# THE EFFECT OF REMANENT MAGNETIZATION ON AEROMAGNETIC INTERPRETATION

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*Summary*

The interpretation of aeromagnetic maps when the source may be approximated to a point dipole or line of dipoles is considered for the general case in which the dipole moment is not necessarily parallel to the Earth's magnetic field. For the line of dipoles, it is shown that even in this general case the depth and location of the source can be found, and in addition the direction of the component of the dipole moment in a plane normal to the line source may be determined. If the Königsberger ratio for the rock constituting the source is large, this is approximately the direction of the remanent magnetization. Such information is important from a palaeomagnetic viewpoint. When the source approximates closely to a point dipole, it is not possible both to locate the source and determine the direction of the dipole moment from an analysis of the aeromagnetic map and the solution of the problem requires further information.

## I. Introduction

Since the development of the " flux-gate " type of total intensity magnetometer as an airborne instrument, large areas have been surveyed using this device. Interpretation of the resulting data in general is difficult, mainly because of the ambiguous nature of all potential fields and the complex (usually) shape of the body of material producing the anomaly. Procedures for interpretation have been developed by various authors.

Vacquier *et al.* (1951) have calculated the magnetic effects of rectangular prisms of different sizes extending downwards indefinitely and for various magnetic latitudes, and the method of interpretation consists of comparing the observed anomalies with the computed data. Zietz and Henderson (1956) have devised a method for the experimental determination of the contours of magnetic prismatic models and, since any irregularly shaped body may be approximated by the correct arrangement of rectangular prisms, the field may be obtained by superimposing the appropriate contour maps.

Smellie (1956) has modified and extended the work of Henderson and Zietz (1948) on elementary approximations, and four cases are considered in which the source of the anomaly is approximated to a point pole, line of poles, point dipole, and line of dipoles respectively. In each case factors are calculated which, when multiplied by the distance from maximum to half-maximum value along the anomaly profile, give the depth of the source. Also the lateral displacement of the anomaly maximum from the point immediately above the source is given in terms of the depth so that the location of the source is fully determined. These simple approximations are more applicable to the problem

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often encountered in mining geophysics of shallow bodies of limited extent, whereas the prism models of Vacquier *et al.* (1951) and Zietz and Henderson (1956) are useful in the interpretation of surveys of sedimentary basins where the anomalies are due to susceptibility contrasts in the basement complex.

In both of these approaches to the problem of interpretation, two fundamental assumptions are made. The first is that the anomalous field is small, in which case the component of the total intensity anomaly in the direction of the total field (Earth's normal field plus anomalous field), which, in effect, is what the airborne magnetometer measures, may be taken as the component in the direction of the Earth's normal field. If this simplification is not made, a theoretical solution to the problem is virtually impossible.

The other assumption is that the polarization of the magnetized body is in the direction of the Earth's field. While this assumption is true in many instances, it is now known, mainly from palaeomagnetic studies (e.g. Nagata 1953), that bodies of igneous rock do exist in which the remanent magnetization, usually acquired when the rock cools through its Curie point in the Earth's magnetic field at that time, is of the same order as or larger than the induced magnetization and not in the same direction. The vector sum of the remanent and induced magnetizations, i.e. the total magnetization vector, for such rocks will differ significantly in direction from the present Earth's field.

In this paper, following Henderson and Zietz (1948) and Smellie (1956), four types of source are considered—point pole, line of poles, point dipole, and line of dipoles—and an investigation is made of the effect that variation of the direction of the total magnetization vector has on the component of the anomalous magnetic intensity in the direction of the Earth's field, i.e. the effect it has on an aeromagnetic map. Consideration is also given to the inverse problem of whether or not it is possible to gain from aeromagnetic maps any information concerning the direction of remanent magnetization of the body producing the anomaly. This, of course, would be of great interest from a palaeomagnetic point of view.

## II. Point Pole and Line of Poles

If remanent magnetization other than in the direction of the Earth's field is absent, a narrow, steeply dipping body extended in depth will approximate to a point pole at high geomagnetic latitudes. If the remanent magnetization is large and is not in the direction of the Earth's field, the point pole approximation becomes less satisfactory, and, as the direction of the total magnetization vector moves further away from the vertical, the source becomes a steeply dipping line of dipoles. If the magnetic body is reversely magnetized so that the total magnetization vector is approximately opposite in direction to the Earth's field, then the approximation to a point pole will again hold but the resulting anomaly will be negative instead of positive.

