A METHOD OF HEATING MATTER OF LOW DENSITY TO TEMPERATURES IN THE RANGE 10^5 TO 10^6 °K

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Summary

A method of confining fully ionized gases entirely by means of an oscillating electromagnetic field is described. The gas is heated by eddy currents induced in it by the oscillating field.

The special case of hydrogen of density $2 \cdot 5 \times 10^{17}$ atom/m³ at a temperature of 4×10^5 °K is treated in detail. It appears practicable for the method to maintain hydrogen in this state indefinitely.

I. Introduction

If gas of controlled composition could be held steadily at temperatures greatly exceeding 10^4 °K an interesting field for experimental research would be open. In particular it would facilitate investigation of the properties of multiply ionized atoms and make possible the duplication of some conditions existing in stellar atmospheres. Current research into the feasibility of constructing fusion reactors adds further interest (Thirring 1955).

At the temperatures and densities considered here, viz. 10^5 to 10^6 °K and 10^{17} to 10^{18} atom/m³, the gas would be almost fully ionized, and its temperature could be maintained by electromagnetic induction. In order to avoid excessive heat loss it is necessary to hold most of the heated gas away from all solid material, and this might also be achieved by electromagnetic means.

One method of obtaining temperatures of the order considered here has recently been published (Kurchatov 1956). It appears to require less power to confine gas at a given pressure than the method described here, but does not confine it in steady, stable equilibrium. It is thought that for general experimental work with rarefied gases, the method proposed here would be more flexible.

II. Description of Method

It is proposed that the configuration of conductors and fields take the form of a spherical, conducting shell with a core of ionized gas at its centre, as indicated in Figure 1. In the cavity between shell and core, standing electromagnetic waves are maintained. Pressure on the core is due to the radiation pressure of the standing waves. The pressure on the core can be calculated by considering

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the force exerted by the electromagnetic field (oscillating in several modes, as will be discussed later) on electrons in the gas.

It is inherent in this method that the pressure exerted on the core surface varies between zero and a maximum value with a frequency twice that of the oscillating field. Assuming that it would not be convenient for the gas to occupy a volume of linear dimensions much greater than 1 m, the field must have a sufficiently high frequency to allow the core surface to move through a distance only very small compared with 1 m in the time between zero and maximum pressure. For hydrogen at a temperature of 10^5 °K, if the linear dimensions of the core are not to fluctuate more than a few centimetres, the minimum frequency is of the order of 10^6 c/s.

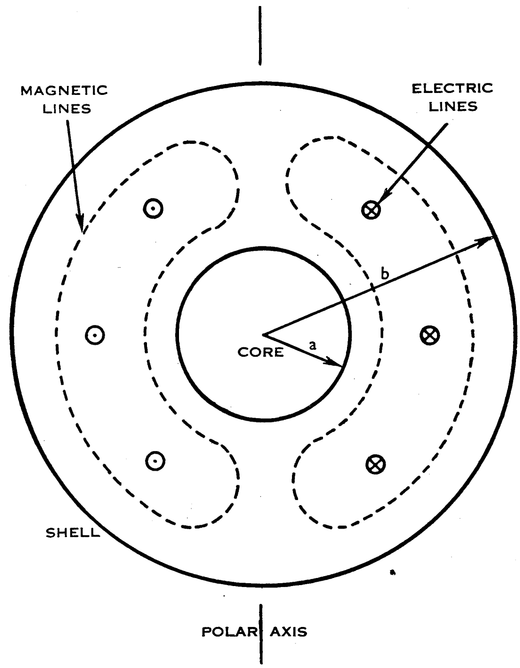

Fig. 1.—Form of the H_{101} mode.

As a first step we select a field configuration suggested by the form of a low frequency field which can support a liquid conductor against its weight (Okress *et al.* 1952), and examine the pressure exerted on the surface of the inner sphere of two concentric conducting spheres by this oscillating field between them. The field, described in equations (**2**), and its lowest mode illustrated in Figure 1, consists of electric lines which are circles centred on the polar axis and in planes normal to it, while the magnetic lines lie in planes through the axis, run close to each surface of the cavity, and loop over near the axis.

Let **E** be the electric and **H** the magnetic vector, ω the angular frequency of the field, ε_0 the permittivity, and μ_0 the permeability of free space.

The mathematical formulation of the oscillating field is obtained from the general equation describing a field resonating in a cavity (Bracewell 1947, pp. 137–8).

$$\left.\begin{aligned}\nabla^2\mathbf{E}+\omega^2\varepsilon_0\mu_0\mathbf{E}&=0,\\ \mathbf{H}&=\frac{\mathrm{j}}{\omega\mu_0}\nabla\times\mathbf{E}.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

In terms of spherical polar coordinates (r, θ, φ), the lowest mode of a field which is symmetrical about the coordinate axis and which has an electric component only in the azimuthal direction (φ), is the H_{101} mode (Lamont 1942) obtained from the general equation for the $H_{10\nu}$ modes :*

$$\left.\begin{aligned}E_r&=0,\\ E_\theta&=0,\\ E_\varphi&=A\omega\mu_0 r^{-1/2}\{\mathrm{J}_{3/2}(\omega r/c)+\gamma\mathrm{J}_{-3/2}(\omega r/c)\}\sin\theta\exp(\mathrm{j}\omega t),\\ H_r&=\mathrm{j}Ar^{-3/2}\{\mathrm{J}_{3/2}(\omega r/c)+\gamma\mathrm{J}_{-3/2}(\omega r/c)\}\cos\theta\exp(\mathrm{j}\omega t),\\ H_\theta&=-\mathrm{j}Ar^{-1}\left\{\tfrac{1}{2}r^{-1/2}+r^{1/2}\frac{\mathrm{d}}{\mathrm{d}r}\right\}\{\mathrm{J}_{3/2}(\omega r/c)+\gamma\mathrm{J}_{-3/2}(\omega r/c)\}\sin\theta\exp(\mathrm{j}\omega t),\\ H_\varphi&=0,\end{aligned}\right\} \quad \ldots (2)$$

where γ, a constant, and ω depend on the boundary conditions, and A, a constant, depends on the amplitude of the field ; c is the velocity of light and t the time.

The pressure on the ionized core of the resonator may be derived by considering the interaction of the magnetic component of the oscillating field with the induced current flowing near the surface of the core. If $|H_\theta|$ be the root-mean-square amplitude with respect to time of H_θ, and P the time-averaged pressure on the core at a point in the vicinity of the element, then,

$$P=\tfrac{1}{2}\mu_0\,|H_\theta|^2_{r=a}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

If the core is not specified, a lower limit only may be set to the total power dissipation ; it is that dissipated in the shell. It is (Bracewell 1947, pp. 153, 123)

$$W_b=\tfrac{1}{2}\mu_0\omega\delta_b\iint|H|^2\mathrm{d}S, \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

where δ_b is the skin depth in the shell, and the integral is taken over the whole surface of the shell.

