out by a computations group at the Massachusetts Institute of Technology. The quantitative data were derived from measurements on a Mann comparator of spicule positions and displacements.

#### III. Results

# (a) General Remarks

Because of the richness, homogeneity, and superior time resolution of films Nos. 4 and 5, we have drawn our conclusions largely from them. Several factors tended to introduce spurious latitude dependences into the spicule distributions. We do not exclude the possibility of real correlations between spicule characteristics and latitude; but the present data have not afforded means of distinguishing any such real dependence from the spurious effects that we know are present. We have therefore assumed that the fairly homogeneous characteristics of the spicules within the middle segment of about 50° of the limb in our pictures are an acceptable sample of phenomena which provisionally we assume to be uniformly distributed over the entire Sun—at least in regions free of sunspot activity. The results that follow are derived from this central segment of the pictures.

		Seminie		1	
Film No.	Date	Latitude at Middle of Picture (+=North)	Duration (min)	Interval between Exposures (sec)	No. of Spicules Observed
1 2 3 4 5	12.xii.43 28.xii.43 21. ii.46 29. i.49 2.vii.49	$+78^{\circ}$ +86^{\circ} +84^{\circ} -86^{\circ} -88^{\circ}	235 67 122 50 42	60 20 20 10 10	361 126 207 458 443

TABLE 1 SUMMARY OF SPICULE FILMS

We wish to emphasize that the spicules observed for this study are faint objects, compared with the usual quiescent prominence. They completely surmount the chromosphere proper. With the exposures used in this work, the chromosphere itself appears as an overexposed luminous band with a rather irregular upper edge. Many of the photographs of spicules taken by others and compared with ours appear to have been taken at somewhat shorter exposures, and thus refer to levels probably lower in the chromosphere.

# (b) Time and Space Distribution of Spicules

Analysis of the position angles and beginning times of the spicules of film No. 4 showed no significant tendency to occur in non-random groups either in time or in space, except the dubious latitude distribution already mentioned. A similar analysis of the distribution of the non-radial spicules only, in film No. 5, showed no non-random tendency.

The density and uniformity of distribution of spicules over the Sun are of fundamental interest. The five films that have been reduced in detail cover only the polar zones; but other films in our files show spicules at low latitudes. Hansen and other observers at Climax (unpublished data) have reported occasions when the entire limb in H $\alpha$  was "bristling" with spicules. Mohler (1951) has reported on the profusion of spicules appearing in white light from pole to equator in Marriott's 1930 eclipse photographs. Dizer (1952) has reported a study of equatorial spicules in Lyot's films. R. B. Dunn has communicated unpublished observations of spicules over wide ranges of latitude. Yet we have seen quiet regions of the limb in H $\alpha$  photographs that were entirely free of spicules, even though the definition of the limb and adjacent prominence detail appeared adequate for the detection of spicules ; and Roberts is of the opinion, based on visual observation, that spicules tend to be most uniform in behaviour—simple, radial, and undisturbed-looking—in large regions centred roughly on the poles.

On the basis of all available evidence, we believe it is reasonable to conclude that spicules occur at one time or another over the entire chromosphere; but the available evidence does not seem sufficient to establish whether or not they are active *continuously* over the entire Sun.

In examining some good copies of Swarthmore College and Lick Observatory eclipse plates,\* we noted that in several instances spicules appeared superimposed on the white-light coronal arches. In each such case the spicules departed from the radial direction and were aligned tangent to the contours of the arches.

If it be assumed that the distribution of spicules we observed in the vicinity of the pole is uniform over the entire Sun, then an upper limit to the number of spicules on the Sun at a given time can be determined, as Mohler (1951) has demonstrated. The method is to assign to each spicule an area of the chromosphere equal to the square of the mean distance observed between spicules at the limb. In each of our films Nos. 4 and 5, an average of 42 spicules per radian were on the limb in the region of interest at a given time. The result, on this basis, for the entire Sun is  $2 \cdot 2 \times 10^4$  spicules.

An independent check of spicule numbers was made on the flash spectrum of the 1952 eclipse. Spicules appeared clearly in  $H\alpha$  at a height of 6300 km above the photosphere, over an arc of the east limb centred at about latitude 65°. The spicule count over the middle 45° of this arc was 46 per radian.