Similarly, a narrow vertical dike which approximates to a line of poles at high geomagnetic latitudes tends to become a vertical sheet of dipoles as the direction of the total polarization vector moves further away from the vertical. The field approximates again to that of a line of poles if the rock is reversely magnetized, and the anomaly is negative.

## III. Line of Dipoles

Consider now a line of dipoles with a total moment per unit length $\mathbf{P}$ in any arbitrary direction. The potential is given by $2(\mathbf{p}.\mathbf{r})/r^2$, where $\mathbf{p}$ is the projection of the vector $\mathbf{P}$ on a plane normal to the line of dipoles and $\mathbf{r}$ is displacement from the source in the same plane (the field is obviously the same for all such planes). Thus the component in the direction of the Earth's field, $\mathbf{F}$, of the anomalous field produced by the line of dipoles is

$$\Delta T = -2\mathbf{f}_0.\text{grad}\,\{(\mathbf{p}.\mathbf{r})/r^2\},$$

where $\mathbf{f}_0$ is the unit vector in the direction of $\mathbf{F}$.
This may be written

$$\Delta T = 2p\{2(\mathbf{f}_0.\mathbf{r}_0)(\mathbf{p}_0.\mathbf{r}_0) - (\mathbf{p}_0.\mathbf{f}_0)\}/r^2,$$

where $\mathbf{r}_0$ is the unit vector in the direction of $\mathbf{r}$ and $\mathbf{p}_0$ is the unit vector in the direction of $\mathbf{p}$.

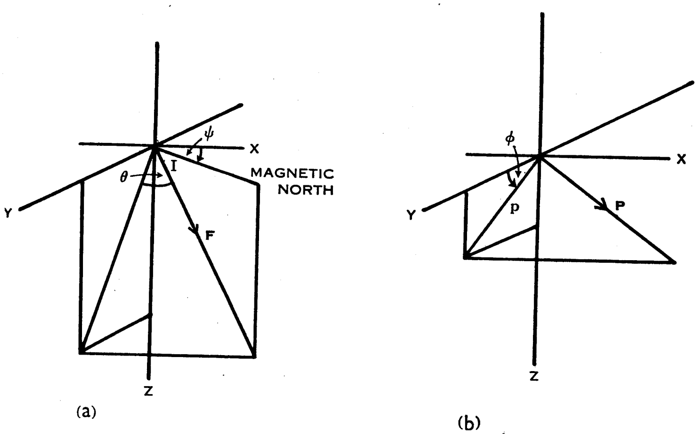


Fig. 1.—Coordinate system showing the direction of (*a*) the Earth's normal field $\mathbf{F}$ and (*b*) dipole moment per unit length $\mathbf{P}$ for a source consisting of a line of dipoles striking along the $X$-direction.

Considering the northern hemisphere and the $Z$-axis vertically downwards, suppose the line of dipoles strikes along the $X$-axis and makes an angle $\psi$ with magnetic north, then, if the line of dipoles is parallel to the plane of observation (the $XY$-plane) and at depth $z=\zeta$ below the plane,

$$\mathbf{f}_0 = \mathbf{i}\cos I\cos\psi + \mathbf{j}\cos I\sin\psi + \mathbf{k}\sin I,$$
$$\mathbf{p}_0 = \mathbf{j}\cos\varphi + \mathbf{k}\sin\varphi,$$

where $I$ is the angle of dip of the Earth's magnetic field, $\varphi$ is the angle between $\mathbf{p}$ and the $Y$-axis, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the $X$, $Y$, and $Z$ directions respectively (see Fig. 1).
Also for a profile along the $Y$-axis

$$\mathbf{r}_0 = (\mathbf{j}y - \mathbf{k}\zeta)/r \quad \text{and} \quad r^2 = y^2 + \zeta^2.$$

Then

$$\Delta T = (2p/r^2)\{2(y\cos I\sin\psi - \zeta\sin I)(y\cos\varphi - \zeta\sin\varphi)/r^2 - (\cos\varphi\cos I\sin\psi + \sin\varphi\sin I)\}.$$

