THE BEHAVIOUR OF A CHAPMAN LAYER IN THE NIGHT F_2 REGION OF THE IONOSPHERE, UNDER THE INFLUENCE OF GRAVITY, DIFFUSION, AND ATTACHMENT

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Summary

It is shown that, in the presence of diffusion, gravity, and attachment, a Chapman layer, no matter what its height, maintains its shape, decaying uniformly with an effective attachment coefficient equal to the true attachment coefficient at the height of the electron density maximum ; and that, at the same time, the layer drifts bodily towards an equilibrium height.

It is then shown that a uniform vertical tidal drift will alter the equilibrium height of a Chapman layer.

I. INTRODUCTION

A number of workers (Martyn 1954 ; Duncan 1956 ; Yonezawa 1956) have suggested that the height gradient of electron decay, and diffusion under gravity should combine to stabilize the height of the night-time F_2 region of the ionosphere. If the layer is raised the increased diffusion under gravity should bring it down again, if it is lowered the increased decay of the lower edge should tend to lift the height of the electron density maximum.

Martyn (1956) gave a quantitative basis to these ideas by showing that, under the action of diffusion, gravity, and attachment, a Chapman layer at a reduced height (Z_m) such that

$$\beta = g\,\frac{\sin^2\psi}{2H\nu} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

decays uniformly without change of shape or height.

Here H is the scale height,
g is the gravitational field strength,
ψ is the geomagnetic dip,
ν is the positive ion collisional frequency,
β is the attachment coefficient.

By a Chapman layer is meant a region in which the height distribution of electron density is described by the relation

$$N = N_m \exp \tfrac{1}{2}[1-(z-z_m)-\mathrm{e}^{-(z-z_m)}], \qquad \ldots\ldots\ldots\ldots \quad (2)$$

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where N is the electron density,
N_m is the maximum electron density,
z is the reduced height,
z_m is the reduced height of the electron density maximum.

For convenience, we shall take the height such that equation (1) is satisfied as the datum level, $z=0$, and the positive ion collisional frequency and attachment coefficient at this level will be denoted by ν_0 and β_0 respectively.

In this paper the conclusions reached by Martyn will be extended.

It will first be shown that, in the presence of diffusion, gravity, and attachment, a Chapman layer, no matter what its height, maintains its shape, decaying uniformly with an effective attachment coefficient (β') equal to the true attachment coefficient at the height of the electron density maximum (z_m); and that at the same time, the layer drifts bodily towards the equilibrium height ($z=0$) with a velocity

$$v=-\frac{2g\sin^2\psi}{\nu_0}\sinh z_m. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

It will then be shown that, if an ionospheric region, in equilibrium with gravity, diffusion, and attachment, is subjected to a vertical tidal drift W, its height is perturbed by an amount

$$\Delta z_m \doteq \text{arc sinh}\frac{W\nu_0}{2g\sin^2\psi}=\text{arc sinh}\frac{W}{4H\beta_0}. \quad \ldots\ldots\ldots (4)$$

II. Solution of the Equation of Continuity for a Night-time Chapman Layer in the Presence of Gravity, Diffusion, and Attachment

We shall assume the attachment coefficient to be proportional to the pressure, that is,

$$\beta=\beta_0 e^{-z}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The continuity equation then becomes

$$\frac{\partial N}{\partial t}=\frac{2g\sin^2\psi}{H\nu_0}e^z\left(\frac{\partial^2 N}{\partial z^2}+\frac{3}{2}\frac{\partial N}{\partial z}+\frac{N}{2}\right)-\beta_0 N e^{-z}; \quad \ldots\ldots (6)$$

Ferraro (1945); Martyn (1956).

Substituting equation (2) into this we get

$$\frac{\partial N}{N\partial t}=\frac{g\sin^2\psi}{2H\nu_0}e^z(e^{-2z}e^{2z_m}-e^{-z}e^{z_m})-\beta_0 e^{-z}. \quad \ldots\ldots\ldots\ldots (7)$$

Now we have chosen our datum level such that

$$g\sin^2\psi/2H\nu_0=\beta_0, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and (7) becomes therefore

$$\frac{\partial N}{N\partial t}=\beta_0(e^{-z}e^{2z_m}-e^{z_m}-e^{-z}). \quad \ldots\ldots\ldots\ldots (9)$$

This may be expanded to

$$\frac{\partial N}{N\partial t}=2\beta_0(e^{z_m}-e^{-z_m})\tfrac{1}{2}\{e^{-(z-z_m)}-1\}-\beta_0 e^{-z_m}, \quad \ldots\ldots \quad (10)$$

or from (8) and (2)

$$\frac{\partial N}{\partial t}=\frac{g\sin^2\psi}{H\nu_0}(e^{z_m}-e^{-z_m})\frac{\partial N}{\partial z}-\beta_0 e^{-z_m}N. \quad \ldots\ldots\ldots\ldots \quad (11)$$

This equation corresponds to the general form

$$\frac{\partial N}{\partial t}=-v\frac{\partial N}{H\partial z}-\beta' N, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where both v and β' are independent of the height z but depend only on the height of the maximum electron density of the layer z_m.

It is evident that the layer decays uniformly with an effective attachment coefficient

$$\beta'=\beta_0 e^{-z_m}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

and that the electron density maximum moves with a velocity

$$v=-\frac{g\sin^2\psi}{\nu_0}(e^{z_m}-e^{-z_m}) \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

towards the height $z_m=0$, i.e. from the way in which we have defined our datum level, towards the height such that $\beta=g\sin^2\psi/2H\nu$.

III. The Behaviour of an Arbitrary Electron Density Distribution under Gravity, Diffusion, and Attachment

Martyn has suggested that, under the influence of gravity, diffusion, and attachment, an ionospheric region with any initial height distribution of electron density, approaches the Chapman form centred at the equilibrium height $z=0$.

Proof of this is still lacking, but we can show that all those height distributions of electron density that can be built up as the sum of a number of Chapman layers of different peak electron densities, and centred at different heights, approach the simple Chapman form. As equation (6) is linear, each of the component layers in such a region can be considered independently of the others and each therefore satisfies equation (12). Each of the component Chapman layers then, approaches the equilibrium height $z=0$, so that the electron distribution as a whole must approach the Chapman form, centred at the height $z=0$.

IV. The Perturbation of a Chapman Region by a Vertical Drift

If a Chapman region in equilibrium with gravity, diffusion, and attachment is subjected to a vertical tidal drift W it will move with the drift until the restoring velocity as given by equation (14) is equal and opposite to the perturbing tidal drift. It follows from these considerations that the perturbation of the equilibrium height of the layer is

$$\Delta z_m = \text{arc sinh}\,\frac{W\nu_0}{2g\sin^2\psi} = \text{arc sinh}\,\frac{W}{4H\beta_0}, \quad \ldots\ldots \quad (15)$$

where ν_0 is still the positive ion collisional frequency and β_0 the attachment coefficient, at the equilibrium height of the electron density maximum of the unperturbed layer.

V. Conclusions

Martyn's suggestion that the observed adoption of an approximately parabolic form by the night F_2 region is due to the action of gravity, diffusion, and attachment has been put on a firmer basis, although a proof that *any* initial electron distribution will adopt the Chapman form is still lacking.

Equation (**15**) should make possible a more concise formulation of the author's lunar tidal theory (Duncan 1956).

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