The boundary conditions require,

$$\left.\begin{aligned}(E_\varphi)_{r=a}&=0,\\ (E_\varphi)_{r=b}&=0.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

When equations (2), (3), (4), and (5) are combined the following expression is obtained, for the relation between pressure on the core to heat dissipated in the shell :

$$\frac{P}{W_b}=\frac{3\sin^2\theta}{8\pi ab\omega\delta_b}\,\frac{\left[\dfrac{\mathrm{d}}{\mathrm{d}r}\{\mathrm{J}_{3/2}(\omega r/c)+\gamma\mathrm{J}_{-3/2}(\omega r/c)\}\right]^2_{r=a}}{\left[\dfrac{\mathrm{d}}{\mathrm{d}r}\{\mathrm{J}_{3/2}(\omega r/c)+\gamma\mathrm{J}_{-3/2}(\omega r/c)\}\right]^2_{r=b}}, \quad \ldots\ldots \quad (6)$$

where b is the shell radius.

* This class of mode is denoted by $H_{nm\nu}$, where n and m refer to the angular, and ν to the radial distribution of field.

If the field impedance* of the ionized gas is very small compared with the intrinsic impedance of free space it will satisfy to a good approximation the boundary conditions given by equation (**5**).

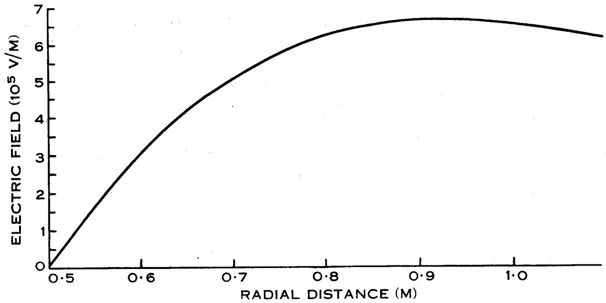

Fig. 2.—Amplitude of the electric field of the H_{101} mode for $W_b = 6 \times 10^4$ W, *v.* distance from the core centre.

For a copper shell of radius $b = 1 \cdot 5$ m and core of radius $a = 0 \cdot 5$ m, equations (**2**), (**5**), and (**6**) yield, in the H_{101} mode, $\omega = 1 \cdot 04 \times 10^9$ sec^{-1}, $\gamma = 0 \cdot 818$, $\delta_b = 0 \cdot 513 \times 10^{-5}$ m (Bracewell 1947, p. 123), and $P/W_b = 0 \cdot 696 \times 10^{-4}$ $\sin^2 \theta$ N m^{-2} W^{-1}. The mean value of P/W_b over the core surface is

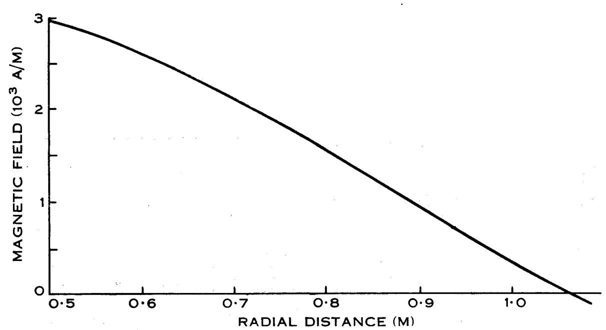

Fig. 3.—Amplitude of the magnetic field of the H_{101} mode for $W_b = 6 \times 10^4$ W, *v.* distance from the core centre.

$0 \cdot 46 \times 10^{-4}$ N m^{-2} W^{-1}. The mean value of P equals the pressure of a gas of 5×10^{17} particles/m^3 at 4×10^5 °K when $W_b = 6 \times 10^4$ W. The amplitudes of E_φ and H_θ are plotted in Figures 2 and 3.

Owing to the variation of pressure over the surface, as given by equation (**6**), a gaseous core would not be in equilibrium. Consider now three resonant fields each giving a $\sin^2 \theta$ pressure variation, exerting the same mean pressure over the

* With hydrogen at a temperature $\sim 4 \times 10^5$ °K, this is at least true for an electron density $\sim 2 \cdot 5 \times 10^{17}$ m^{-3} and field frequency $\sim 3 \times 10^8$ sec^{-1}. The electron collision frequency with other particles (Spitzer 1956, p. 82) is very much less than the field frequency, therefore the field impedance, E_φ / H_θ (Booker 1947), can be obtained by considering a single current-carrying electron. Its velocity, v_φ, can be expressed in terms of E_φ or H_θ, and if v_φ is eliminated between these two expressions the field impedance can be obtained.

surface, and having their respective axes, $\theta=0$, mutually perpendicular. Let their frequencies be all different such that in a time not longer than the inverse of the difference frequencies only negligible movement of the core surface occurs. The total time-averaged pressure on any point on the core surface is the sum of the pressures independently exerted on that point by the component fields. The total pressure is

$$P_1=P_0\,(\sin^2\theta_1+\sin^2\theta_2+\sin^2\theta_3)=2P_0, \qquad \ldots\ldots\ldots \quad (7)$$

as $\cos\theta_1$, $\cos\theta_2$, $\cos\theta_3$ are the direction cosines of the point considered referred to three mutually perpendicular axes. P_0 is the time-averaged pressure exerted by a component field at points on the core surface where $\theta=\frac{1}{2}\pi$, and is constant. The conditions above can be satisfied when the three component fields are the H_{101}, H_{102}, H_{103} modes (Lamont 1942).

The configuration is now an equilibrium one and the question of its stability must be considered.

III. Stability

(a) *Change of Core Size*

If the cavity always oscillates at the resonant frequencies of the modes used, the equilibrium is unstable against change of core size. It can be made stable if the cavity is driven to oscillate at frequencies slightly higher than the resonant frequencies. In this case, if the core volume increases, the resonant frequencies increase and become more nearly equal to the driving frequencies, thus allowing the component field intensities to increase, which tends to reduce the core volume.

(b) *Change of Core Shape*

In the following the stability of the core against small changes of shape is discussed. The change in pressure distribution over the core surface is evaluated only in the case of the simplest distortion associated with a single H_{101} mode. To each of equations (2) is added a small term which satisfies equation (1) and the shell boundary conditions. The core shape which is needed to satisfy the remaining boundary condition is then found and finally the magnetic field at the core surface.