This procedure assumes that all of the observed spicules are actually on the limb. Since they must be distributed in the line of sight, the actual number on the Sun is substantially less than the above result. A rough estimate of the actual distribution, based on the argument developed later (Section III (d)), suggests that 5000 is a reasonable number. In appraising this result, it must be kept in mind that the visibility of small spicules is very sensitive to seeing conditions and other factors.

## (c) Size and Brightness of Spicules

The reduction programme included a qualitative estimate of the relative size and brightness of each spicule. The size so estimated was a composite of maximum height and breadth, an index of the apparent bulk of the spicule. No significant correlation was found between size and brightness.

<sup>\*</sup> Through the courtesy of Dr. Donald H. Menzel of Harvard College Observatory.

The actual diameters of spicules are quite uncertain because of their diffuse boundaries and the limited resolution in the photographs. For the great majority of spicules, a typical diameter at the chromosphere of the order of 2000 km is indicated.

# (d) Heights of Spicules

The maximum heights reached by spicules were difficult to determine, because of the faintness of the tip of the spicule, uncertainty in locating the level of the chromosphere, and inconsistencies caused by variations in seeing. Because of such difficulties, it was impracticable to determine the heights of the great majority of spicules, those less than two or three thousand km in height above the photosphere.



Fig. 1.—Distribution of heights above the chromosphere of 263 spicules.

The distribution of observed heights from films Nos. 4 and 5 is shown in Figure 1. Probably the distribution is influenced significantly by observational selection in the reduction process; the heights of only one-fourth of the spicules were measured, and the decision whether to try to measure a spicule or omit it was necessarily somewhat arbitrary. It appears virtually certain that the decline of the curve below 5000 km is due to such selection, and that the distribution actually continues upward somewhat as indicated in the figure. This distribution is similar to that of lifetimes (Section III (e)); but the data were insufficient to determine its form with comparable precision.

The distribution of spicules in the line of sight obviously affects their apparent heights, since they suffer partial obscuration depending on their distances from the limb. If a spicule of actual height h is located at an angle  $\alpha$ 

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from the limb, its apparent height  $h^*$  is given with sufficient accuracy by the approximation

$$h^* = h - \frac{R\alpha^2}{2},$$

where R is the radius of the limb. Thus the apparent heights  $h^*$  of a set of spicules of a given actual height h are dispersed througout the range from 0 to h.

The actual height distribution N(h) may be approximated from the observed distribution  $N(h^*)$  by a simple geometrical analysis (Appendix I). The dashed curve in Figure 1 is the corrected distribution we obtained in this way from the solid curve representing the observed distribution. The value 15,000 km was taken as the maximum height H, and computations were based on 1000-km ranges from 5000 to H km. Evidently the effect of distribution in the line of sight does not greatly alter the form of the distribution, which after correction still approximates a hyperbola or an exponential.



# (e) Lifetimes of Spicules

We define the lifetime T of a spicule as the time interval from its first appearance to its final disappearance. During this period a typical spicule rises vertically at roughly constant velocity from the chromosphere to its maximum height; then it may remain stationary while fading from visibility, or it may appear to descend back into the chromosphere at a velocity comparable to the upward velocities.

The frequency distribution of lifetimes for 400 spicules is given in Figure 2. The mean lifetime for the range covered in the figure is  $3 \cdot 3 \min$ . Similar distributions were obtained from films Nos. 1, 2, and 3, yielding an overall mean lifetime for all five films of about  $3 \cdot 5 \min$ . These means agree well with Roberts's (1945) preliminary result of 4–5 min from film No. 1, which included no lifetimes less than 1 min.

The hyperbola  $(N \% = 16 \cdot 6/T - 0 \cdot 7)$  approximates the data much more closely than does an exponential. The continuing upward trend of the curve at the short-lifetime end indicates that the maximum is to be found below our present 0.3-min limit of time resolution, and that large numbers of short-lived spicules must occur below this limit. This result lends some support to the view that the chromosphere is composed entirely of radial spikes or filaments.

