THE ALBEDO FOR THE ATOMIC SCATTERING OF OPTICAL RADIATION

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Summary

A general method for evaluating scattering is discussed for a multilevel atom. It is shown that, in the special case when the gas is either opaque or highly transparent to every other spectral line, the effects of interlocking on a given spectral line can be disregarded and the scattering readily computed.

I. INTRODUCTION

The transfer of radiation through an emitting and absorbing medium can be described by the so-called equation of transfer, which in turn is dependent on the monochromatic source function B_ν defined by $B_\nu=E_\nu/\alpha_\nu$, E_ν and α_ν being the emission and absorption coefficients. If emission in a given spectral line arises as a result of previous absorption in the same spectral line, it is referred to as scattering, and is said to be coherent if the re-emission is of the same frequency as that absorbed, non-coherent if not. The albedo for single scattering, denoted by $1-\lambda$, is defined as the fraction of absorbed radiation which is subsequently scattered.

Thus for isotropic, coherent scattering the source function may be written

$$B_\nu=(1-\lambda)\frac{J_\nu}{4\pi}+b_\nu, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.1)$$

where J_ν is the total intensity of the radiation and b_ν is a term dependent on collisional excitation and absorption of radiation in other spectral lines.

For non-coherent scattering

$$B_\nu=\frac{1-\lambda}{4\pi\alpha_\nu}\int J_{\nu'}\alpha_{\nu'}g_{\nu',\nu}\mathrm{d}\nu'+b_\nu, \qquad \ldots\ldots\ldots\ldots \quad (1.2)$$

where $(1-\lambda)g_{\nu',\nu}\mathrm{d}\nu'\mathrm{d}\nu$ is the fraction of energy absorbed in the frequency range ν' to $\nu'+\mathrm{d}\nu'$ which is scattered into the range ν to $\nu+\mathrm{d}\nu$. In a simple case of non-coherent scattering, the scattered radiation is redistributed proportionally to α_ν, so that

$$g_{\nu',\nu}=\frac{\alpha_\nu}{\int\alpha_\nu\mathrm{d}\nu},$$

and so

$$B_\nu=\frac{1-\lambda}{4\pi}\,\frac{\int J_\nu\alpha_\nu\mathrm{d}\nu}{\int\alpha_\nu\mathrm{d}\nu}+b_\nu. \qquad \ldots\ldots\ldots\ldots \quad (1.3)$$

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Another important case occurs in the scattering of continuous radiation which is distributed as $\exp(-h\nu/k\theta)$ for $\nu \geqslant \nu_0$, the frequency of the beginning of the continuum. Here

$$B_\nu = \frac{1-\lambda}{4\pi\alpha_\nu}\left(\frac{h}{k\theta}\right)\exp\left[\frac{-h(\nu-\nu_0)}{k\theta}\right]\int \alpha_\nu J_\nu \mathrm{d}\nu + b_\nu. \quad \ldots\ldots \ (\mathbf{1.4})$$

In general, owing to the dependence of b and λ on radiation intensities, simultaneous equations of transfer are involved in describing the radiant intensity and the parameter of importance is not an individual λ—which then has little meaning—but some interlocking parameter obtained by elimination of the other radiation intensities.

Most attention has been devoted to scattering in connexion with the formation of absorption lines by atmospheres in local thermodynamic equilibrium, and a detailed account of techniques applicable has been given by Woolley and Stibbs (1953).*

A method will now be given for simplifying such problems when, for any line other than the one under discussion, the medium is either opaque or transparent. In these cases it will be shown that the quantities b_ν and λ depend only on collisions and external irradiation. As an example of the application of the method general expressions for these two quantities are obtained for the case of an atom with four energy levels.

II. Effect of Interlocking

The equation of secular equilibrium for the population N_j of the j state may be written as

$$\sum_s P_{js}N_j = \sum_s P_{sj}N_s, \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (\mathbf{2.1})$$

where P_{sj} is the rate of transition $s \to j$ per atom in the s state, and is the sum of a radiative term A_{sj} (which would be zero for a forbidden transition) and a collision term R_{sj}.