In equations (2) let all factors

$$-\{J_{3/2}(\omega r/c)+\gamma J_{-3/2}(\omega r/c)\}\sin\theta$$

be replaced by

$$-\{J_{3/2}(\omega r/c)+\gamma J_{-3/2}(\omega r/c)\}\sin\theta+\alpha\{J_{7/2}(\omega r/c)+\gamma' J_{-7/2}(\omega r/c)\}\frac{d}{d\theta}P_3\,(\cos\theta),$$

where $\alpha\ll 1$ and

$$J_{7/2}(\omega b/c)+\gamma' J_{-7/2}(\omega b/c)=0. \qquad \ldots\ldots\ldots\ldots \quad (8)$$

Equations (2) now describe the $H_{10\nu}$ modes for a core whose surface is described by

$$r=a_0+h(\theta), \qquad \ldots\ldots\ldots\ldots \quad (9)$$

where

$$J_{3/2}(\omega a_0/c)+\gamma J_{-3/2}(\omega a_0/c)=0, \qquad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

and $h(\ll a_0)$ is obtained from

$$E_\varphi(a_0+h)=0. \qquad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{11})$$

The magnetic field intensity over the core surface can be expressed as

$$H_\theta(r)=H_\theta(a_0)+k(\theta), \qquad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{12})$$

where $k \ll H_\theta(a_0)$.

On expanding equations (**11**) and (**12**) and neglecting all terms of order higher than the first in h/a_0, α,

$$\frac{k}{H_\theta(a_0)}\bigg/\frac{h}{a_0}=\frac{\omega a_0}{c}\left[\frac{\mathrm{J}''_{3/2}(\omega a_0/c)+\gamma\mathrm{J}''_{-3/2}(\omega a_0/c)}{\mathrm{J}'_{3/2}(\omega a_0/c)+\gamma\mathrm{J}'_{-3/2}(\omega a_0/c)}-\frac{\mathrm{J}'_{7/2}(\omega a_0/c)+\gamma'\mathrm{J}'_{-7/2}(\omega a_0/c)}{\mathrm{J}_{7/2}(\omega a_0/c)+\gamma'\mathrm{J}_{-7/2}(\omega a_0/c)}\right]-\tfrac{1}{2}.$$

$$\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{13})$$

Equation (**13**) was evaluated for the cases $b=1\cdot5$ m, $a_0=0\cdot32$, $0\cdot50$, $0\cdot55$, $0\cdot7$, $1\cdot03$ m, and gave $[k/\{H_\theta(a_0)\}]/[h/a_0]$ positive for $a_0<0\cdot66$ m, and negative for $a_0>0\cdot66$ m. This shows that for the simple distortion of core shape con-

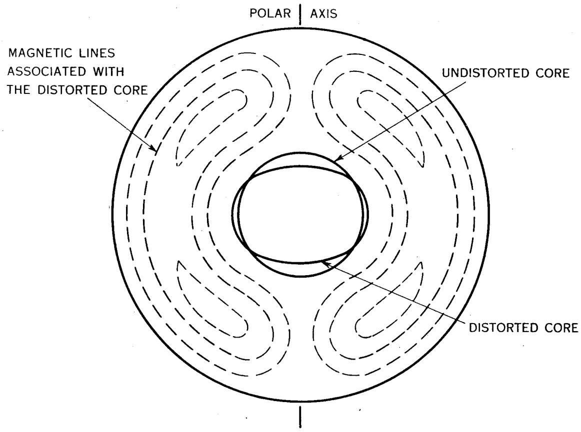

Fig. 4.—Form of the H_{101} mode for the distorted core shown.

sidered above, where $a_0<0\cdot66$ m the field intensity increases where the core surface moves outwards and decreases where it moves inwards from the equilibrium spherical shape; and vice versa for $a_0>0\cdot66$ m.

The change of field intensity during distortion of the core can be broken into two parts: (i) change in the overall field pattern of the type illustrated in Figure 4, which becomes negligible for small enough cores, and (ii) local change in the field near the core, which is independent of core size.

In Figure 4 it is seen that, near where the core surface moves outwards, magnetic lines are removed from the space between core and shell. Therefore, where the surface has moved out, pressure on it should decrease, leading to instability when (i) predominates. When (ii) predominates, the core should be stable against general changes of shape, for the magnetic lines can be considered to resist bending, the net effect of which, together with the tendency for the pressure to remain constant throughout the core, is to keep the curvature of the surface and the lines adjacent to it constant. That is, the core tends to return to its equilibrium spherical shape after any distortion from it. The result of the detailed calculation given above indicates that the ratio of core radius to shell radius does not have to be excessively small for (ii) to predominate; less than a number of the order of 0·4.

(c) *Change of Core Position*

Stability against change of shape implies stability against change of core position, which can be looked on as a special case of change of shape. It should therefore be possible to support a core of finite weight.

IV. Heat Balance in a Hydrogen Core

A particular case is taken for more detailed study. The shell has a radius $b=1\cdot5$ m and is of copper. The core has a radius $a=0\cdot5$ m and is of hydrogen at a temperature $T=4\times10^5$ °K and a density $n=2\cdot5\times10^{17}$ atom/m³. In all of the following discussion only the H_{101} mode is treated, but the general conclusions should be valid for the combination of H_{101}, H_{102}, H_{103} modes. In order that the field pressure balance the hydrogen pressure, the power dissipated in the shell must be $W_b=6\times10^4$ W.

The size is taken as large as would be practicable, in order that near the centre of the core the confining current be as small as possible. It will be shown later that neutral hydrogen has to exist in appreciable density outside the core. Near the centre of the core the density of neutral hydrogen will also be smaller, the larger the core size. However, smaller apparatus should be capable of yielding useful results.

(a) *Radio-frequency Power Dissipation in the Core*

The root-mean-square current per unit length in the core surface, which terminates the oscillating field, is $|H_\theta|_{r=a}$. Its magnitude can be calculated using equations (**3**), (**6**): it is $2\cdot09\times10^3$ A/m.

The magnetic field does not excessively bend the electron paths at the core surface,* therefore an expression given by Spitzer (1956, pp. 50, 51) can be used to calculate the depth at which the field is reduced to 0·368 of its intensity at the surface. The depth is $\delta=1\cdot06\times10^{-2}$ m. Current distribution in the core surface can be approximated by a uniform layer $1\cdot06\times10^{-2}$ m thick and of current density $i=1\cdot97\times10^5$ A/m².

Resistivity of the core material can be calculated from the following expression (Spitzer 1956, p. 84)

$$\eta=65\cdot3T^{-3/2}\ln\Lambda \quad (\Omega\mathrm{m}), \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{14})$$

where $\ln\Lambda$ is a slowly varying function of T and n (Spitzer 1956, p. 73). In the present case $\eta=3\cdot61\times10^{-6}$ Ωm. The expression is based on the assumption

* In the worst deviation from the assumption that the electron paths, associated with motion due to the current, are straight lines, the electron only traverses 20 per cent. of a complete circle, so the effect should not make any appreciable difference to the order of magnitude calculations performed here.

that the electron energy associated with the current is small compared with its thermal energy. Here, this assumption is not strictly valid, as the two energies are comparable, but the result obtained, although too high, should still be of the right order.

