SHORT COMMUNICATIONS

ON THE DIPOLE RESONANT MODE OF AN IONIZED GAS COLUMN*†

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In a recent paper, Makinson and Slade (1954) examined theoretically the scattering of a transverse-electric radio wave by an ionized gas column. In their treatment, a smooth radial distribution of ion density was replaced by a stepped approximation and the quasi-static equations (Kaiser and Closs 1952) were used to consider scattering by this stepped column. They claim that there exists a multiplicity of resonant dipole modes for a small cylindrical column in which the radial distribution of ionization is described by a Gaussian function. Previously, Kaiser and Closs (1952) had found a single, broad resonance peak by an approximate numerical solution of the quasi-static equations, using the Gaussian distribution itself. The purpose of this note is to suggest that the multiple resonances found by Makinson and Slade were created by the discontinuities in their approximation to the Gaussian function, and that the actual Gaussian distribution exhibits only one resonance.

For transverse polarization of a plane radio wave normally incident on a small column of ionized gas, the dipole mode (cos θ variation of the scattered far fields around the column), is most significant. The scattered fields may be considerably greater than predicted by the Born approximation (which assumes that the incident wave is not attenuated appreciably in passing through the column), due to the effect of charge separations within the column. The charges separate whenever there is an electric field across a volume in which the density of ionization is not constant. The term " resonance " is used here to indicate that the transverse scattering coefficient exceeds the parallel scattering coefficient.

On the basis of a five-step approximation, Makinson and Slade found that the column will exhibit five resonant dipole modes, at values of $\lambda=1\cdot17$, $1\cdot83$, $3\cdot44$, $8\cdot73$, and $29\cdot5$, where $(1-\lambda)$ is the relative dielectric constant at the column axis. The authors claim that " there is no reason to believe that the stepped distribution manufactures resonances with no counterparts for the smooth distribution ".

However, there is sufficient evidence to cast grave doubt on these claims. Preliminary calculations using stepped distributions made by Eshleman several years ago (personal communication) showed that a plasma resonance peak

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would be obtained for each step. Each resonance appeared to be due to surface charges which build up at the discontinuities in the ionization distribution, with each surface being most effective at distinct size-wavelength ratios. These resonances were very sharply peaked. Therefore this method of approximating transverse scattering by a column of ionization with a smooth radial distribution was discarded.

The exact evaluation of reflection coefficients of a Gaussian column of ionization has been performed at Stanford University using the wave-matching technique (Keitel 1955). The differential equations which describe the electromagnetic fields within the column were numerically integrated using a high-speed digital computer, the SWAC (National Bureau of Standards Western Automatic Computer). In order to be able to integrate past the singular point of the differential equation for transverse polarization, collisional losses were included in the expression for the relative dielectric constant. Line densities of ionization of 10^{13}, 10^{15}, and 10^{17} electrons per metre were considered, for various column sizes and collisional frequencies. The results of these computations, described elsewhere (Keitel 1955), did not exhibit the multiple resonance behaviour claimed by Makinson and Slade.

Subsequently, a special series of calculations has been made for a small column of fixed size, $(2\pi r_0/\lambda_0)=0\cdot1$, for various values of λ (corresponding to the notation of Makinson and Slade). The parameter r_0 occurs in the Gaussian distribution function, $\exp[-(r/r_0)^2]$, and λ_0 is the free-space wavelength. The complex relative dielectric constant is $\{1-\lambda(1-i\delta)\exp[-(r/r_0)^2]\}$. In terms of the parameters describing the column of ionization,

$$\lambda=\frac{\Gamma q}{[2\pi r_0/\lambda_0]^2(1+\delta^2)},$$

where $\Gamma=e^2/\pi\varepsilon_0 mc^2=1\cdot1271\times10^{-14}$ m,

q=line density of ionization, electrons per metre,
$\delta=\nu/\omega$=loss ratio,
ν=collisional frequency,
ω=angular frequency of the incident wave,
e=electron charge,
m=electron mass,
ε_0=permittivity of free space.

A loss ratio δ of 10^{-4} was used since this has been found to be small enough to have a negligible effect on the scattering coefficient while making it possible to integrate past the singular point at the critical radius (i.e. the radius at which the real part of the dielectric constant is zero).

The values of λ were chosen sufficiently close together so that there could be no question as to any resonance points being missed. The results are shown in Figure 1. The curve for the parallel reflection coefficient was obtained from the zero-order mode, which is sufficient for the size trail considered. The transverse reflection coefficient curve considered only the dipole mode, the zero-order and higher order modes being insignificant for this size trail. There is only one very broad resonance centred at approximately $\lambda=2\cdot2$, corresponding to a relative dielectric constant of $-1\cdot2$ on the axis of the column. This agrees very well with the approximate results of Kaiser and Closs. The locations of the resonances claimed by Makinson and Slade are indicated in the figure. Three of their five resonances are seen to fall in the range of the calculated resonance. However, separate resonances are not observed. It is doubtful that the transverse reflection coefficient would drop down to the parallel reflection coefficient between any two calculated points in the resonance region.

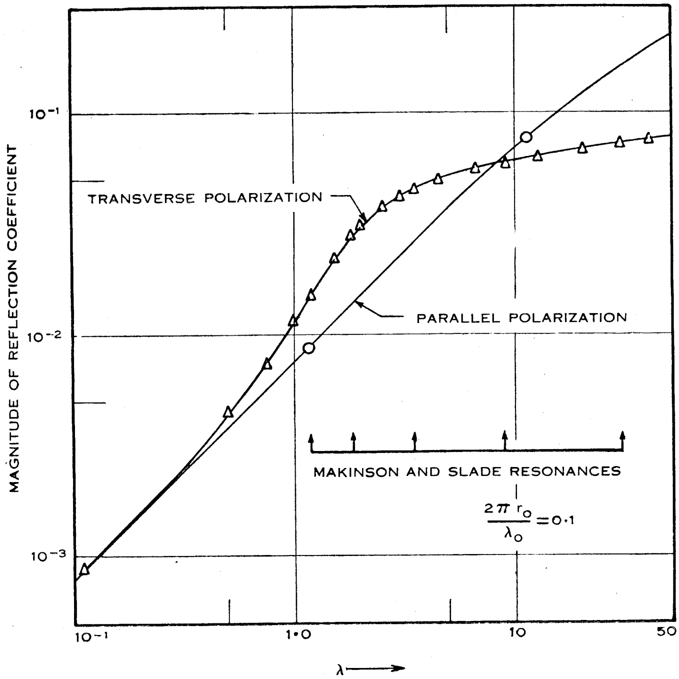

Fig. 1.—Parallel and transverse reflection coefficient of a small Gaussian column as a function of λ, where $(1-\lambda)$ = relative dielectric constant at column axis.

Makinson and Slade have based their claim of multiple resonances on their approximate solution of scattering by a stepped approximation to the Gaussian column. But this type of approximation introduces infinite gradients of electron density at each step. If one examines the differential equations which describe the internal fields, the ratio of the gradient of the dielectric constant to the value of the dielectric constant is found to be very important. For the stepped column, this ratio becomes infinite at each step because of the infinite gradient. For the smooth Gaussian distribution, this ratio becomes large only at the critical radius, where the real part of the dielectric constant is zero. Kaiser and Closs considered the gradient term for the Gaussian column in the neighbourhood of the critical radius. This author has considered this term throughout the Gaussian column. In both cases, only one resonance was found. Thus, it would appear that the multiple resonances found by Makinson and Slade *are* manufactured by the discontinuities in the stepped approximation and are not present for the smooth distribution.

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