If $y$ is expressed in terms of the depth of burial, i.e. $y=\alpha\zeta$, then

$$\Delta T=(2p\cos I\sin\psi\cos\varphi)f(\alpha)/\zeta^2, \qquad (1)$$

where

$$f(\alpha)=\{(\alpha^2-1)(1-q\tan\varphi)-2\alpha(q+\tan\varphi)\}/(1+\alpha^2)^2, \qquad (2)$$
$$q=\tan I\operatorname{cosec}\psi.$$

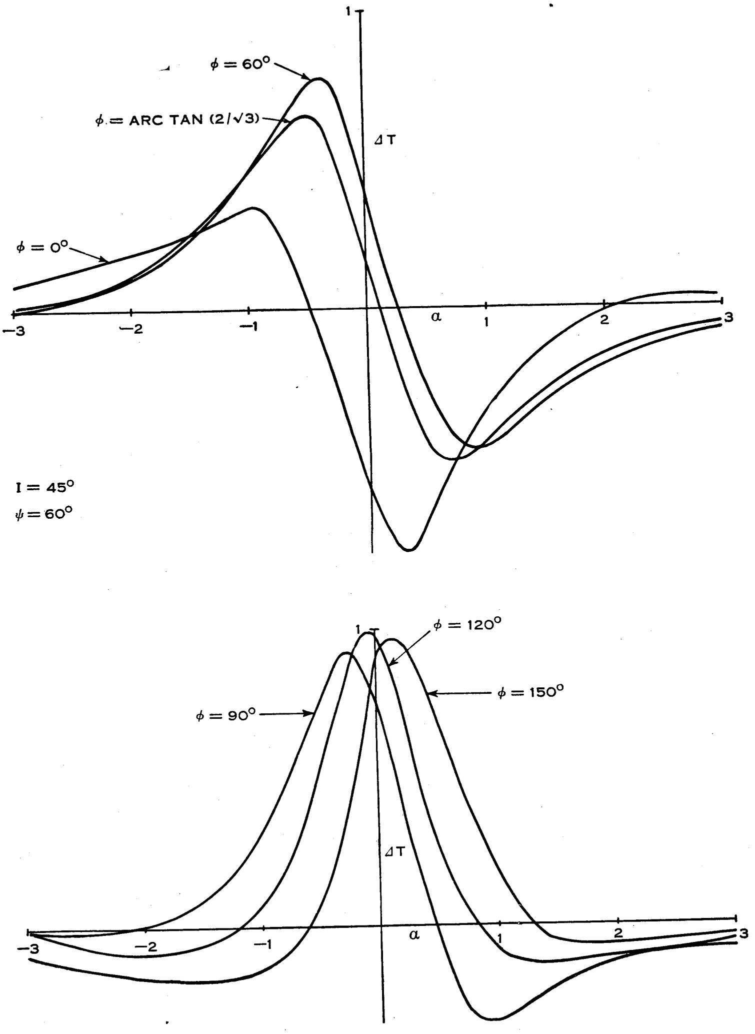


Fig. 2.—Profiles along an axis normal to a line source of dipoles for different directions of the dipole moment per unit length. $\varphi$ is the angle between the $Y$-axis and **p**, the component of the dipole moment per unit length in the plane normal to the line source. ($I=45°$, $\psi=60°$, i.e. $q=2/\sqrt{3}$.) $\Delta T$ in units of $2p/\zeta^2$.

Variations in $\varphi$ thus change the shape of a profile along an axis perpendicular to the line source as is shown in Figure 2 (for $I=45°$, $\psi=60°$).

If the dipole moment is parallel to the Earth's field the direction cosines of the projection of the dipole moment in the $YZ$-plane are $\cos\varphi = \cos I \sin\psi/\cos\theta$, $\sin\varphi = \sin I/\cos\theta$, i.e. $\tan\varphi = q$, where $\theta$ is the angle between the Earth's field and the $YZ$-plane. Then, since $p = P\cos\theta$, equations (1) and (2) become

$$\Delta T = 2P\cos^2 I \sin^2\psi\, f_1(\alpha)/\zeta^2,$$

where

$$f_1(\alpha) = \{(\alpha^2-1)(1-q^2)-4\alpha q\}/(1+\alpha^2)^2. \quad \ldots\ldots\ldots\ldots \quad (3)$$

This is the expression given by Smellie (1956).

