THE MEASUREMENT OF THE DRIFT VELOCITY OF ELECTRONS THROUGH GASES BY THE ELECTRON SHUTTER METHOD

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Summary

The behaviour of an electron shutter apparatus such as that used by Neilsen and Bradbury for the measurement of electron velocities is analysed, taking into account the effect of electron diffusion. A relation between shutter frequency and drift velocity, and a relation giving the accuracy obtainable by the method in terms of the chamber length and gas pressure are derived. It is thus shown that neglect of the influence of diffusion can lead to large errors, but that these can be reduced to any desired degree by increasing the chamber length or the gas pressure. Numerical examples are given.

I. Introduction

An important quantity in the theory of electrical conduction in gases is the drift velocity of electrons through the gas, under the influence of an electric field. Nielsen and Bradbury (1936, 1937) have measured this velocity by using electron shutters. Figure 1 is a schematic diagram of an electron shutter apparatus. Electrons drift from the electrode A to the electrode B under the influence of a uniform electric field. Two shutters S_1 and S_2 are placed in the path of the electrons and these admit electrons only in periodic pulses, the two shutters operating in synchronism. From simple considerations electrode B would be expected to collect a maximum current when the shutter frequency is such that a bunch of electrons admitted by the first shutter (S_1) has just drifted to the second shutter (S_2) by the time the shutters are ready to open again.

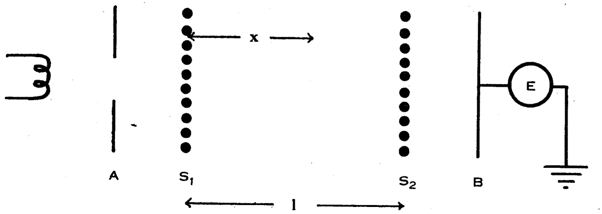

Fig. 1.—An electron shutter apparatus for the measurement of the drift velocity of electrons through a gas.

Thus, if f is the shutter frequency, W is the electron drift velocity, and l is the distance between the shutters, we should expect a maximum current at a frequency f_0 such that

$$W = lf_0, \qquad (1)$$

and other maxima at integral multiples of the frequency f_0.

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In this paper electron diffusion is considered and it is shown that equation (**1**) is not strictly accurate. A more precise relation between shutter frequency and drift velocity and an estimate of the accuracy realizable by the method are derived. It is thus shown that the effect of diffusion can be reduced to any desired degree by increasing the chamber length or the gas pressure.

II. An Analysis of the Behaviour of an Electron Shutter Apparatus

The shutter S_1 (Fig. 1) is assumed to admit sharp pulses of electrons at time intervals T or $1/f$ where f is the shutter frequency. These electrons will drift and diffuse down the chamber and an indication of the distribution of electron density n with distance down the chamber, x, at any instant is given in Figure 2. There will be a series of pulses, the centres of which will be a distance W/f apart and, if the first shutter is a point source admitter, the electron density distribution in each pulse will be given by the expression (Carslaw and Jaeger 1947)

$$n=\{N/(4\pi Kt)^{3/2}\}\exp\{-(x-x_0)^2/4Kt\}, \quad \ldots\ldots\ldots \quad (\mathbf{2})$$

where W is the electron drift velocity,
f the shutter frequency,
n the electron density,
N the number of electrons per pulse,
K the electron diffusion constant,
x the distance from the first shutter,
x_0 the position of the pulse centre,
t the age of the pulse.

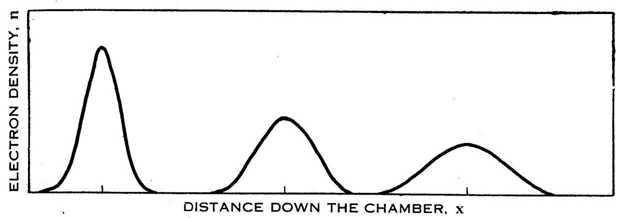

Fig. 2.—The theoretical variation of electron density at some particular instant, with distance down chamber.

Most practical shutters permit electrons to pass for a given fraction of the pulsing period so that the number of electrons in each pulse, N, is inversely proportional to the shutter frequency f,

We may write therefore $N=N_0/f$ and hence

$$n=\frac{N_0}{f(4\pi Kt)^{3/2}}\exp\left\{-\frac{(x-x_0)^2}{4Kt}\right\}. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{3})$$

If the second shutter operates in synchronism with the first, it will admit electrons when the age of the mth pulse in Figure 2 is m/f, and when the coordinate of the mth pulse centre is mW/f. While the shutter S_2 is transmitting, therefore, the electron density at the shutter will be

$$n=\sum_{m=1}^{\infty}\frac{N_0}{f\{4\pi Km/f\}^{3/2}}\exp\left\{-\frac{(l-mW/f)^2}{4Km/f}\right\},$$

and putting $W=lf_0$ we get

$$n=\sum_{m=1}^{\infty}\frac{N_0 f^{\frac{1}{2}}}{(4\pi mK)^{3/2}}\exp\left\{-\frac{l^2}{4K}\,\frac{(f-mf_0)^2}{mf}\right\}. \quad \ldots\ldots\ldots \quad (4)$$