The density of power dissipation in the core surface is $\eta i^2=1\cdot 4\times 10^5$ W/m³. The volume of the surface layer is $4\pi a^2\delta=3\cdot 33\times 10^{-2}$ m³. Therefore, the total power dissipated in the core is $W=4\cdot 66\times 10^3$ W.

(b) *Heat Radiation from the Core*

Heat radiated from the core will now be calculated. It will be found to be many orders of magnitude smaller than the power dissipated in the core, so the following crude approximations are justified.

For the bound-bound transitions, only the Lα intensity is estimated. In estimating the order of magnitude of the density, N_{1S}, of atoms in the $1S$ state, only the transitions ionized$\rightarrow 1S$ and $1S\rightarrow$ionized are considered. The values given by Jefferies (1953) for the rate of these transitions were extrapolated to $T=4\times 10^5$ °K and used to determine N_{1S}. For the order of magnitude of the density, N_{2P}, of atoms in the $2P$ state, only the transitions $1S\rightarrow 2P$ and $2P\rightarrow 1S$ are considered. The value for the rate of $1S\rightarrow 2P$ was obtained by the extrapolation of Jefferies' values, and the rate of $2P\rightarrow 1S$ from a value given by Giovanelli (1949). The above yields

$$\left.\begin{aligned} N_{1S}&=3\cdot 13\times 10^{-7}n \quad (\mathrm{m}^{-3}),\\ N_{2P}&=1\cdot 32\times 10^{-29}n^2 \quad (\mathrm{m}^{-3}).\end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots \quad (15)$$

The total rate of transitions $2P\rightarrow 1S$ is $6\cdot 32\times 10^8 N_{2P}$, and the energy radiated per transition is $1\cdot 62\times 10^{-18}$ J. The volume of the core is $4\pi a^3/3=0\cdot 523$ m³, therefore the power radiated in the Lα line is approximately $0\cdot 44\times 10^{-3}$ W.

For free-bound radiation only the Lymann continuum is considered. Using a formula given by Giovanelli (1949), the total power radiated is found to be $1\cdot 62\times 10^{-3}$ W.

The power radiated in free-free transitions was calculated from a formula given by Woolley and Stibbs (1953, p. 236) and is $2\cdot 95\times 10^{-3}$ W.

(c) *Heat Removal from the Core*

It is seen that power radiated from the core is negligible compared with power dissipated in it. Therefore in order to obtain steady conditions some other means of energy removal must be provided.

One method of increasing power loss from the core is to place a metal surface in contact with it. Protons impinging on the surface acquire electrons, and, as neutral atoms, leave the core. Electrons also flow into the surface, and maintain its charge constant. The neutral atoms give rise to a gas of hydrogen in the space between shell and core, and this permeates the outer part of the core. Their reionization in the outer part of the core makes up for the loss due to recombination at the metal surface.

Let the metal surface take the form of a spherical cap coinciding with part of the core surface. (It could be supported by a dielectric post without excessive modification of the oscillating field.) It will be assumed that a proton, having arrived at the metal surface, captures an electron, with emission of radiation, and rebounds as a neutral atom with kinetic energy equal to the kinetic energy of the impinging proton. The lifetime against ionization of a hydrogen atom in the core is approximately 10^{-4} sec,* therefore most of the neutral atoms escape from the core and collide with the shell.

The above assumption is justified as follows. If the hydrogen atoms are absorbed on the cooling surface for some time before being ejected, their velocity of ejection should be small (as the surface would be kept at only a few hundred degrees centigrade), and most of them would be reionized in the core before they could escape. In this case the core energy loss per atom would be slightly lower than, and heating of the surface would be about double that in the previous case, but the density of molecular hydrogen would be much less. The assumpton made in the preceding paragraph, therefore, defines the worst case, and this will be treated in more detail.

If the proton temperature equals the electron temperature (4×10^5 °K), the energy lost to the core per neutral atom formed is $3kT$ (k is Boltzmann's constant) plus the ionization and molecular dissociation energy per atom, the total amounting to $1\cdot9\times10^{-17}$ J. In order to remove energy at the rate of $4\cdot66\times10^3$ W the collision rate of atoms with the cooling surface must therefore be $2\cdot45\times10^{20}$ sec^{-1}.

However, the proton temperature could be much lower than the electron temperature ; if this occurs the gas pressure in the core is about half of the value previously dealt with, and the field pressure, and therefore the power dissipation in the core, must be reduced to about half of the previous values. Energy lost to the core per neutral atom formed is also about half of the previous value, so that the turnover of protons and the neutral gas density remain the same.

It is seen that a low proton temperature does not radically alter the operation of the device, and for the time being it will be assumed that the proton and electron temperatures are equal. The relation between proton and electron temperatures will be dealt with in the concluding section.

In a layer of ionized material adjacent to the cooling surface, conditions are very different from those in the main body of the core. The thickness of this layer is of the order of the Debye shielding distance (Spitzer 1956, p. 17), approximately 10^{-4} m, which is negligible compared with the core radius.

Using the Langmuir probe theory (Cobine 1941, Section 6.6 ; also Spitzer 1956, p. 17) the potential across the layer adjacent to the cooling surface is found to be of the order of 40 V, and the flow of protons to the surface $3\cdot84\times10^{22}$ m^{-2} sec^{-1}. The area of the surface in contact with the core must therefore be $6\cdot38\times10^{-3}$ m^2. This is $0\cdot2$ per cent. of the core surface area.

The neutral atoms are assumed to form a gas of molecular hydrogen at the shell temperature. If this temperature is taken to be 400 °K (it may have

* Extrapolation of Jefferies' (1953) table for ionization rates, to $T=4\times10^5$ °K gives the rate of $1S\rightarrow$ionized as $4\cdot8\times10^{-14}\,N_{1S}\,n$ (m^{-3} sec^{-1}). Therefore the rate for a single atom is $4\cdot8\times10^{-14}n$ (sec^{-1}). On inserting the value of n, the mean life of the atom is found to be $0\cdot833\times10^{-4}$ sec.

to be above 110 °C to prevent the formation of cuprous hydride), the rate of arrival of hydrogen at the core surface is (Roberts 1940, p. 71) $5 \cdot 15 \times 10^{2} \nu$ (molecule sec^{-1} m^{-2}), where ν is the molecular density of the neutral hydrogen far from the core. This rate multiplied by the surface area of the core must equal the rate of loss of hydrogen from the core. This gives the density as $\nu = 0 \cdot 76 \times 10^{17}$ molecule/m^{3}. (The corresponding pressure is $3 \cdot 1 \times 10^{-6}$ mm Hg.)

The velocity inwards of molecules at the surface is $1 \cdot 03 \times 10^{3}$ m/sec. With a mean life against ionization in the core of $0 \cdot 833 \times 10^{-4}$ sec* the mean depth of penetration is approximately $0 \cdot 09$ m, which is small compared with the core radius.

