THE CLOCK PARADOX AND THE GENERAL THEORY OF RELATIVITY*

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The clock paradox (Dingle 1956; McCrea 1956) which arises when the Special Theory of Relativity is applied to the problem of two identical clocks having different histories in the space time diagram is resolved by the General Theory of Relativity (G.T.R.) (Tolman 1934).

This suggests that an experiment equivalent to the clock paradox problem would constitute an experimental, as distinct from observational, test of the G.T.R. and its rival theories.

Such a test appears to be quite feasible using charged unstable particles (e.g. π-mesons and emulsions as detectors of π^- stars or π–μ decay) describing a circular path of radius R in a magnetic field of induction B.

If the numbers of π-mesons are N_0 and N_x at the same point (in the laboratory frame) but separated by one revolution, a path length of $x=2\pi R$ in the laboratory frame, we have

$$N_x=N_0 \exp\left\{-\frac{x}{\beta\gamma\tau c}\right\}, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

provided the time dilatation applies throughout their motion (i.e. if it depends on the speed only), with βc=speed=constant, $\gamma=(1-\beta^2)^{-\frac{1}{2}}$, and τ=proper mean life of the π-meson.

If the time dilatation effect should be cancelled during the accelerated motion (Hill 1947) (i.e. two identical clocks once synchronized remain so irrespective of their histories) we have

$$N_x'=N_0 \exp\left\{-\frac{x}{\beta\tau c}\right\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

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The differences between equations (**1**) and (**2**) are appreciable at available energies, magnetic fields, and radii of orbits as Table 1 shows where $m_{\pi}c^2 = 140$ MeV and $\tau = 2 \cdot 6 \times 10^{-8}$ sec (Thorndike 1952).

The above experiment is actually equivalent to the clock paradox problem because an inertial frame M exists such that mesons are at rest at a point P_0 for time $t < t_0$ (the number of mesons at time $t = t_0$ is N_0), then are accelerated and retarded and come to rest at a point P_1 at time t_1 (N_x mesons remaining) as measured by a local clock which is synchronized with the clock at P_0 in the usual way (Einstein 1905).

Table 1

RATIO OF N_x TO N_x' AT AVAILABLE ENERGIES, ORBITS, AND MAGNETIC FIELDS

Kinetic Energy of Mesons (MeV)	Radius of Orbit (m)	Magnetic Induction (Wb m^{-2})	N_x/N_x'
20	1	0·26	1·23
140	1	0·81	1·59
560	2	1·14	3·72

An alternative method is to determine the number of π-meson decays per unit length of path, $\mathrm{d}N/\mathrm{d}x$. Equation (**1**) gives

$$\frac{\mathrm{d}N}{\mathrm{d}x}/N_x = \frac{1}{\beta\gamma\tau c}, \qquad \text{(1a)}$$

while equation (**2**) gives

$$\frac{\mathrm{d}N'}{\mathrm{d}x}/N_x' = \frac{1}{\beta\tau c}. \qquad \text{(2a)}$$

The symmetry of a circular orbit implies that $(\mathrm{d}N/\mathrm{d}x)/N_x$ should be independent of the particular arc and one can therefore use an arc instead of a complete circle and correspondingly higher energy. For instance, for 1260 MeV mesons, $\gamma = 10$ and the results should decide between (**1a**) and (**2a**) even if it might be difficult to distinguish π–μ decay from π scattering.

Although a straight line is obtained if one lets the radius R of the orbit tend to infinity, equation (**2a**) does not go over into equation (**1a**). This in itself implies a discontinuity in the decay rates and suggests that equation (**2a**) and therefore (**2**) cannot hold for circular motion with constant speed if equations (**1**) and (**1a**) hold for uniform velocity motion. But equations (**1**) and (**1a**) are consequences of the Special Theory of Relativity and have even been verified experimentally for μ-mesons describing slightly curved paths in the Earth's magnetic field (Janossy 1948). We therefore conclude that two identical clocks synchronized initially do not necessarily remain synchronized; whether their readings are given by equation (**1**) only experiment can decide.

It should be noted that this time dilatation effect corresponds to the " red shift " effect of the G.T.R., but, whereas the " red shift " has been explained in terms of the energy loss of photons in the Sun's gravitational field (McCrea 1954; Papapetrou 1956), the above effect cannot be explained in these terms.

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ANGULAR DISTRIBUTION OF α-PARTICLES FROM ^{7}Li(d,α)^{5}He†

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Introduction

The deuteron bombardment of ^{7}Li can result in the production of two α-particles and a neutron by one of the following processes:

$$
\begin{aligned}
{}^7\mathrm{Li}+{}^2\mathrm{H}\rightarrow{}^9\mathrm{Be}^*&\rightarrow{}^4\mathrm{He}+{}^5\mathrm{He},\quad {}^5\mathrm{He}\rightarrow n+{}^4\mathrm{He} && (a)\\
&\rightarrow n+{}^8\mathrm{Be},\quad {}^8\mathrm{Be}\rightarrow{}^4\mathrm{He}+{}^4\mathrm{He} && (b)\\
&\rightarrow n+{}^4\mathrm{He}+{}^4\mathrm{He} && (c).
\end{aligned}
$$

It is possible to distinguish between (a) and (b) by means of groups present in the spectrum of particle energies. A mono-energetic α-particle group has been observed (Ajzenberg and Lauritsen 1955), which can only be due to a two-body break up of ^{9}Be leaving ^{5}He in the ground state. Two neutron groups have been observed (Ajzenberg and Lauritsen 1955), corresponding to the formation of ^{8}Be in the ground and first excited states. No substantial evidence exists for reaction (c).

The angular distribution of the mono-energetic group in the α-particle energy spectrum with respect to the deuteron beam has been measured previously (Treacy 1951) at a bombarding energy of 900 keV and found to be isotropic to within an experimental error of 10 per cent. Using this information, the calculations in a recent study (Riviere 1956) of reaction (a) at the same

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