# METEOR RADIANT DETERMINATION FROM HIGH ECHO-RATE OBSERVATIONS 

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Summary
A simplified analysis is given of the Clegg method for delineating meteor radiants from radar observations. A further analysis reveals a new and faster method of interpreting the data contained in meteor echo records. This method is applicable when sensitive equipment is employed and the resulting echo rate is very high.

## I. Introduction

When an appreciable increase is made in the sensitivity of radar apparatus used for the detection of meteors a large increase in the total number of echoes is recorded. If meteors fainter than magnitude +6 are detectable this increase in numbers makes record analysis extremely difficult and much too prolonged. It has recently been shown that sporadic meteors appearing at low sensitivity are resolved into radiant type groupings at higher sensitivity (Ellyett and Keay 1956), making it essential to be able to look for radiants in high rate activity.

This problem became acute during a survey of meteoric activity that was made early in 1956, at Christchurch, New Zealand, using equipment with a limiting magnitude exceeding $+7 \cdot 5$. The fundamental method for determining the radiant coordinates of meteor streams (Right Ascension $\alpha$, declination $\delta$ ) was that due to Clegg ( $1948 a, 1948 b$ ) which possesses the great advantage of requiring only a single observing station. However, Clegg's method of analysis and his technique of using range-time envelopes in the reduction of the meteor plots became unsuitable when the meteor rates were high. Whenever the average rate of detection exceeded about 50 echoes per hour from a single aerial array the amount of data in the daily film records was too great to handle by Clegg's analytical techniques. With such high average rates it is still practicable to draw diurnal rate curves for successive days, from which a reasonable amount of information on shower activity can be found, but the resolution of separate showers is very poor, as may be expected.

In this paper a new approach is presented, whereby the fundamental Clegg method may be made very much easier to analyse. A simple extension of the analysis shows that it is possible to make rate curves for a narrow range band (which will be called partial rate curves) yield complete radiant coordinates. As a result it becomes practicable to reduce records containing even as many as 500 echoes per hour.

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## II. Assessment of Clegg's Method

Clegg's method for determining meteor radiants by radio echo techniques uses the directive properties of an aerial array to discriminate between meteors which arrive from different radiant points. This is made possible by the fact that most meteor trails are markedly aspect sensitive at the radio frequencies used ( $72 \mathrm{Mc} / \mathrm{s}$ for Clegg and $69 \mathrm{Mc} / \mathrm{s}$ for the New Zealand equipment) ; and that meteoric ionization occurs in a limited range of heights centred on 95 km . By alternating the aerial azimuth from day to day Clegg obtained sufficient information to be able to delineate a meteor radiant with reasonable accuracy.

Clegg's method of analysing the film records is essentially graphical and relies on a range extrapolation from the maximum observed range to the theoretical horizon range of 1100 km . By making the theoretical curves fit the echo distribution at lower ranges a value can be obtained for the time of reception of echoes corresponding to the horizon range. Unless this extrapolation is madea time correction has to be applied. This correction is difficult to calculate exactly and may lead to considerable error.

In order to delineate a radiant from a single day's records Aspinall, Clegg, and Hawkins (1951) replaced Clegg's single rotatable array by two fixed arrays equally spaced in azimuth about the east-west direction. With neither beam directed at an azimuth of $90^{\circ}$ from north the time of reception of echoes from the horizon range no longer corresponds closely to the time of transit of the radiant. This necessitates a more complicated approach in the theoretical treatment, of which Aspinall, Clegg, and Hawkins only quote the results (in graphical form). A graph was given showing how much the observed time of reception of echoes should alter for two selected values of range ( $800,900 \mathrm{~km}$ ), other than the horizon range, when using their two-aerial system. In order to obtain this graph Aspinall, Clegg, and Hawkins regarded each aerial beam as having a finite spread only in the vertical plane (Weiss, personal communication 1952 to Dr. C. D. Ellyett). This simplification enabled transit times to be calculated directly, provided that the range exceeded 700 km . Below this range the correction involved became too complicated and the original method of analysis developed by Clegg, using sensitivity contours, had to be reverted to.

The accuracy of Clegg's method is considerably reduced when a meteor shower does not stand out plainly against the background activity. If the background rate is fairly high it becomes very difficult to fit the theoretical range-time envelopes and to extrapolate to maximum range. This results in determinations of Right Ascension which tend to be several degrees too low if the aerials are directed to westerly azimuths (and vice versa for easterly directed arrays). The radiant coordinates previously published by Ellyett and Roth (1955) are for this reason from 0 to $5^{\circ}$ too low in Right Ascension, depending on the radiant elevation at transit. The values for the declinations are almost unaffected.