(d) *Power Flow into the Cooling Surface*

An upper limit for the power flowing into the cooling surface is obtained by assuming that all the energy lost to the core is given initially to the surface. This yields a power density for energy flowing from the core into the surface of $7 \cdot 30 \times 10^{5}$ W/m^{2}. If the cooling surface coincides with part of the outer surface of the core, there will also be energy flowing into it from the field ; the rate is $0 \cdot 15 \times 10^{5}$ W/m^{2}. The total rate is $7 \cdot 45 \times 10^{5}$ W/m^{2}, which is not excessive.

V. Effect of Neutral Hydrogen

(a) *Power Loss from the Current-carrying Layer*

At the core surface the density of neutral hydrogen is half of the value far from the core. The presence of neutral gas of density $0 \cdot 76 \times 10^{17}$ atom/m^{3} in the current-carrying layer gives rise to extra dissipation, due to ionization and scattering.

At a great distance from a hydrogen atom the electrostatic fields of its component electron and proton cancel. Therefore, only " close " encounters of a current-carrying electron with the bound electron-proton pair are important, and the mean free path for large-angle deflection is an order of magnitude lower than that for " distant " encounters, which are the main cause of power dissipation by current flowing in a fully ionized gas (Spitzer 1956, p. 68).

The extra power flow into the core, due to the presence of neutral hydrogen at an atomic density less than the density of protons in the core, is small compared with the flow ($4 \cdot 66 \times 10^{3}$ W) assumed when calculating the density of neutral hydrogen. The results obtained will therefore not be much upset by these secondary effects, and the general picture should remain valid.

(b) *Power Loss due to Charge Exchange, Excitation, and Dissociation in the Core*

The presence of neutral hydrogen in the core gives rise to energy loss via charge exchange, for in this process a neutral H atom having the kinetic energy of a H^{+} ion is formed. This energy is then lost to the core. (The worst case occurs when the proton temperature is a maximum, i.e. equal to the electron temperature.)

* See footnote on p. 573.

For thermal energies, the mean free path for charge exchange near the core surface is greater than twice the value to be used later, viz. 41·5 m. This leads to an estimate of the power loss, via charge exchange, of less than $0 \cdot 584 \times 10^3$ W, which is small, but not inappreciable compared with $4 \cdot 66 \times 10^3$ W.

Excitation and dissociation, by thermal electrons, of neutral molecules flowing into the core can result in loss of thermal energy. An upper limit to the rate of loss can be obtained by assuming that all the energy lost by the electrons, other than that needed for ionization, is lost to the core. On extrapolating to 4×10^5 °K the values given by Jefferies for the $1S \rightarrow$ionized transition rates, the upper limit to the rate of loss is found to be $0 \cdot 7 \times 10^3$ W, which is again small, but not inappreciable compared with $4 \cdot 66 \times 10^3$ W.

(c) *Ionization between Core and Shell*

For an electron outside the core, the time-averaged force tending to drive it inwards can be calculated using Figures 2 and 3. Also, the electron is oscillating in a direction perpendicular to the radius vector, and centrifugal force arising out of the curved electric field tends to move it outwards. The net inward force is plotted in Figure 5.

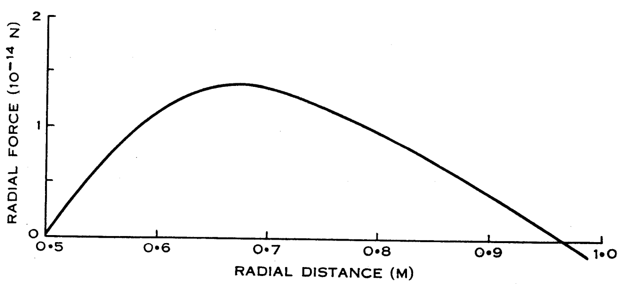

Fig. 5.—Time-averaged, radial force inward on an electron, due to the oscillating electromagnetic field, *v.* distance from the core centre. Corrected for centrifugal effect.

In order to escape, an electron at the core surface has to move through a region in which there is a time-averaged force tending to drive it back to the core. The integral of this force with respect to radial distance represents an energy barrier which an electron at the surface has to surmount in order to escape from the core. It is $4 \cdot 04 \times 10^{-15}$ J.

The more massive protons have a much smaller barrier, viz. $2 \cdot 20 \times 10^{-18}$ J.

The mean thermal energy is $8 \cdot 28 \times 10^{-18}$ J. Therefore very little leakage of electrons is likely, but there is no insurmountable barrier to most of the protons. However, as the protons escape, negative electrostatic charge builds up until the rates of escape of electrons and protons are equal. The equilibrium potential difference, V_0, between core and shell and the final escape rate can be calculated using kinetic theory formulae (Roberts 1940, pp. 63–5).

The magnitude of V_0 is found to be $1 \cdot 41 \times 10^4$ V, and the fractional rate of escape of material from the core is of the order of 10^{-170} sec^{-1}, which is quite negligible. The excess electrons are distributed just outside the core and are part of a general distribution of electrons and ions between core and shell, which will now be considered.

Electrons or protons which in some way have arrived in the space between core and shell are accelerated by the field present. They are eventually driven out of this space, but while they are there they can ionize the neutral hydrogen present, and ionization may build up. However, the limiting electron and ion densities and ionization rate are not sufficient to appreciably modify the situation treated earlier, which leads to the maintenance of a stable core, as is shown in Appendix I.

VI. Establsihing the Core

The inward radial force exerted by the oscillating field acts directly mainly on the electrons. However, as the electrons move, a space charge field appears which tends to hold electrons and ions together. Changes in the distribution of ionization are brought about by the forces acting on the electrons effectively acting on the ions as well. If a force of the order of the forces considered here is applied to a hydrogen ion, the ion will move over a distance of the order of the linear dimensions of the ionized region (1 m, say) in a time of the order of 10^{-6} sec. In this time there are some 10^2 cycles of the field, so it is legitimate to consider only the time-averaged forces exerted on the ions.

The time-averaged force on electrons and ions in a central region of linear dimensions somewhat greater than 1 m (with or without a core present) has an inward radial component. Ions and electrons produced in this region will tend to accumulate towards the centre. Outside of the region ions and electrons will be driven to the shell where they recombine and replenish the neutral hydrogen.

The numbers of ions and electrons in the central region where they are accumulating will be approximately equal. Recombination will limit the process when the recombination rate equals the ionization rate. The limit at any point near the centre is reached at the earliest (since accumulation of ions which have been produced further out is neglected) when

$$\alpha n^2 = \zeta n. \qquad (16)$$

ζ, the rate of production of ion pairs per electron, can be taken as $3\times10^3\ \text{sec}^{-1}$ (see Fig. 9), since the initial neutral gas density is not much higher than the density of the neutral gas when the steady state has been reached.