The evaluation of T is complicated by two important factors. First, the spicules seen in profile on the limb presumably are distributed randomly in the line of sight; and no simple theoretical correction to the distribution for obscuration of spicules by the limb can be applied, because T is a composite of the interval during which the spicule is ascending (or descending) and the interval during which it is stationary. The former interval is affected by limb obscuration, but the latter is not, and the two do not appear to be systematically related.

As we showed in Section III (d), however, the effect of obscuration by the limb is to elevate the height distribution curve by a factor that, qualitatively speaking, varies inversely as the height. Only a weak correlation exists between heights and total lifetimes of the spicules whose heights were measured; but there is good reason to believe that the correlation would be stronger if the The heights of most spicules were heights of all spicules could be measured. too small for reliable measurement, and these were predominantly in the shortlifetime region. If we assume such a rough overall correlation between spicule heights and lifetimes, it is evident that obscuration by the limb of spicules distributed along the line of sight will tend to elevate the lifetime distribution curve in the same sense as the height curve, so that the short-lived spicules appear relatively too numerous. Although this effect cannot be accurately evaluated from our data, it is evident that its influence on the distribution of lifetimes must be less than on that of heights, and that the effect cannot alter the general form of the distribution.

Second, intervals of bad seeing and possibly other factors prevent a spicule from being seen on every frame during its lifetime. Only continuity of position and development permits the identification of such an interrupted sequence as a single spicule. Obviously, such interruptions must be particularly effective in cutting off observation of a spicule at the beginning and end of its lifetime, when it is least conspicuous. This loss of terminal frames results in a systematic reduction of the observed lifetimes below the true values.

In this case, the effect on the distribution curve can be estimated from the number of frames in which a spicule was not seen, between its first and last appearance. The resulting corrections to the observed distribution are small, amounting to about 8 per cent. for the most numerous lifetime class. But this effect of lost terminal frames upon the actual lifetime distribution curve is in the opposite sense to that of obscuration in the line of sight, because it preferentially lowers the short-lifetime end of the curve. Lacking a way to evaluate the line-of-sight effect, we can only note that these two effects tend to compensate. We believe that the empirical distribution curve of Figure 2 is a good

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approximation to the actual distribution of spicule lifetimes; and, in particular, that the maximum of the distribution is to be found at shorter lifetimes than we have resolved. The relatively low value of the last point (T=0.5 min) may mean that we are approaching the maximum; but it may equally well be ascribed to failure to detect some of the very short-lived spicules.

### (f) Velocities of Spicules

The velocities of some larger spicules were determined from plots of the heights measured on successive frames. The results are summarized in Table 2. In this summary, only those spicules have been included whose development was fairly regular and continuous. Such a spicule typically rose with approximately constant velocity and stopped abruptly at its maximum height. In

Film No.	No. of Typical Spicules	Mean Lifetime* (min)	$v_a \; (\mathrm{km/sec})$		
			Mean	Median	Mode
1	17	11.3	33	30	
2	6	$7\cdot 2$ .	30		
3	15	$15 \cdot 0$	29	32	34
4	35	10.5	30	26	28
5	28	$10 \cdot 8$	33	24	25
All	101	11.1	31		

TABLE 2

\* Mean lifetime of those spicules whose entire lifetimes were observed. Note that these lifetimes for spicules with measured velocities are substantially longer than the mean lifetimes of all spicules on these films.

view of the many uncertainties and irregularities involved, the agreement among the mean velocities derived from the five films is remarkably close. These values and the mean of all of the velocities, 31 km/sec, confirm the value of 30 km/sec estimated by Roberts (1945) in a preliminary study of film No. 1. They agree also with the mean of Dizer's (1952) values of 20 and 40 km/sec for equatorial spicules in "quiet" and "agitated" regions of the chromosphere.

It should be noted, however, that the values listed in Table 2 are the means of a broad range of individual velocities. The distribution of velocities derived from films Nos. 4 and 5 is presented in Figure 3.