From equation (3), factors $k$ and $k'$ may be determined which are such that the depth to the source may be given as $\zeta = k\eta = k'\eta'$, where $\eta$ is the distance from the anomaly maximum to the half-maximum value in a northerly direction, $\eta'$ in a southerly direction (in the northern hemisphere). These directions are reversed in the southern hemisphere. The half-maximum distance ratio $\eta/\eta'$ and $\alpha_0$, the displacement of the peak of the anomaly from a line vertically above the source may also be determined. Smellie (1956) has calculated these various factors for different values of $I$ and $\psi$ and expressed his results as a family of curves. Thus, since in any practical case $I$ is known and $\psi$ can be determined by inspection of the aeromagnetic map, the values of the various factors appropriate to the particular problem can be read from the family of curves and hence the location and depth of the source can be found.

In the general case when the total dipole moment is not necessarily in the direction of the Earth's field (equations (1) and (2)) the factors mentioned above depend for a given value of $I$ and $\psi$ (i.e. fixed $q$) on the value of $\varphi$, e.g. Figure 3 shows these factors as a function of $\varphi$ when $q=2$. $\alpha_0$ is considered positive if the peak of the anomaly is displaced in the northerly half-plane. For values of $\varphi$ other than those shown, the anomaly is a low and the peak will be negative. If this is taken into account then, since $\tan(180+\varphi) = \tan\varphi$, the factors for angles $(180+\varphi)$ will be the same as for $\varphi$. Since the half-maximum distance ratio may be read directly from the aeromagnetic map, the corresponding value of $\varphi$ may be found from such a graph showing $\eta/\eta'$ as a function of $\varphi$ (for the appropriate value of $q$). Total intensity depth factors and peak displacement (e.g. from Fig. 3) then yield depth and location of source.

Thus, if we have a source that can be represented truly by a line of dipoles, both the location of the source and the direction of the component of the dipole moment in a plane normal to the line source can be determined. In practice, of course, the difficulty lies in knowing when, in fact, the source can be truly represented by a line of dipoles. A comparison between the observed profile and a theoretical one obtained from equations (1) and (2) using the known value of $q$ and the value of $\varphi$ determined as above would decide whether or not the approximation were a good one.

From a palaeomagnetic point of view, even in the case of a perfect source, the problem remains unsolved, since there is still no information about the direction of remanent magnetization. The direction of the component of the total magnetization in a plane normal to the line source and the direction of the induced magnetization are known but any information concerning the direction of the remanent magnetization requires a knowledge at least of the ratio of intensity of remanent magnetization ($J_n$) to intensity of induced magnetization ($J_i$). This ratio is called the Königsberger ratio and in general for igneous rocks has values from 2 to 10 although sometimes it may be much greater and for a few cases is slightly less than 1 (Nagata 1953). For rocks with a high Königsberger ratio the direction of $\mathbf{J}_n$ is very nearly that of the total magnetization, so that the above analysis would yield approximately the direction of the projection of $\mathbf{J}_n$ on a plane normal to the line source (except when the total magnetization vector is almost in the direction of the line source, in which case the projection of $\mathbf{J}_n$ and $\mathbf{J}_i$ in a plane normal to the line source will be of the same

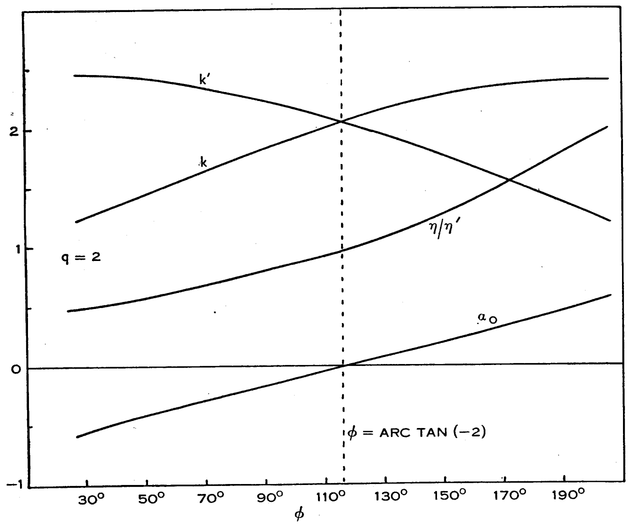


Fig. 3.—Depth factors $k$ and $k'$, half-maximum distance ratio $\eta/\eta'$, and peak displacement $\alpha_0$ as functions of $\varphi$ for a line of dipoles source when $q=2$.