Clearly, a method is required which avoids the need for such corrections and overcomes the difficulty of application to high rate data.

## III. Pencil-beam Approach to Radiant Determination

In this section it will be shown that if a narrow but finite aerial beam is regarded as being an infinitely narrow pencil beam (subject to certain conditions) then the whole of the theoretical treatment of the Clegg method is greatly simplified, and almost all sources of error are avoided. In practice this is made possible by adopting a new way of using the data contained in experimental range-time plots, such that a reliable reference point for defining transit times can be found.

Let us first examine more closely Clegg's original method of constructing range-time plots which is partly illustrated in Figure 1 (this figure is similar to Figures 3 and 4 of Clegg's (1948a) paper). The contour lines in Figure 1 (a) represent the relative sensitivity of the system, plotted on the surface of a hypothetical dome that lies 95 km above the surface of the Earth. For the


Fig. 1.-Construction of theoretical range-time plots. $P Q$ represents the locus of the most probable time to receive echoes from any given range. $X Y$ represents the locus of the most probable range from which to receive echoes at any given time.
purpose of this discussion consider all meteors belonging to a given shower whose magnitudes are such that they can be detected at some point within the outermost contour. The lines $S_{0} T_{0} H_{0}$ etc., are the loci of points on the 95 km dome where a meteor from the radiant $R$ would satisfy the conditions for detection at the observing station $O$. Positions of $S T H$ at successive times are shown in Figure 1 (a), and from their points of intersection with the outermost sensitivity contour the theoretical range-time envelopes of Figure 1 (b) can be drawn. The dotted line $X Y$, which appears in all of Clegg's original envelopes, represents the most probable range from which echoes are received at any given time : it is simply the locus of the point along the line $T H$ where the system sensitivity is highest. The line $X Y$ crosses the azimuth direction $O E$ at the point where the sensitivity is the maximum possible, i.e. at the range from which the maximum number of echoes is received. This defines a unique reference point $A_{m}$ on the range-time plot from which the time of transit could be calculated, and will be referred to again later. Although Clegg included the line $X Y$ in his theoretical range-time envelopes he did not appear to use the property just described.

However, a re-examination of Figure 1 reveals that it is possible to define a reference point at any given range on the range-time plot provided the locus
$P Q$ is drawn instead of $X Y$. In Figure 1 (b) $P Q$ represents the locus of the most probable time to receive echoes from any given range. In Figure 1 (a) $P Q$ is coincident with the azimuth direction $O E$; this is so because at any given range the aerial sensitivity is highest at the central azimuth of the beam. The locus $P Q$ may be drawn on the actual meteor plots to quite a high degree of accuracy, and the point of intersection with a range-marker line $r$ (known accurately) yields a convenient reference point $A$ from which fairly precise transit times $t_{0}+\tau$ may be found.

The point $A$ in Figure 1 corresponds to the collecting area of a hypothetical pencil-beam aerial, directed to the azimuth $O E$, and of such elevation as to intersect the echo region ( 95 km dome) at the range $r$. This correspondence greatly simplifies the mathematical treatment, and, with one stipulation, enables a relation to be computed which gives the time difference $\tau$ between the reception of echoes from $A$ and the true time of local transit, as a function of the radiant declination $\delta$. The accuracy of the $\delta, \tau$ relationship is subject to the mean height of detection being known for the meteors concerned, but fortunately this does not critically affect the result. Unless positive evidence to the contrary is forthcoming it seems best to regard 95 km as being the mean height for maximum ionization and therefore maximum detection probability. This is in accordance with recent data by Elford and Robertson (1953) and with the data from Jodrell Bank (Kaiser 1953). However, a height correction, to be applied if needed, will be mentioned later.

