isotropic distribution. The points obtained at the largest angles are seen to lie well below the line. These two spectra were recorded for an increased gain in the counter I amplifier to spread the counts over a larger number of channels, and this may have introduced an error.

We conclude that the angular distribution of the α-particles contributing to the formation of the ground state of ^{5}He in reaction (a) is isotropic at a deuteron energy of 900 keV to within an experimental error of 2 per cent. There is no evidence for a "knock-on" reaction to within this accuracy. The most simple explanation of the result is that primarily s-wave deuterons are responsible for the reaction. The necessary spin assignments under this assumption are discussed by Riviere (1956).

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SOLUTION OF FLOW PROBLEMS IN UNIDIMENSIONAL LAGRANGIAN HYDROMAGNETICS*

By R. E. Loughhead†

Introduction

A problem of considerable interest in many branches of astrophysics is that of the subsequent behaviour of a current which at an initial time $t=0$ is largely concentrated within a given region of an ionized gas of infinite extent. In particular, it has been suggested by Alfvén (1950) that a high current discharge in an ionized gas is likely to constrict because of the electromagnetic attraction between parallel currents and that this constriction effect may be involved in the formation of solar prominences. Similar considerations may also be of importance in studies of magnetic fields in the spiral arms of the Galaxy.

The solution of an initial value problem of this type is greatly complicated by the non-linear character of the hydromagnetic equations governing the motion of the ionized gas. However, in the simple case of the unidimensional motion of an ionized gas in which the magnetic field is everywhere at right angles to the direction of motion of the fluid, a numerical solution can be carried through using a finite difference scheme along the lines proposed by the author

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in a previous paper (Loughhead 1955). The complexity of the numerical computations can be reduced if, instead of using the hydromagnetic equations in their usual Eulerian form, the problem is formulated in terms of the Lagrangian equations of motion. The process of numerical solution then comes well within the capacity of a desk machine.

Lagrangian Equations for Unidimensional Flow

It is convenient to introduce Cartesian axes Ox, y, z such that the magnetic field in the fluid is everywhere directed in the y-direction and the motion of the fluid occurs parallel to the x-axis and depends on the single spatial coordinate x. The analysis is especially facilitated by the use of the Lagrangian representation in which a number h is attached to each plane section of particles normal to the x-axis, so that the changing position of each section is given by a function $x(h, t)$. If $\rho(x, t)$ denotes the density of the fluid at the point x and time t, then h may be conveniently defined by the relation

$$h=\int_{x(0,t)}^{x(h,t)} \rho \mathrm{d}x. \qquad (1)$$

According to this definition h is equal in magnitude to the mass per unit cross section area between the plane $x(h, t)$ and a " zero " plane $x(0, t)$. Another important result

$$\rho(h, t)\frac{\partial x(h, t)}{\partial h}=1 \qquad (2)$$

can be obtained by differentiating (1) with respect to h.

Then the Lagrangian equations governing the motion of the fluid may be stated in the form (cf. Loughhead 1955):

$$(H\tau)_t=0, \qquad (3)$$

$$(p\tau^{\gamma})_t=0, \qquad (4)$$

$$v_t+\left(\frac{H^2}{8\pi}+p\right)_h=0, \qquad (5)$$

$$v_h=\tau_t, \qquad (6)$$

where H is the component of the magnetic field in the y-direction, $\tau=1/\rho$ is the specific volume, v is the velocity of the fluid in the x-direction, p is the scalar pressure, γ is the ratio of the specific heats of the gas, and the subscripts t and h denote partial differentiation with regard to t and h respectively. The units employed are Gaussian. In this form the equations are identical with those given by Kaplan and Stanyukovich (1956) in a recent paper.

Equations (3) and (4) lead to the two integrals of motion

$$H\tau=a(h), \qquad (7)$$

$$p\tau^{\gamma}=b(h), \qquad (8)$$

where a and b are functions of h specified in the initial conditions of the problem. Using (7) and (8), equations (5) and (6) may then be written in the forms

$$v_t = k^2\tau_h - \psi(\tau, h) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$v_h = \tau_t, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where

$$\psi(\tau, h) = \left(\frac{a \cdot a_h}{4\pi\tau^2} + \frac{b_h}{\tau^\gamma}\right), \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

and

$$\tau k = \sqrt{\left(\frac{H^2}{4\pi\rho} + \frac{\gamma p}{\rho}\right)} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

is the local value of the hydromagnetic wave velocity V.