The field becomes appreciably attenuated in a distance comparable with the linear dimensions of the region from which ions are accumulating when $n=4\cdot5\times10^{14}\ \text{m}^{-3}$ (Spitzer 1956, pp. 50, 51).

At this stage $\alpha n<13\cdot5/\text{sec}\ll3\times10^3/\text{sec}$, the value of ζ, so that recombination is not yet limiting the ionization build-up. Attenuation of the field towards the centre allows the electron temperature there to drop, and the field to compress the embryo core, which extends over the region from which ions are accumulating. This can proceed until recombination becomes appreciable, when equation (**16**) yields $n=10^{17}\ \text{m}^{-3}$, and the corresponding temperature needed for the core pressure to equal the field pressure is $T=10^6$ °K. (The value adopted above for ζ is valid for electron energies down to about 400 eV, which corresponds to a temperature not far above 10^6 °K.) α, the recombination coefficient, is less than 3×10^{-14} m^3 sec^{-1} (Persson and Brown 1955).

The minimum size of the above accumulation of electrons can be estimated by taking the total number of ions in the region from which they were collected and finding the volume in which they would give the above limiting value of n. The linear dimensions of the region from which ions are being collected is of the order of 1 m ; assume it to be a sphere of radius $0\cdot5$ m. The ion density initially is of the order of $4\cdot5\times10^{14}$ m^{-3}, so that in order to bring the density up to 10^{17} m^{-3} the sphere must shrink to a radius of $0\cdot08$ m. The neutral gas density is appreciably attenuated inside the core in a distance of this order, so that a true core has been formed.

The values of n and T in the small core are close to the equilibrium values for a larger core and the small core should therefore grow in size (preserving roughly constant n, T) until it makes contact with the cooling surface and equilibrium is established.

Not much faith can be placed in any details of the above picture of the establishing of the core, but it appears that the field is adequate to form a core as well as maintain it. Starting may involve nothing more than filling the shell with hydrogen at the appropriate pressure ($3\cdot4\times10^{-6}$ mm Hg at 400 °K for equilibrium $n=2\cdot5\times10^{17}$ m^{-3}, $T=4\times10^5$ °K) and switching on the field, always keeping the driving frequencies just above resonance, however.

The rate of ionization per electron is approximately 3×10^3/sec. If the build-up starts from one electron ion pair, the time needed for an embryo core to form is of the order of $x/3\times10^3$ sec, where $2^x=4\cdot5\times10^{14}$. That is, the time needed is of the order of $0\cdot016$ sec.

In order to make a rough estimate of the time needed for the embryo to cool, assume that the excitation rate into the first excited state of hydrogen is comparable with the ionization rate. The rate of loss of energy by radiation is then obtained by multiplying the number of electrons present by the rate per electron, of ionization of neutral hydrogen, and the energy loss due to radiation from the excited state. The rate of loss is $1\cdot2$ W. The total energy contained in the embryo is found by multiplying the number of electrons present by their average energy. Taking the average energy as 1 keV (see Fig. 7), the total energy is $0\cdot038$ J. The time needed for the embryo to cool, and a true core to form, is therefore of the order of $0\cdot03$ sec.

The time needed for the core to reach full size can be estimated by dividing the number of atoms in a full-sized core by a mean rate of flow of neutral atoms into the core. The mean rate of flow can be found by multiplying the inward velocity of neutral hydrogen molecules by the atomic density of the neutral hydrogen and a mean area for the growing core. The time needed for the core to reach full size is of the order of 10^{-3} sec. Finally, therefore, the core should become established in a total time of the order of $0\cdot1$ sec.

VII. Conclusion

The temperature and density of the core need not be limited to the range considered here. In the main the limiting factor is the maximum pressure which the field can exert, which, in practice, would be determined by the power generated at the appropriate frequencies. In practice, the pressure considered here could be obtained, but it is near the limit.

Keeping the above pressure constant, a lower limit to the temperature of the core is set by the power which would be dissipated in it. The limit can be taken where dissipation in the core becomes comparable with that in the shell, for, if the dissipation were much greater, oscillators at present available could not give enough power to maintain the field.

Ln Λ is a slowly varying function of T and n (Spitzer 1956, p. 73), so to a good approximation $\eta \propto T^{-3/2}$. At constant pressure $i \propto \delta^{-1} \propto n^{1/2}$ (Spitzer 1956, pp. 50, 51) and $n \propto T^{-1}$. Therefore the power dissipated in the core is $4\pi a^2 \delta \eta i^2 \propto T^{-2}$. Thus at $T=10^5$ °K the power dissipated in the core would be $7 \cdot 5 \times 10^4$ W, so that 10^5 °K would be approximately the lower limit of temperature attainable for a core of density 10^{18} atom/m^3. Lower temperatures are attainable, however, at lower densities or with smaller cores.

The upper limit to temperature would be set by the difficulty of maintaining a low enough density of neutral hydrogen for steady state running. A core of temperature 4×10^5 °K and density $2 \cdot 5 \times 10^{17}$ atom/m^3 requires a density of neutral hydrogen corresponding to a pressure of $3 \cdot 1 \times 10^{-6}$ mm Hg at 400 °K. This low pressure should be attainable in a cavity of the dimensions considered here, but the required pressure becomes lower, and therefore more difficult to attain, as the core temperature (at constant core pressure) is raised, being $1 \cdot 2 \times 10^{-7}$ mm Hg at 400 °K when the core has a temperature 10^7 °K and density 10^{16} atom/m^3.

After the steady state has become established, most of the hydrogen towards the centre of the core is completely ionized. Conditions at the centre, however, are not strictly the same as if the core were immersed deeply in a large amount of hydrogen at the specified temperature and density. The dilution of the radiation field would be very great, the density of the neutral atoms present, though small (about 10^{-3} of the density of the ionized atoms), would be considerably above the equilibrium density, and there would be fast ions from outside oscillating through the core, together with some small current due to the confining field.

Furthermore, if only electron-proton collisions in the body of the core are important, the turnover of protons in the core is sufficiently fast so that the protons have only a few per cent. of the electron thermal energy. Although they might gain energy much more rapidly at the core boundary (Gabor, Ash, and Dracott 1955, p. 919) the proton temperature could not be guaranteed greater than 10^4 °K. Even if the proton temperature were as low as the value just given, the overall picture would not need serious modification. The main difference would be that the field pressure (and therefore the power dissipated in the shell) would be less by a factor of 2.

For many purposes only the electron temperature is important, and in some cases a low ion temperature would be an advantage. For example, there would be a reduction of Doppler broadening of the light from bound-bound transitions occurring in the ions, and a reduction in energy spread of a beam of ions drawn from the core.

Finally, it should be said that, if a high temperature core were wanted only momentarily, and the field therefore pulsed, conditions could be very different from those considered here. In some cases the ion temperature could even go higher than the electron temperature.