The mathematical treatment is now straightforward. The elevation angle of the hypothetical pencil-beam aerial may be found from the relation

$$
\begin{equation*}
\sin \theta=\left(h^{2}+2 \rho h-r^{2}\right) / 2 \rho r \tag{1}
\end{equation*}
$$

where $r=$ slant range,
$h=$ height of echo region ( 95 km ),
$\rho=$ radius of the Earth (mean 6370 km ), $\theta=$ elevation angle.
An aerial plane may now be defined as the plane drawn through the observing station $O$ perpendicular to the aerial beam. When an echo is received the meteor trail must, due to the property of specular reflection, lie in a plane parallel to the aerial plane and, because all meteors from a given radiant travel in effectively parallel paths, the radiant (at infinity) can be considered to lie in some direction within the aerial plane. Now the aerial plane will, during the course of a sidereal day, sweep across most of the celestial hemisphere above $O$, and any radiant $R$ lying within this coverage will be detectable. This is illustrated in Figure 2, with the usual assumption that the Earth is fixed in space and that the celestial sphere, containing $R$, rotates about $O P$ with constant angular velocity. The aerial beam is shown directed to an azimuth $90^{\circ}+a$, and the intersection of the aerial plane with the celestial hemisphere is the great semicircle $J C B L$. The path of a radiant is a small circle about $P$ and when $R$ and $C$ coincide echoes can be received at $O$ from meteors originating from $R$.

Thus far only one aerial has been considered. With a second aerial directed to an azimuth of $90^{\circ}-a$ the same arguments apply, except, of course, that a different aerial plane is involved.

The methods of spherical trigonometry may now be used to find an expression relating the declination $\delta$ of a radiant to the hour angle $\tau$ between the passage of a radiant through the aerial plane and the time of its upper culmination


Fig. 2.-Diagram showing radiant coverage of pencil-beam aerial.


Fig. 3.-Horizontal projection of celestial hemisphere showing aerial planes.
(local transit). To simplify the analysis it is best to project Figure 2 onto the horizon plane through $O$, as shown in Figure 3. Both aerial planes are now included, and are shown symmetrically disposed about the east-west direction.

Considering the aerial directed south of east (azimuth $90^{\circ}+a_{s}$ ) the spherical triangle $Z B_{s} K_{s}$ yields

$$
\begin{align*}
\tan \zeta_{0} & =\tan \theta \operatorname{cosec} a_{s},  \tag{2}\\
\cos \chi & =\cos \theta \cos a_{s}, \tag{3}
\end{align*}
$$

where $Z B_{s}=\theta=$ aerial elevation,
$a_{s}=$ aerial azimuth spacing from $90^{\circ} \mathrm{E}$.,
$Z K_{s}=\zeta_{0}\left(=Z K_{n}\right.$ by symmetry if $\left.a_{s}=a_{n}\right)$.
The values of $\zeta_{0}$ and $\chi$ enable the spherical triangle $P K_{s} C_{s}$ to be solved, but before the hour angle $\tau_{s}$ can be found the parallax angle $\psi_{s}$ must be evaluated over the range of declination required. For the triangle $P K_{s} C_{s}$

$$
\begin{equation*}
\sin \psi_{s}=\left\{\cos \left(\varphi+\zeta_{0}\right) \sin \chi\right\} / \cos \delta \tag{4}
\end{equation*}
$$

where $\left(90^{\circ}-\varphi\right)=P Z=$ co-latitude of $O$.
Using this relation a table for $\psi_{s}$ may be computed. It is then possible to find the values of $\tau_{s}$ by using the relation

$$
\begin{equation*}
\tan \frac{1}{2} \tau_{s}=\tan \frac{1}{2}\left(\chi+\psi_{s}\right)\left\{\sin \frac{1}{2}\left(\delta-\zeta_{s}\right) / \cos \frac{1}{2}\left(\delta+\zeta_{s}\right)\right\} \tag{5}
\end{equation*}
$$

where $\zeta_{s}=\varphi+\zeta_{0}$.
The same procedure is followed when computing a table for $\tau_{n}$, except that $\zeta_{n}=\varphi-\zeta_{0}$.


Fig. 4.-Relation between the hour angles $\tau_{n}$ and $\tau_{s}$ and the declination $\delta$; assuming

$$
a_{n}=a_{s}=22 \frac{1}{2}^{\circ} \text { from due east, } \varphi=43 \frac{1}{2}{ }^{\circ} \mathrm{S} \text {. }
$$

Typical values of $\tau_{s}$ and $\tau_{n}$ for various slant ranges $r$ are shown in Figure 4, assuming an azimuth separation $\pm a$ of $22 \frac{1}{2}^{\circ}$ from $90^{\circ}$ (or $270^{\circ}$ ) for aerials situated at a latitude $(\varphi)$ of $43 \frac{1}{2}{ }^{\circ} \mathrm{S}$. Figure 5 represents $\tau_{n}-\tau_{s}$ plotted against $\delta$, and is used, in practice, to obtain $\delta$ when $\tau_{n}-\tau_{s}$ is known. The value of $\delta$ so obtained is used in turn to determine the true transit time from Figure 4, thus completely delineating the position of the radiant.