order). If the Königsberger ratio is low then no further information can be obtained, since the ratio of the projections of $\mathbf{J}_n$ and $\mathbf{J}_i$ on the above plane will be in general quite different from the Königsberger ratio $J_n/J_i$. In no case is it possible to learn anything about the component of $\mathbf{J}_n$ in the direction of the line source.

## IV. Point Dipole

In the case where the source may be approximated to a dipole of moment $\mathbf{p}$, the field at a point distant $\mathbf{r}$ from the dipole is given by

$$-\text{grad}\,\{(\mathbf{p}.\mathbf{r})/r^3\}.$$

The component of this anomalous field in the direction of the total field $\mathbf{F}$ of the Earth is

$$\Delta T=-\mathbf{f}_0.\text{grad}\,\{(\mathbf{p}.\mathbf{r})/r^3\},$$

where $\mathbf{f}_0$ again is the unit vector in the direction of $\mathbf{F}$. Then

$$\Delta T=p\{3(\mathbf{f}_0.\mathbf{r}_0)(\mathbf{p}_0.\mathbf{r}_0)-(\mathbf{f}_0.\mathbf{p}_0)\}/r^3.$$

Again considering the northern magnetic hemisphere, take the $Z$-axis vertically downwards and the $Y$-axis in the direction of magnetic north. If the $XY$-plane is the plane of observation and the dipole is located at $z=\zeta$, then

$$\mathbf{r}_0=(\mathbf{i}x+\mathbf{j}y-\mathbf{k}\zeta)/r.$$

Also

$$\mathbf{f}_0=\mathbf{j}\cos I+\mathbf{k}\sin I,$$

and

$$\mathbf{p}_0=\mathbf{i}\sin\theta+\mathbf{j}\cos\theta\cos\varphi+\mathbf{k}\cos\theta\sin\varphi,$$

where $\theta$ is the angle between $\mathbf{p}$ and the $YZ$-plane and $\varphi$ is the angle between the $Y$-axis and the projection of $\mathbf{p}$ on the $YZ$-plane, i.e. the direction cosines of $\mathbf{p}$ are $\sin\theta$, $\cos\theta\cos\varphi$, and $\cos\theta\sin\varphi$ respectively (see Fig. 4).

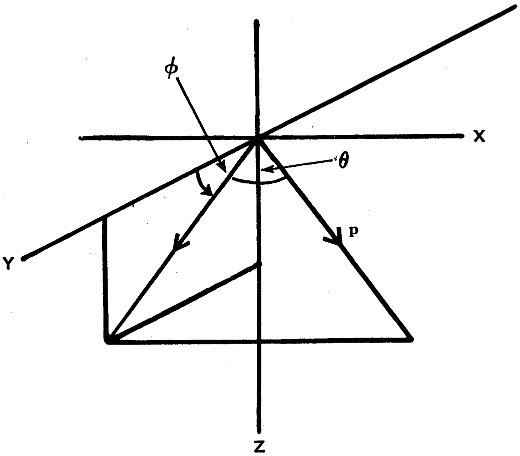


Fig. 4.—Coordinate system showing the direction of the dipole moment $\mathbf{p}$ for a source consisting of a point dipole.