Some spicules exhibited apparent variations in velocity, occasionally reaching 200 km/sec, and sometimes stationary phases, before attaining their maximum height. In a few cases spicules seemed to ascend in a succession of surges. We did not attempt to summarize the velocities with which some spicules appeared to descend. These descents sometimes were regular and measurable; more often they were abrupt and irregular, suggesting that an apparent descent was actually a progressive loss of visibility. It might be thought that variations in seeing from frame to frame would affect the apparent height of a spicule, thus introducing spurious irregularities into its progress; but we found a systematic variation of only about 10 per cent. in height between the best and worst seeing.

The generality of the results in Table 2 is limited also by the fact that satisfactory measurements of velocity could be made only on relatively large, long-lived spicules. Almost all of these exceeded 5000 km in height, and their mean lifetime of 10 min contrasts with a mean of about 3.5 min for all spicules that were observed.

The individual velocities showed no correlation with lifetimes. They were, however, correlated with maximum heights by a coefficient of 0.70. This relation (Fig. 4) is essentially linear, with its origin approximately at the top of the chromosphere. These relations appear to contradict those reported by Dizer (1952), who found uniform speeds of ascent and descent, and heights directly proportional to lifetimes. The scatter of the data is such that a curve originating at the photosphere is not entirely precluded ; but such a curve, to fit the data acceptably, would have to be non-linear.



Fig. 3.—Distribution of ascending velocities of 59 spicules from films Nos. 4 and 5.

It is obvious that the correlation between heights and velocities, implying that these spicules must rise to maximum height in about 4.5 min, cannot be generalized to the large class of spicules whose total lifetimes are less than 4 min. Yet we found no basis for attributing the correlation to any preferential selection of data. Though the velocity measurements were limited of necessity to a class of generally large, long-lived spicules, the range of heights, velocities, and total lifetimes within this body of data was great.

Figure 4 shows no evidence of influence of line-of-sight obscuration on the height distribution. The scatter of heights about the visually-located line is essentially random; the points show no tendency to concentrate in the upper portion of the plot, as they would be expected to do if obscuration in the line of sight were a dominant factor. It seems probable that the necessity of selecting relatively large spicules for velocity measurements insured that most of those selected would be situated near the limb, thus practically eliminating the complications that would have resulted from an extensive distribution in the line of sight.

#### IV. THEORETICAL IMPLICATIONS

One of the most interesting of our results is the finding that the linear relation between velocity and height apparently converges to an origin approximately at the top of the chromosphere. As it does not appear probable that the jet, which emerges as a spicule, originates at so high a level, the simplest conclusion is that while the spicule is rising through the chromosphere it remains in condition to radiate effectively in  $H\alpha$ ; it might be regarded as a moving sample of the chromosphere. But, as it rises above the chromosphere, it enters a region in which, because of increasing ionization, the radiation from neutral hydrogen is rapidly attenuated. The radiation from the rising material should, however,



Fig. 4.—Ascending velocities and maximum heights of 61 spicules from films Nos. 4 and 5. Slope of curve :  $\Delta h/\Delta v = 4 \cdot 5$  min.

require some time to decline to a negligible level, and the height above the chromosphere to which it remains visible should then be proportional to its velocity, to a first approximation.

On this interpretation, the observation that a spicule typically rises at approximately uniform velocity and then abruptly stops becomes intelligible. The interpretation we propose is that the material continues to flow upward beyond its visible terminus at a constant velocity during the entire period in which the spicule remains apparently stationary. Then, if the velocity decreases, the column of material appears to descend; or, if the supply of material diminishes, the spicule fades away without appearing to descend. The anomalous cases of spicules that appear to rise and fall repeatedly in an irregular series of surges can be explained by assuming corresponding fluctuations in density of material or in upward velocity alone.

This interpretation of a spicule implies that the motion of the jet is essentially independent of the gravitational field. Our measurements of spicule heights in successive frames indicated that the upward velocity of a typical spicule is essentially constant. The fluctuations appeared to be random, with no evidence of systematic deceleration. It appears that spicules, like eruptive prominences on a larger scale, ascend at approximately constant velocities despite the gravitational field.