## IV. The Height Correction

The values for the hour angles $\tau$ that have been obtained in the previous section are exact, except for a single small correction which could in some cases be necessary. This correction only arises because the mean height of detection of meteor trails cannot be determined as precisely as the other parameters. Furthermore this height varies for different meteor velocities and magnitudes.


Fig. 5.-Relation between the time difference of the hour angles, $\tau_{n}-\tau_{s}$, and the declination $\delta$.

By keeping $r$ constant and differentiating equation (1) we obtain

$$
\begin{equation*}
\frac{\partial \theta}{\partial h}=\frac{\rho+h}{\rho r \cos \theta} \approx \frac{1}{r \cos \theta}, \tag{6}
\end{equation*}
$$

since $\rho \gg h$. This shows that a change of height causes least error when $r$ is large. In practice, however, too few echoes are found at the larger ranges, so a compromise is reached, and $r$ is made no smaller than is necessary to ensure adequate numbers of echoes.

In order to obtain a table of corrections it is best to recalculate some values of $\tau$ corresponding to the changed values of $\theta$. Further differentiation of equations (2) to (5) is too cumbersome. Tables 1 and 2 have been derived for the case represented by the solid line in Figures 4 and 5 (i.e. for $r=500 \mathrm{~km}, h=95 \mathrm{~km}$ assumed).

## V. Extension of Pencil-beam Method for High Rate Data

The main importance of the method described in Section III lies in the fact that it is readily extendible to cope with records containing large meteor echo rates, whereas the Clegg analysis becomes impracticable to apply. By drawing only partial rate curves (for a range band $50-100 \mathrm{~km}$ wide) it is possible to use the peaks in the curve to define transit times. This is illustrated in Figure 6. By plotting the range distribution of received echoes (a function of the vertical polar diagram of the aerial system) the range $r_{m}$ of maximum occurrence of echoes may be found. The peak in the echo distribution curve
will be fairly symmetrical for a reasonable distance either side of this range. Provided the line $P Q$ on the range-time plot does not bend too sharply in this region, a sampling range band may be defined with the property that the peak of the resulting partial rate curve will coincide closely in time with the transit at the point $A_{m}$ (Fig. 6). The width chosen for the sampling range will depend

Table 1
MINUTES TO BE ADDED TO $\tau_{n}$

| Height $h$ (km) | Declination $\delta$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $+40^{\circ}$ | $+20^{\circ}$ | $0^{\circ}$ | $-20^{\circ}$ | $-40^{\circ}$ | $-60^{\circ}$ |
| 110 | 0 | -2 | -4 | -6 | $-10$ | -15 |
| 105 | 0 | -1 | -2 | -4 | $-7$ | -10 |
| 100 | 0 | 0 | -1 | -2 | $-3$ | $-5$ |
| 95 | 0 | 0 | 0 | 0 | 0 | 0 |
| 90 | 0 | 0 | $+1$ | $+2$ | $+3$ | $+5$ |
| 85 | 0 | +1 | $+2$ | +4 | $+6$ | $+10$ |
| 80 | 0 | $+2$ | $+4$ | $+6$ | $+9$ | $+14$ |

on the number of echoes required to give a significant peak on the partial rate curve. A table of transit times must be computed for the range of maximum occurrence of echoes, $r_{m}$, and from this and the times of the peaks in the partial rate curve the radiant coordinates may be found. It will be observed that all

Table 2
MINUTES TO BE ADDED TO $\tau_{s}$

| $\begin{gathered} \text { Height } \\ h \\ (\mathrm{~km}) \end{gathered}$ | Declination $\delta$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $+40^{\circ}$ | $+20^{\circ}$ | $0^{\circ}$ | $-20^{\circ}$ | $-40^{\circ}$ | $-60^{\circ}$ |
| 110 | 0 | -3 | -6 | $-9$ | $-13$ | $-20$ |
| 105 | 0 | -2 | -4 | -6 | $-8$ | $-13$ |
| 100 | 0 | -1 | -2 | $-3$ | $-4$ | $-6$ |
| 95 | 0 | 0 | 0 | 0 | 0 | 0 |
| 90 | 0 | $+1$ | +2 | +3 | +4 | +6 |
| 85 | 0 | +2 | $+3$ | $+5$ | $+7$ | $+12$ |
| 80 | 0 | +3 | $+5$ | $+7$ | $+11$ | $+18$ |

of the essential information in the film record is used; both range and time data are implicit in the partial rate curve provided that the sampling range is not too wide.