When $H=0$, the quantity τk becomes simply the sound velocity $q=\sqrt{(\gamma p/\rho)}$ and the equations (9) and (10) reduce to the well-known gas dynamical equations for an unionized fluid. Also, if the state of the fluid is uniform at time $t=0$, then $\psi(\tau, h)=0$, and τk is a function of τ alone. In this case the hydromagnetic equations for an ionized fluid are identical with the corresponding gas dynamical equations except that τk represents the hydromagnetic wave velocity and not the velocity of sound. Under these circumstances the hydromagnetic equations may be solved by methods identical to those used to treat the corresponding equations of gas dynamics. This fact has been previously pointed out by Kaplan and Stanyukovich (1954).

For other problems where the initial state of the fluid at time $t=0$ is not uniform, τk is in general an explicit function of h as well as of the specific volume τ, and the whole process of solution is greatly complicated. Kaplan and Stanyukovich (1956) have attempted to find particular solutions for the case of non-uniform initial conditions. Their method is based directly on equations (5) and (6), and consists essentially in replacing the total pressure $P=H^2/8\pi+p$ by a postulated analytical expression, which then makes the equations mathematically tractable.

While the particular solutions obtained by Kaplan and Stanyukovich are useful in providing some insight into the nature of the hydromagnetic flow, it is important to point out that a full numerical solution can be obtained for any given initial value problem by a finite difference method based on the characteristic forms of equations (9) and (10). The advantage of using the Lagrangian representation is that, instead of having to solve four partial differential equations simultaneously as in the Eulerian scheme (cf. Loughhead 1955), one has now to deal only with two, and the amount of numerical computation is much reduced. This reduction of effort is due to the existence in the Lagrangian representation of the two integrals of motion (7) and (8).

To obtain the Lagrangian equations in characteristic form one merely adds and subtracts k times equation (10) to and from equation (9), yielding the relations

$$v_t + kv_h = k(\tau_t + k\tau_h) - \psi(\tau, h), \quad \ldots\ldots\ldots\ldots \quad (13)$$

$$v_t - kv_h = -k(\tau_t - k\tau_h) - \psi(\tau, h). \quad \ldots\ldots\ldots\ldots \quad (14)$$

In each of the equations (**13**) and (**14**) the variables v and τ are both differentiated in the same direction along a common curve in the (h-t) plane, whose slope is given by

$$\mathrm{d}h/\mathrm{d}t = \pm k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{15})$$

respectively. When $H=0$, the equations (**13**) and (**14**) become the normal Lagrangian equations describing the unidimensional adiabatic motion of an unionized fluid.

Difference Equations

To proceed with the numerical computation in any particular initial value problem consider a rectangular mesh in the (h-t) plane whose (l, m)th point is defined by

$$h_l = l\,.\,\Delta h, \quad t_m = m\,.\,\Delta t, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{16})$$

where

$$r = \Delta t/\Delta h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{17})$$

is the mesh ratio. The mesh ratio must be chosen to satisfy the stability condition

$$r\,.\left|\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{\text{max.}} < 1 \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{18})$$

throughout the region of integration.

The technique is now to replace derivatives along characteristics by finite differences, the choice of difference quotients (forward or backward) being made so as to preserve the domain of dependence. If $v_{l,m}$ and $\tau_{l,m}$ denote the values of the solutions of the difference equations at the net point (h_l, t_m), the characteristic equations (**13**) and (**14**) are replaced by the difference equations

$$v_{l,m+1} = v_{l,m} - rk_{l,m}[v_{l,m} - \tfrac{1}{2}(v_{l-1,m} + v_{l+1,m})] + \tfrac{1}{2}rk^2_{l,m}(\tau_{l+1,m} - \tau_{l-1,m})$$
$$-\Delta t\,.\,\psi_{l,m}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{19})$$

$$\tau_{l,m+1} = \tau_{l,m} - rk_{l,m}[\tau_{l,m} - \tfrac{1}{2}(\tau_{l-1,m} + \tau_{l+1,m})] + \tfrac{1}{2}r(v_{l+1,m} - v_{l-1,m}).$$
$$\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{20})$$

Equations (**19**) and (**20**) determine the values of the variables v and τ along the line t_{m+1}=constant in terms of the values along the preceding line t_m=constant, and hence enable the solution to be stepped off given the initial values of the variables at time $t=0$.

The Lagrangian method may also be adopted for axially and spherically symmetric flows, but, in these cases, it does not afford the considerable reduction of numerical computation outlined in the analysis above.

Note added in Proof (*February* 6, 1957).—Trial computations performed by Miss J. Ward have confirmed the utility of the numerical method described above.

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