Thus

$$\Delta T=(p\cos\theta/r^3)\{3(y\cos I-\zeta\sin I)(x\tan\theta+y\cos\varphi-\zeta\sin\varphi)/r^2 -(\cos I\cos\varphi+\sin I\sin\varphi)\}. \quad \ldots\ldots\ldots\ldots (4)$$

If $x$ and $y$ are expressed in terms of the depth of burial, i.e. $y=\alpha\zeta$ and $x=\beta\zeta$, this becomes

$$\Delta T=F(\alpha\beta)p\cos\theta/\zeta^3, \quad \ldots\ldots\ldots\ldots (5)$$

where

$$F(\alpha\beta)=(A\alpha^2+B\alpha\beta+C\beta^2+D\alpha+E\beta+F)/(1+\alpha^2+\beta^2)^{5/2}, \quad \ldots (6)$$

and $A=2\cos I\cos\varphi-\sin I\sin\varphi$,
$B=3\tan\theta\cos I$,
$C=-(\cos I\cos\varphi+\sin I\sin\varphi)$,
$D=-3(\cos I\sin\varphi+\sin I\cos\varphi)$,
$E=-3\tan\theta\sin I$,
$F=2\sin I\sin\varphi-\cos I\cos\varphi$.

(1) If the dipole moment $\mathbf{p}$ is in the $YZ$-plane, i.e. $\theta=0$, but $\varphi\neq I$ then $F(\alpha\beta)$ becomes

$$(A\alpha^2+D\alpha+F+C\beta^2)/(1+\alpha^2+\beta^2)^{5/2}.$$

Thus symmetry exists about the $Y$-axis, indicating that the anomaly maximum is situated somewhere along the $Y$-axis. For such a profile, $\beta=0$ and the expression for $\Delta T$ becomes

$$\Delta T=f_2(\alpha)p/\zeta^3, \quad \ldots\ldots \quad (7)$$

where

$$f_2(\alpha)=\{\alpha^2\,(2\cos I\cos\varphi-\sin I\sin\varphi)-3\alpha\,(\cos I\sin\varphi+\sin I\cos\varphi)+2\sin I\sin\varphi-\cos I\cos\varphi\}/(1+\alpha^2)^{5/2} \quad \ldots \quad (8)$$

In particular when $\mathbf{p}$ is parallel to the Earth's field, i.e. $\varphi=I$,

$$\Delta T=f_3\,(\alpha)p/\zeta^3,$$

where

$$f_3(\alpha)=\{\alpha^2\,(3\cos^2 I-1)-6\alpha\sin I\cos I+3\sin^2 I-1\}/(1+\alpha^2)^{5/2}.$$

This is the value of $\Delta T$ given by Smellie (1956) (equations (**24**) and (**25**)). Since equations (**7**) and (**8**) represent a meridional profile through the anomaly maximum, it is possible to determine, for any given value of $I$, the way in which the various factors (total intensity depth factors etc.) depend on the angle $\varphi$. Then by measuring the half-maximum distance ratio on the aeromagnetic map, the corresponding value of $\varphi$ and hence the depth and location of the source could be determined in the same way as for the line of dipoles.

(2) In the general case when $\mathbf{p}$ does not lie in the $YZ$-plane, the problem is more difficult, since the peak of the anomaly is no longer over the $Y$-axis. If $\alpha_0$ and $\beta_0$ are the values of $\alpha$ and $\beta$ at the peak of the anomaly, then $(\alpha_0, \beta_0)$ is a solution of

$$\frac{\partial F(\alpha,\beta)}{\partial\alpha}=0 \quad\text{and}\quad \frac{\partial F(\alpha,\beta)}{\partial\beta}=0.$$

In general it is not possible to solve these two equations for $\alpha_0$ and $\beta_0$, and therefore to locate the peak of the anomaly it is necessary to draw a series of profiles in the vicinity of the peak. The location is given approximately by

$$\frac{\partial F(\alpha,0)}{\partial\alpha}=0 \quad\text{and}\quad \frac{\partial F(0,\beta)}{\partial\beta}=0.$$

The amount of work therefore involved in calculating a profile through the anomaly maximum is considerable and, furthermore (for fixed $I$), the shape of the profile depends on $\theta$ and $\varphi$. Thus, if the location of the source is not known, solution of the problem requires a knowledge of the direction of the dipole moment. The coordinates of the peak of the anomaly can then be determined. This locates the origin on the aeromagnetic map, which is vertically above the source, and a meridional profile over the source gives the depth (as in the following).