To apply this method a count is made over 10 -min time intervals of all meteor echoes whose ranges lie within the sampling band. Ten minutes is suggested as a suitable time interval because it is small compared to the width of a significant peak in the rate curve. Random fluctuations in the rate can be
smoothed out to a large extent by taking sliding groups of three values, thereby obtaining an effective half-hourly rate at 10 -min intervals. Even so, it is found that the amount of fluctuation becomes excessive if the echo rate drops below 100 echoes per hour (for the total rate), so this is a method only suitable for the analysis of high rate records. By widening the sampling range lower rates may be tolerated but the accuracy decreases rapidly.


Fig. 6.-Construction showing how a partial rate curve can yield a transit time for a radiant, with respect to the reference point $A_{m}$.

## VI. Expertmental Resulis

In order to test the validity of the foregoing discussion some experimental work was done, the results of which will now be given.

During the early part of August 1956 new equipment (Ellyett and Fraser 1955), together with higher gain aerials, was used to obtain a high rate of metéor echo detection. The results of some preliminary observations have been analysed. For the night of August $4 / 5$ only, the sensitivity was reduced intentionally to allow one set of full range-time plots to be drawn (Figs. 7 (a) and 7 (b)). Through the main concentrations of echoes the lines $P Q$, representing the most probable time to receive echoes at each given range, are drawn by eye on the basis of previous experience with range-time envelopes. From the intersection of these


Fig. 7 (a).-Full range-time plot showing the lines $P Q$. August 4/5, 1956, aerial azimuth $67 \frac{1}{2}^{\circ} \mathrm{E}$. (The dotted line refers to an earlier concentration of echoes ; cf. Figure 8.)


Fig. 7 (b).-Full range-time plot showing the lines $P Q$. August 4/5, 1956, aerial azimuth $112 \frac{1}{2}^{\circ} \mathrm{E}$.
lines with the 500 km range marker line the following times of maximum reception of 500 km echoes were obtained :

Aerial azimuth $67 \frac{1}{2}^{\circ}$ E. 0350 hr N.Z.S.T.
Aerial azimuth $112 \frac{1}{2}^{\circ}$ E. 0222 hr N.Z.S.T.,
yielding for the radiant coordinates

$$
\alpha=343^{\circ} ; \quad \delta=-16^{\circ} .
$$

These coordinates agree well with published figures for the $\delta$-Aquarid shower, viz. Ellyett and Roth, 338/-18; Hoffmeister, 345/-20; Lindblad, 339/-12 ; McKinley, 340/-17.


Fig. 8.-Partial rate curves. (a), (b) August 1/2, 1956 ; (c), (d) August 2/3, 1956 ; (e), (f) August $3 / 4,1956$.

In order to use the partial rate curve method the range of maximum occurrence of echoes $\left(r_{m}\right)$ must be known. This can be determined quite simply, and only needs to be done once for any given aerial system. Range distribution curves drawn for both aerial arrays reveal that $r_{m}$ is $400 \pm 10 \mathrm{~km}$ (estimated) for the aerial directed to azimuth $112 \frac{1}{2}{ }^{\circ} \mathrm{E}$. ; and $410 \pm 10 \mathrm{~km}$ (estimated) for
the aerial directed to azimuth $67 \frac{1}{2}^{\circ}$ E. Accordingly a sampling band between 350 and 450 km was chosen, and 400 km regarded as $r_{m}$ for both aerials.

In Figure 8 partial rate curves, obtained during the nights preceding August 4, are shown. The times corresponding to the peaks yield the following radiant coordinates:

| August 1/2 | $\alpha=344^{\circ}$ | $\delta=-20^{\circ}$ |
| :--- | :--- | :--- |
| August 2/3 | $\alpha=3433^{\circ}{ }^{\circ}$ | $\delta=-21^{\circ}$ |
| August 3/4 | $\alpha=345^{\circ}$ | $\delta=-20^{\circ}$. |

## VII. Acknowledgments

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