Thomas (1948) estimated that the kinetic energy transported by the spicules from the photosphere is approximately  $2 \times 10^{32}$  ergs/sec, of which about 10 per cent. is dissipated in the chromosphere. This estimate was based on the assumption of gravitational motion of the spicule material, which implied an initial velocity of about 90 km/sec at the photosphere to project a spicule to a typical height of 7500 km above the chromosphere. Our findings, however, indicate that the velocities at the photosphere must be substantially the same as those observed at the top of the chromosphere. Our mean velocity of 31 km/sec therefore requires a reduction of Thomas's estimate of the kinetic energy in the spicule system by a factor of 9; but, on our interpretation, all of this energy would be available for heating the chromosphere and corona.

We do not believe, however, that our mean of 31 km/sec for the few velocities we measured is representative of all spicules. The relation between height and velocity, if it has any validity for the numerous unmeasured small spicules, implies that these must have velocities of less than 10 km/sec. At the same time, the distribution of lifetimes indicates the existence of many still shorterlived small spicules not yet observed individually. The low energies of these small jets may be effectively compensated by their great numbers. We do not believe that the new data are necessarily inconsistent with Thomas's estimate of energy transport; but they raise questions that can be resolved only by further research.

The possibility of a direct relation to the photospheric turbulence granules is implicit in any discussion of spicules. The chief obstacle to direct identification of the two phenomena is the wide disparity between the mean turbulence velocities observed in the granulation and the observed velocities of the larger spicules. Yet the relation we have cited between heights and velocities, as well as the scarcity of spicules having velocities great enough to measure and the large scatter of velocities in this group, all suggest a broad statistical distribution of spicule velocities, only a few of which are as great as 30 or even 10 km/sec. Similarly, the mean Doppler velocity of the order of 1 km/sec observed in the granulation by Richardson and Schwarzschild (1950) and others does not preclude a statistical dispersion of velocities such that perhaps one granule in 10,000 might exceed 10 km/sec.

Nor does the disparity between the observed numbers of spicules and of granules appear to offer any insuperable difficulty. We found a value of the order of  $10^4$  for the total number of observable spicules on the Sun, under certain

dubious assumptions; Mohler (1951), from eclipse plates of extraordinarily good definition, found an upper limit of  $4 \times 10^5$ . The number of small granules, from observations by Keenan (1938, 1939), Macris (1953), W. A. Miller (personal communication), and others, appears to be about 10<sup>6</sup>. Both the "grasslike" appearance of the upper chromosphere and the trend of the lifetimes distribution argue for the existence of additional small unresolved spicules which may quite possibly be more numerous than those we have observed by a factor of 100.

We believe, therefore, that the evidence at hand supports the hypothesis that the quiet-Sun chromosphere—or at least its visible components—is composed of radial jets arising from the photospheric turbulence. On this view, each convection cell of ascending gas, marked by a bright granule on the photosphere, projects a jet beyond the photosphere. The velocities of these jets, derived from the underlying turbulence, are distributed statistically over a range of perhaps 0–100 km/sec. The uneven top of the chromosphere is defined statistically as the region in which the great majority of small, low-velocity jets encounter conditions that render them invisible. The positive temperature gradient in the chromosphere is maintained, as Thomas (1948) proposed, by the dissipation of kinetic energy from the rising jets into the surrounding gas. The jets of exceptionally great bulk and velocity, the "tail" of the statistical distribution, remain visible to various heights above the general level of the majority, and appear individually as spicules.

This hypothesis is admittedly tentative and subject to some unresolved difficulties; but we believe that it fits the observational situation sufficiently well to merit intensive study.

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### **VI.** References

Allen, C. W. (1944).-Mon. Not. R. Astr. Soc. 104: 13.

ATHAY, R. G., BILLINGS, D. E., EVANS, J. W., and ROBERTS, W. O. (1953).—Astr. J. 58: 210.

DIZER, M. (1952).-C.R. Acad. Sci., Paris, 235: 1016.

DIMOCK, D. L., BILLINGS, D. E., ATHAY, R. G., and THOMAS, R. N. (1953).—Paper presented before the American Astronomical Society in Boulder, Colorado, August 29, 1953. Abstract, Astr. J. 58: 213.

HEDEMAN, E. R. (1949).—Publ. Astr. Soc. Pacif. 61: 224.

KEENAN, P. C. (1938).—Astrophys. J. 88: 360.

KEENAN, P. C. (1939).—Astrophys. J. 89: 604.

MACRIS, C. (1953).—Ann. Astrophys. 16: 19.