VIII. Acknowledgments

Thanks are due to Mr. R. I. Skinner and Mr. J. E. Drummond* of the Dominion Physical Laboratory for much constructive criticism; also to the latter for the solution of equation (**24**).

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Appendix I

Limit of Charge Density between Core and Shell

A stable distribution of excess electrons between core and shell would be such that the electrostatic field due to the excess electrons produces a force on any electron in the distribution equal and opposite to the time-averaged force due to the oscillating field, corrected for centrifugal effects.

Let n_1 be the excess electron density at any point. Now

$$\nabla\cdot(\varepsilon_0\mathbf{E}_1)=n_1e, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{17})$$

where $\mathbf{E}_1$ is the electrostatic field due to the excess electrons. The electrostatic force must balance the time-averaged force due to the oscillating field, i.e.

$$-\mathbf{E}_1e=\mathbf{F}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{18})$$

$\mathbf{E}_1$ is everywhere taken in the radial direction, as, near the core, $\mathbf{F}$ is approximately a purely radial force, therefore only the magnitudes of these vectors need be

* Now of the Applied Mathematics Laboratory.

considered in equation (**18**). With a combination of three modes as discussed previously, the vector field of **F** would be approximately spherically symmetrical. On combining equations (**17**) and (**18**),

$$n_1=\frac{\varepsilon_0}{e^2}\cdot\frac{1}{r^2}\cdot\frac{\mathrm{d}}{\mathrm{d}r}(r^2F), \qquad (\mathbf{19})$$

and is solved using the graph of F versus r in Figure 5. The result is plotted in Figure 6.

Electrons in the space between core and shell oscillate about equilibrium positions* and ionize the neutral hydrogen present. The kinetic energy of an

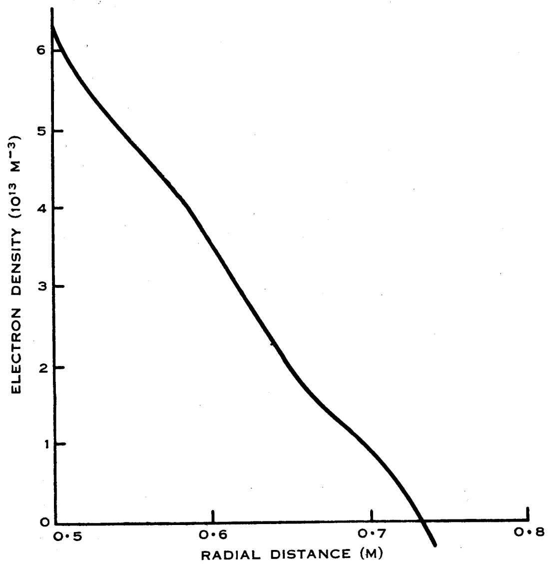


Fig. 6.—Density of excess electrons, *v*. distance from the core centre.

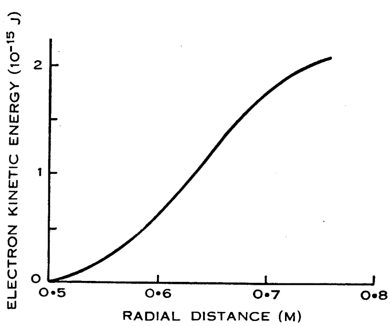
electron is oscillatory, being caused by both the oscillating electric component of the field and the fluctuating radial force on the electron. The mean kinetic energy was calculated using Figures 2, 5, and 6. It is plotted in Figure 7.

The distance travelled in unit time by an electron oscillating sinusoidally is $(4/\pi)\sqrt{(\xi_1/m)}$, where ξ_1 is its time-averaged energy. In the case here, the motion is approximately sinusoidal (since the contribution from the oscillating electric component is much greater than the contribution from the fluctuating radial force) and the distance traversed in unit time is plotted in Figure 8.

The rate of energy loss for air at S.T.P. for electrons with the above energies was first obtained, using the table on page 8 and the equations on

* The following treatment is valid, as the variation of position of an electron is small. The largest radial excursion of interest occurs at $r=0\cdot73$ m, and is somewhat less than $0\cdot01$ m, which is small compared with $0\cdot23$ m, the range of distribution of ions and electrons. The largest excursion perpendicular to the radius vector also occurs at $r=0\cdot73$ m, and is $0\cdot18$ m, which is small compared with the circumference of a circle of radius $0\cdot73$ m.

pages 9 and 19 given by Wilkinson (1950). On multiplying by 0·2 (the stopping power of hydrogen relative to air)* and the ratio of the number of hydrogen atoms present in unit volume of air at S.T.P., and dividing by the



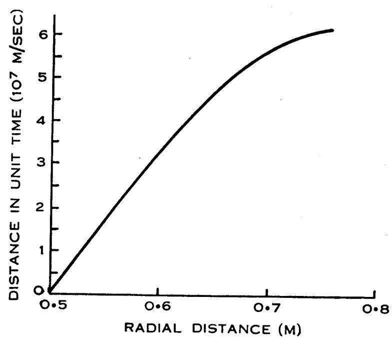


Fig. 7.—Mean energy of oscillation of electrons, *v.* distance from the core centre.
Fig. 8.—Distance traversed in unit time by an oscillating electron, *v.* distance from the core centre.

energy absorbed per ion formed, the number of ion pairs produced by an electron per unit length of path is obtained. When multiplied by the path traced out in unit time by the electron, the rate of production of ion pairs per electron, $\zeta(r)$, is obtained. This is plotted in Figure 9.

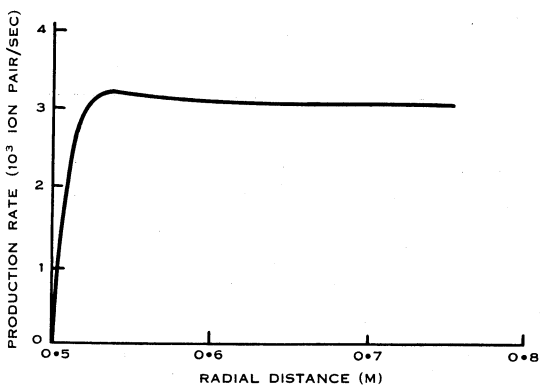


Fig. 9.—Rate of production of ion pairs per electron, *v.* distance from the core centre.

* The stopping power for $\frac{1}{2}H_2$ relative to $\frac{1}{2}N_2$ for 6 MeV α-particles is approximately 0·2 (Livingstone and Bethe 1937, p. 272). That is (ibid., p. 263),

$$\frac{\log\,(2mv_\alpha^2/I_H)}{7\cdot 22\,\log\,(2mv_\alpha^2 I_N)}=0\cdot 2,$$

where v_α is the velocity of the α-particle and I_H and I_N the respective average excitation potentials of a hydrogen and a nitrogen atom. Inserting $I_N=80\cdot 5$ eV (ibid., p. 267), yields $I_H=15\cdot 6$ eV.