When $\beta=0$, equation (**4**) becomes

$$\Delta T=f_2(\alpha)p\cos\theta/\zeta^3, \quad \ldots\ldots \quad (9)$$

where $f_2(\alpha)$ is given by equation (8). This means that the *shape* of a profile along the $Y$-axis (i.e. a meridional profile over the source) is independent of the value of $\theta$ (Fig. 5). Thus it would be possible to draw graphs showing the various

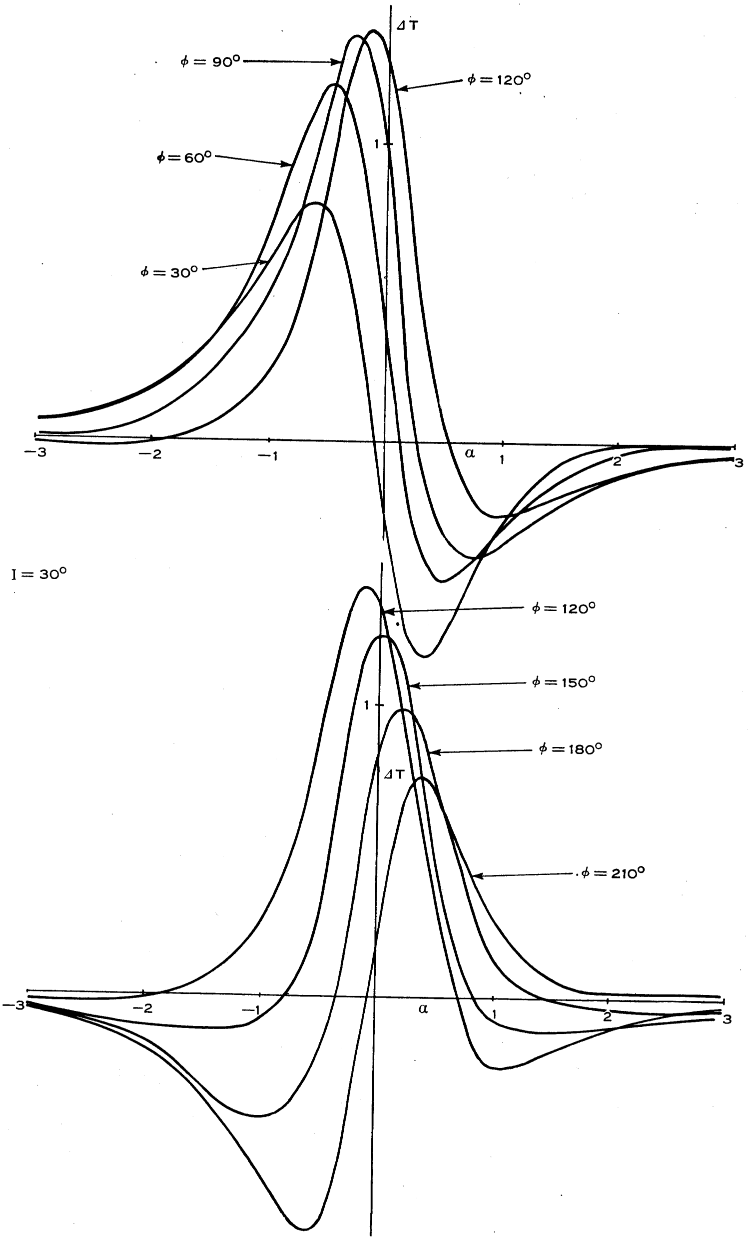


Fig. 5.—Curves showing profiles along the meridian passing over a dipole source for different $\varphi$ ($I = 30°$). In this case the shape of the profile is independent of $\theta$. $\Delta T$ in units of $p \cos \theta / \zeta^3$.

factors as functions of $\varphi$ (appropriate to that particular profile). If, therefore, such a profile were known (which in turn means that *the point in the plane of observation vertically above the source must be known*), both the value of $\varphi$ and the depth to the source could be determined in the same way as for the line of dipoles source.