If we treat the second shutter as a mere geometrical plane, the current across it will be

$$i=nW-K\partial n/\partial x.$$

From (3)

$$\frac{\partial n}{\partial x}=-\frac{N_0(x-x_0)}{2f(4\pi Kt)^{3/2}Kt}\exp\left\{-\frac{(x-x_0)^2}{4Kt}\right\},$$

and thus at the second shutter when it opens

$$\frac{\partial n}{\partial x}=\sum_{m=1}^{\infty}-\frac{N_0(l-mW/f)}{2f(4\pi Km/f)^{3/2}Km/f}\exp\left\{-\frac{(l-mW/f)^2}{4Km/f}\right\},$$

that is,

$$\frac{\partial n}{\partial x}=\sum_{m=1}^{\infty}-\frac{N_0 lf^{\frac{1}{2}}(f-mf_0)}{2(4\pi)^{3/2}(mK)^{5/2}}\exp\left\{-\frac{l^2}{4K}\,\frac{(f-mf_0)^2}{mf}\right\}. \quad \ldots\ldots \quad (5)$$

Hence

$$i=\sum_{m=1}^{\infty}\left[\frac{N_0 f^{\frac{1}{2}} lf_0}{(4\pi mK)^{3/2}}+\frac{N_0 lf^{\frac{1}{2}}(f-mf_0)K}{2(4\pi)^{3/2}(mK)^{5/2}}\right]\exp\left\{-\frac{l^2}{4K}\,\frac{(f-mf_0)^2}{mf}\right\}, \quad \ldots \quad (6)$$

or

$$i=\frac{N_0 lf^{\frac{1}{2}}}{(4\pi K)^{3/2}}\sum_{m=1}^{\infty}\frac{f+mf_0}{2m^{5/2}}\exp\left\{-\frac{l^2}{4K}\,\frac{(f-mf_0)^2}{mf}\right\}. \quad \ldots\ldots \quad (7)$$

This relation gives the instantaneous current through the second shutter during the time it is open. If this shutter is open for a given fraction of the pulsing period, the average current collected by the electrode B will be proportional to the instantaneous current and we do not need to involve the shutter frequency further.

Now, we are not interested in the absolute magnitude of the current but in its variation with shutter frequency so that it is convenient to multiply equation (7) by the constant factor $(4\pi K)^{3/2}/N_0 lf_0^{3/2}$ and thus reduce it to the form

$$i=\sum_{m=1}^{\infty}\left(\frac{f+mf_0}{2m^2 f_0}\right)\left(\frac{f}{mf_0}\right)^{\frac{1}{2}}\exp\left\{-\frac{l^2}{4K}\,\frac{(f-mf_0)^2}{mf}\right\}. \quad \ldots\ldots \quad (8)$$

This, then, is the relation which determines the variation of current with shutter frequency in an electron shutter apparatus.

Equation (8) has been used to plot curves for the following cases :

Hydrogen $Z=5$, $p=5$, (Figs. 3 and 4)
Hydrogen $Z=1$, $p=1$, (Fig. 5)

where Z is the electric field in volts per centimetre and p is the pressure in millimetres of mercury. We have assumed l the distance between the shutters to be 5 cm in each case and we have adopted the coefficients of diffusion and

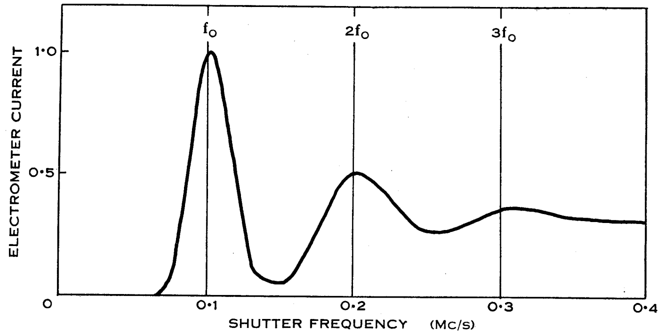

Fig. 3.—The theoretical variation of electrometer current with shutter frequency, for hydrogen at a pressure of 5 mm in a 5 cm chamber and under a field of 5 V/cm.

drift velocities given by Crompton and Sutton (1952) and Nielsen and Bradbury (1936, 1937). It is apparent from these curves that the current maxima are displaced to the right of the positions mf_0.

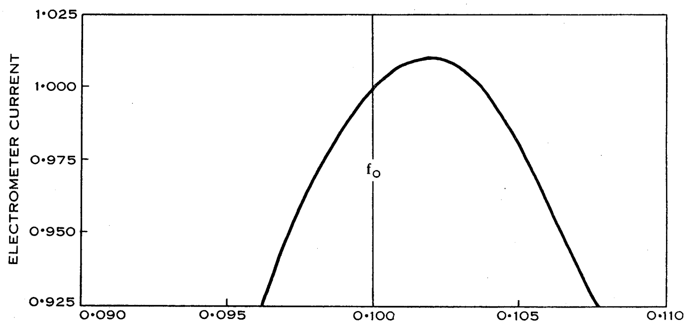

Fig. 4.—The theoretical variation of electrometer current with shutter frequency, for hydrogen at a pressure of 5 mm in a 5 cm chamber and under a field of 5 V/cm.