MENZEL, D. H. (1931).—Publ. Lick Obs. 17: 287.

MOHLER, O. (1951).-Mon. Not. R. Astr. Soc. 111: 630.

REDMAN, R. O. (1942).—Mon. Not. R. Astr. Soc. 102: 140.

REDMAN, R. O. (1953).-Accad. Naz. Lincei, Conv. Volta No. 11.

RICHARDSON, R. S., and SCHWARZSCHILD, M. (1950).—Astrophys. J. 111: 351.

ROBERTS, W. O. (1945).—Astrophys. J. 101: 136.

ROBERTS, W. O., BRENTON, V., SHAPLEY, M. B., and KOPAL, S. (1949).—Publ. Astr. Soc. Pacif. 61: 160.

ROBERTS, W. O., GRENCHIK, R., and BILLINGS, D. E. (1953).—Paper presented before the American Astronomical Society in Boulder, Colorado, August 29, 1953. Abstract, Astr. J. 58: 225.

RUSH, J. H., and ROBERTS, W. O. (1953).—Trans. Inst. Radio Engrs., Professional Group on Communications Systems CS-2: 24.

Тномая, R. N. (1948).—Astrophys. J. 108: 130.

WILDT, R. (1947).—Astrophys. J. 105: 36.

WOOLLEY, R. V. D. R., and ALLEN, C. W. (1950).-Mon. Not. R. Astr. Soc. 110: 358.

#### APPENDIX I

### Effect of Obscuration in the Line of Sight

In Figure 5, a, b, c, d, e represent an arbitrary series of equal ranges  $\Delta h$  of actual spicule heights, from the limb L to the maximum height H above the limb. The angles A, B, C, D, E are the successive angular increments from the limb over which the curvature of the limb away from the tangent L is equal to  $\Delta h/2$ ,  $3\Delta h/2$ ,  $5\Delta h/2$ ,  $7\Delta h/2$ ,  $9\Delta h/2$  respectively. These angles are small enough that the reduction of h by foreshortening can be neglected. Also, the ratio H/R is sufficiently small that the curvature away from the tangent over any angle we are concerned with can be considered constant for all radii from R to R+H.

An observer whose line of sight is parallel to the tangent to the limb L will see spicules apparently in range a arising throughout the angular range A+B+C+D+E; but it is evident from the figure that this apparent class of heights is actually a composite derived from all values of h. Heights actually in range a are observed in this range over the angle A; heights actually in range bare observed as being in range a throughout angle B; those actually in cappear in a throughout angle C; and so on. Thus the heights appearing in range a actually are derived from all height ranges. The apparent range b, however, receives no contribution in angle E; c receives none in angle D or E; and so on. It is therefore evident that, if spicule heights are randomly distributed over the surface of the Sun, the distribution observed in the line of sight will be a distortion of the actual distribution, and will show disproportionately large numbers of spicules in the lesser height ranges.

By examination of the observed distribution curve  $N(h^*)$ , a somewhat arbitrary determination may be made of the maximum height that occurs with any significant frequency. This height is taken as the maximum actual height

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*H* attained by the spicules; but the correction is not very sensitive to the choice of *H*, because of the relatively small numbers of spicules in this part of the distribution. The angles *A*, *B*, *C*, *D*, *E* are then computed within which the mean height of range *e* falls in the arbitrary height ranges *e*, *d*, *c*, *b*, *a* respectively. Since the maximum range *e* can receive no apparent contributions from the lesser ranges, all of the  $N_e^*$  spicules appearing in range *e* are assumed to be located



Fig. 5.—Effect of obscuration by the limb on apparent heights of spicules distributed along the line of sight.

in the angle A, so that  $N_e^* = N_e$ . Now, the  $N_d^*$  spicules appearing in range d include  $(B/A)N_e$  spicules from actual range e distributed over angle B; and the difference between this term and  $N_d^*$  gives the actual residue  $N_d$  in angle A. Similarly,  $N_c = N_e^* - (B/A)N_d - (C/A)N_e$ ; and so on. The numbers  $N_a, N_b, \ldots$  of spicules occurring over the same angular range A therefore constitute the actual height distribution.