A profile along the $X$-axis ($\alpha=0$) will vary with both $\theta$ and $\varphi$ (assuming $I$ fixed), since $\Delta T$ then becomes

$$g(\beta)p\cos\theta/\zeta^3,$$

where

$$g(\beta)=\{-\beta^2(\cos I\cos\varphi+\sin I\sin\varphi)-3\beta\tan\theta\sin I +2\sin I\sin\varphi-\cos I\cos\varphi\}/(1+\beta^2)^{5/2}. \quad \ldots\ldots\ldots \quad (10)$$

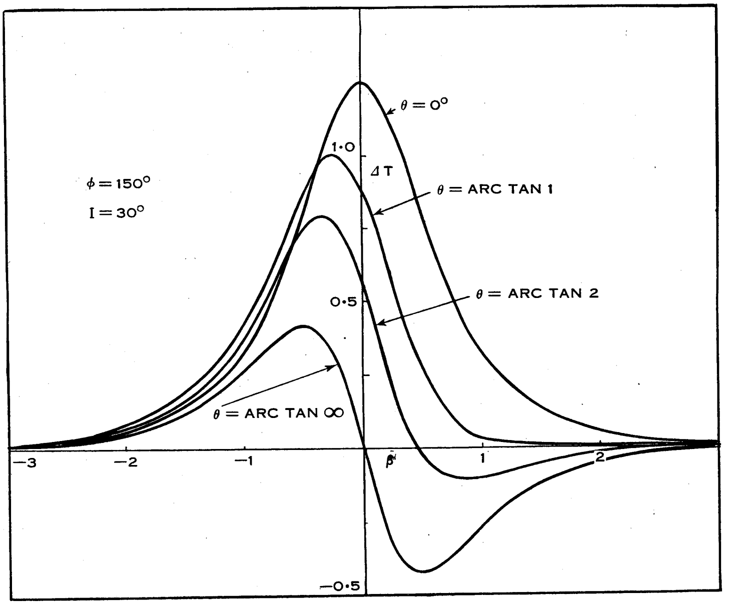


Fig. 6.—Profile along a line normal to the meridian and passing over a dipole source, for different $\theta$ ($\varphi=150°$, $I=30°$). $\Delta T$ in units of $p/\zeta^3$.

If $\varphi$ has been determined ($I$ fixed) then $g(\beta)$ will depend only on $\theta$, e.g. Figure 6 shows the way in which $\Delta T$ changes with increasing $\theta$ ($\varphi=150°$ and $I=30°$). Thus a graph showing any of the factors (this time measured along a profile normal to the meridian and passing over the source) as a function of $\theta$, would allow $\theta$ to be determined.

Thus, if the point in the plane of observation vertically above the source is known, both the depth to the source and the direction of the total magnetization vector can be determined. If, then, the Königsberger ratio is known for the type of rock constituting the source, the direction of the remanent magnetization may be found.

## V. Discussion

The application of elementary approximations of the type above is necessarily limited because in general the body producing the anomaly is complex in shape or too extensive to be represented by a point or a line source. When such approximations can be applied, previous authors have shown how measurements along a meridional profile (or in the case of a line source, a profile normal to the source) yield the depth and location of the source, on the assumption that, for the point dipole or line of dipoles, the moment is in the direction of the Earth's field.

If the direction of the dipole moment (i.e. the direction of the total magnetization vector) is not necessarily parallel to the Earth's field, the problem is more difficult, since another variable has been introduced. In spite of this, the analysis above shows that, for a line of dipoles, the depth and location of the source can still be determined and, in addition, the direction of the component of the total magnetization vector in a plane normal to the line source. If the Königsberger ratio is large this will approximate to the direction of the component of remanent magnetization. If the ratio is small, little information of interest from a palaeomagnetic point of view can be obtained.

For problems in which the source can be represented truly by a point dipole and the total magnetization vector is parallel to the Earth's field, the peak of the anomaly is displaced from the point vertically above the source in a meridional direction and a meridional profile through the anomaly maximum does in fact pass over the source. In the general case the peak is displaced in some arbitrary direction depending on the orientation of the dipole and analysis is virtually impossible without further information. Both the depth to the source and the direction of the dipole moment can be determined from a meridional profile and one at right angles to the source, *both passing over the source.* Again it should be stressed that these elementary aeromagnetic approximations and an analysis of the above type are applicable only to the interpretation of those anomalies where the approximation to point dipole or line of dipoles is a good one. In all cases the ultimate check lies in a comparison of the actual profile with that calculated to arise from a dipole (or line of dipoles) at the depth and orientation determined by such an analysis.

## VI. Acknowledgments

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