We can obtain an approximate expression for the difference between the shutter frequency giving maximum current (f_m) and the frequency f_0, if this difference is small. In equation (8) put $2K/l^2=\alpha$ and $(f-f_0)/f_0=\beta$. If α is small we need only consider the first term in the series ; that is, we may assume that at the shutter frequency at which the first current maximum is obtained there is no overlapping of pulses at the second shutter. This is by no means generally true, but, unless it is true, the difference between the frequency at which maximum current is obtained and the frequency f_0 will not be small.

From (**8**) then

$$i = \frac{f+f_0}{2f_0}\left(\frac{f}{f_0}\right)^{\frac{1}{2}} \exp\left\{-\frac{(f-f_0)^2}{2\alpha f}\right\}$$

$$\simeq (1+\tfrac{1}{2}\beta)(1+\tfrac{1}{2}\beta)\exp(-f_0\beta^2/2\alpha),$$

hence

$$\partial i/\partial\beta \simeq [1-(\beta f_0/\alpha)(1+\beta)]\exp(-f_0\beta^2/2\alpha).$$

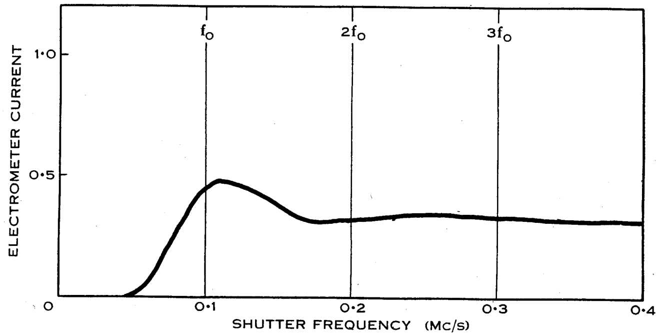

Fig. 5.—The theoretical variation of electrometer current with shutter frequency, for hydrogen at a pressure of 1 mm in a 5 cm chamber and under a field of 1 V/cm.

At the current maximum $\partial i/\partial\beta$ is zero so that

$$1-\beta f_0/\alpha \simeq 0,$$

that is,

$$\beta \simeq \alpha/f_0,$$

that is,

$$f_m - f_0 \simeq \alpha \simeq 2K/l^2, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where f_m is the frequency at which the first current maximum is obtained. Thus a more accurate relation between shutter frequency for maximum current and electron drift velocity is

$$W = l(f_m - 2K/l^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Even this relation is only useful if $2K/l^2$ is small, and the means of making it small for a given value of drift velocity W is to use a long chamber, high gas pressure (p), and large electric fields (Z) (W is a function of Z/p). A comparison of Figures 3 and 5 illustrates this point.

In our analysis we have ignored the fact that the collection of electrons by the electrode B will perturb the electron density. The error introduced by this simplification is probably not serious, but, if anything, the lowered electron density near the collecting electrode must accelerate diffusion down the chamber and the difference between the shutter frequency for maximum current (f_m) and the frequency f_0 must be greater than we have indicated.

The finite width of the current peaks will limit the accuracy which may be obtained in the measurement of drift velocity by the shutter method under given experimental conditions. This problem may be investigated.

From (8)

$$i \simeq \frac{f+f_0}{2f_0}\left(\frac{f}{f_0}\right)^{\frac{1}{2}} \exp\left\{-\frac{(f-f_0)^2}{2\alpha f}\right\}.$$

The exponential factor in this expression has the greatest influence on the shape of the pulse. Considering this alone we have

$$i \simeq \exp\{-(f-f_0)^2/2\alpha f\} \simeq \exp\{-(f-f_0)^2/2\alpha f_0\}.$$

This is the standard form for the normal distribution. Maximum current occurs in our simplified expression at the shutter frequency f_0, and the current will have fallen by 1 per cent. of its maximum value when

$$f-f_0 = \pm 0\cdot 14\sqrt{(\alpha f_0)}.$$

If a 1 per cent. drop in current can just be detected, therefore, the possible relative error in the measurement of the frequency for maximum current, and hence in the measurement of the drift velocity W, will be

$$(f-f_0)/f_0 = \pm 0\cdot 14\sqrt{(\alpha/f_0)} = \pm 0\cdot 14\sqrt{(2K/lW)}. \quad \ldots\ldots \quad \mathbf{(11)}$$

For hydrogen at a pressure of 1 mm Hg, in a 5 cm chamber and under a field of 1 V/cm (the case shown in Fig. 5), this is equal to 4 per cent. It will be noted, however, that diffusion becomes much less important when higher gas pressures and chamber voltages are used, and this fact is of importance in the design of experiments of this nature.

III. Acknowledgments

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IV. References

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