By solving the set of simultaneous equations (**2.1**), the ratios of the populations may be obtained in terms of the transition rates. Thus in a four-level atom it is found that

$$\frac{N_b}{N_a} = \frac{P_{ab}(1-p_{cd}p_{dc}) + P_{ac}(p_{cb}+p_{cd}p_{db}) + P_{ad}(p_{db}+p_{dc}p_{cb})}{P_{ba}(1-p_{cd}p_{dc}) + P_{bc}(p_{ca}+p_{cd}p_{da}) + P_{bd}(p_{da}+p_{dc}p_{ca})}, \quad \ldots \ (\mathbf{2.2})$$

where

$$p_{rs} = \frac{P_{rs}}{\sum_t P_{rt}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (\mathbf{2.3})$$

This equation shows how N_b/N_a, and hence the source function for (b,a) radiation, depends on the intensity of each line emitted by the atom.

Now, if the medium is optically thin $(\tau_0 \ll 1)$ in any line, self-absorption in this line can be neglected with respect to other excitation processes, and radiative excitation in this line can be due only to irradiation from outside the atmosphere.

* Woolley, R. v. d. R., and Stibbs, D. W. N. (1953).—" The Outer Layers of a Star." Ch. 8. (Oxford Univ. Press.)

Again, if the atmosphere is of sufficient optical depth then

$$A_{rs}N_r = A_{sr}N_s. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.4)$$

On cancelling these terms in (**2.1**), except for the transition (a,b), P_{rs} and P_{sr} are replaced by R_{rs} and R_{sr} respectively, with corresponding changes in (**2.2**) and (**2.3**).

If for each spectral line other than the line (a,b) itself, either of the two above conditions applies, then the effects of interlocking on the line (a,b) may be disregarded.

III. Determination of λ and *b*

In such a case, N_b is the sum of a term $\mathbf{N}_b$ proportional to the rate of absorption of (a,b) radiation, and a term n_b which is explicitly independent of absorption. Then

$$A_{ba}\mathbf{N}_b = (1-\lambda_{ab})A_{ab}N_a. \quad \ldots\ldots\ldots\ldots \quad (3.1)$$

Thus

$$1-\lambda_{ab} = \frac{A_{ba}(1-p_{cd}p_{dc})}{D}, \quad \ldots\ldots\ldots\ldots \quad (3.2)$$

where D is the denominator on the right-hand side of (**2.2**).

Hence

$$\lambda_{ab} = \frac{R_{ba}(1-p_{cd}p_{dc})+P_{bc}(p_{ca}+p_{cd}p_{da})+P_{bd}(p_{da}+p_{dc}p_{ca})}{D}. \quad \ldots \quad (3.3)$$

The other quantity, b, involved in the source function can also be obtained from (**2.2**), for

$$b_{ab} = \frac{\rho_{ab}n_b}{4\pi N_a}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

where ρ_{ab} can be found from expressions for the emission and absorption coefficients. Thus

$$b_{ab} = \frac{\rho_{ab}}{4\pi}\cdot\frac{R_{ab}(1-p_{cd}p_{dc})+P_{ac}(p_{cb}+p_{cd}p_{db})+P_{ad}(p_{db}+p_{dc}p_{cb})}{D}, \quad \ldots \quad (3.5)$$

where in both (**3.3**) and (**3.5**) for lines of high optical depth P_{rs} is replaced by R_{rs} and for lines of low optical depth self-absorption may be neglected.

IV. Conclusions

The solution of the transfer equations is simplest in the case of a gas in which the values of R and P in (**3.3**) and (**3.5**) can be taken as constant throughout. This will be so in a uniform density isothermal atmosphere of small optical depth in the resonance lines.

Where the optical depth is high in the resonance lines, condition (**2.4**) breaks down near the boundaries, but this will not usually have a serious effect ; the main result will be a reduction in the central intensity of the first resonance line, and a corresponding increase in the central intensity of some of the other lines.

When the atmosphere is of intermediate optical thickness in one or more lines, or near the boundaries of the gas, equation (**3.3**) still gives formally the effective scattering parameter, though in these cases λ varies with the intensities in the other lines. Unless these can be shown to be negligible, the radiation intensities in such cases are to be found only by solving the equations of transfer simultaneously.