Electron energies of interest range from 10^2 eV to 10^4 eV, the relative stopping power then ranges from 0·28 to 0·18, so that 0·2 is a fair estimate.

The rate of production of ion pairs per unit radial length is now obtained by multiplying the number of electrons per unit radial length by ζ. This product is $4\pi r^2 n_1 \zeta$ and is plotted in Figure 10.

So far, no effect removing electrons from the ionizing region has been revealed. However, there is a mechanism which produces drift of positive ions into the core, and an equal number of electrons follows, if the equilibrium electrostatic field is maintained. The rate of drift and the effective densities of protons and electrons will now be calculated. It will be found that the overall situation requires no serious modification.

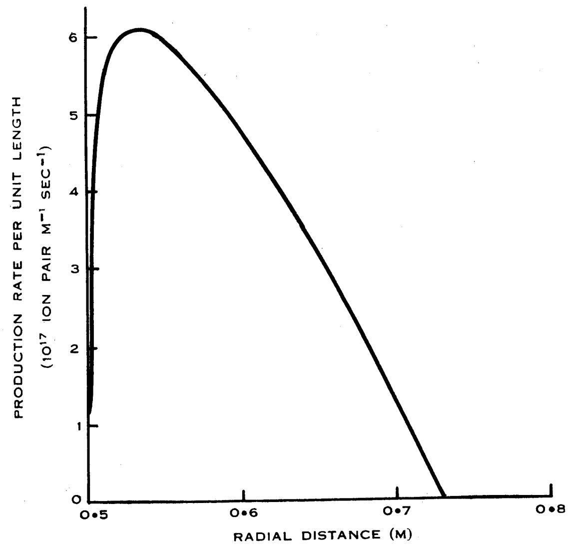


Fig. 10.—Rate of production of ion pairs per unit length, *v*. distance from the core centre.

Positive ions, under the influence of the electrostatic field due to the excess electrons, are accelerated into the core, pass through it (as their mean free path is long compared with the core diameter),* are brought to rest on the other side, and are then accelerated back through the core, and so on. Owing to charge exchange the amplitude of oscillation decreases discontinuously with time, giving rise to a drift of ions into the core.

The potential energy of a proton in the electrostatic field is plotted in Figure 11. For positive ions with this range of energy (up to $15 \cdot 3 \times 10^3$ eV) charge exchange is at least 10 times more probable than ionization (Keene 1949)

* The mean electron velocity at $T = 4 \times 10^5$ °K is $3 \cdot 93 \times 10^6$ m/sec. Together with the collision frequency (Spitzer 1956, p. 82), this defines a mean free path for large-angle deflection, $1 \cdot 55 \times 10^2$ m. The proton mean free path for large-angle deflection will also be approximately this value.

Ions accelerated outside the core, and passing through it, have energies greater than the protons comprising the core, therefore $1 \cdot 55 \times 10^2$ m is a lower limit to the mean free path of these ions. The value is much greater than the linear dimensions of the core.

so that an ion generally will be very close to the core before producing an ion pair. In the following, ionization by positive ions is neglected. Recombination is also neglected.* Also the calculation is carried through only for H^+ ions, but H_2^+ ions behave much the same in exchange and ionization (Keene 1949), and their velocities in the electrostatic field will be not much different from those of the H^+ ions. Therefore the conclusions should remain valid for a mixture of H^+ and H_2^+ ions.

Let the density of H^+ ions with zero velocity at any distance r from the core centre be $N(r)$.

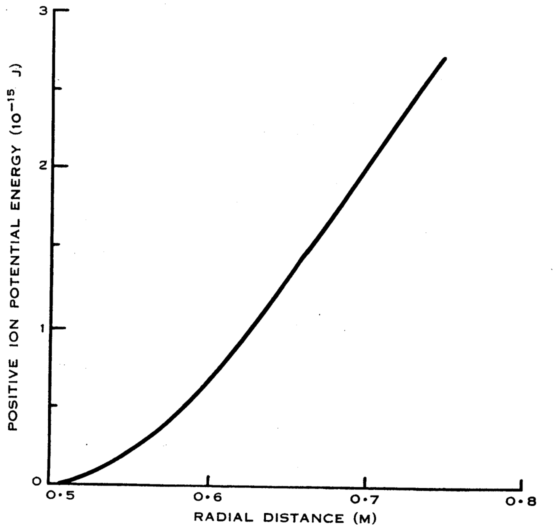


Fig. 11.—Potential energy of a singly charged positive ion in the space charge field, *v.* distance from the core centre (10^{-15} J $=6\cdot 25$ keV).

The effective charge in the region between r, $r+\mathrm{d}r$ due to protons with zero velocity between $r'(r'>r)$ and $r'+\mathrm{d}r'$ is

$$4\pi r'^2\mathrm{d}r'N(r')e\frac{4\mathrm{d}r}{\tau(r')v(r',r)},$$

where $\tau(r')$ is the period of oscillation of protons with zero velocity at r', and $v(r',r)$ is their velocity at r. This effective charge will be balanced by electron charges, and the number of electrons contributing to the balancing between r, $r+\mathrm{d}r$ is

$$4\pi r^2 n_2(r)\mathrm{d}r=4\pi\mathrm{d}r\int_r^{r_1}\frac{4r'^2N(r')}{\tau(r')v(r',r)}\mathrm{d}r', \qquad \ldots\ldots\ldots \quad (20)$$

where $n_2(r)$ is the density at r of these electrons, and r_1 is the maximum value of r for which the proton distribution exists. Owing to the drift inwards of balanced

* On using the values for electron and positive ion densities derived in this section and the recombination coefficient given by Persson and Brown (1955) ($<3\times 10^{-14}$ m^3 sec^{-1}), neglect of recombination is seen to be justified.

charges, the proton distribution is controlled by the excess electrons, so that r_1 is the maximum value of r for which the excess electron distribution exists. This is $0 \cdot 73$ m.

The total rate of production of ions by electrons between r, $r+\mathrm{d}r$ is

$$4\pi r^2 \mathrm{d}r\{n_1(r)+n_2(r)\}\zeta(r)=4\pi r^2 n_1(r)\zeta(r)\mathrm{d}r+4\pi r^2\int_r^{r_1} 4\left(\frac{r'}{r}\right)^2\frac{N(r')\zeta(r)}{\tau(r')v(r',r)}\mathrm{d}r'\mathrm{d}r. \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

These protons which appear in the region between r, $r+\mathrm{d}r$ have zero velocity.

Let $\sigma(r', r)$ be the probability per unit time spent in the region between r, $r+\mathrm{d}r$, of charge exchange at r by a proton with zero velocity at r'.

The probability per unit time of a proton with zero velocity at r' exchanging charge between r, $r+\mathrm{d}r$, is then

$$\frac{4\mathrm{d}r\,.\,\sigma(r',r)}{\tau(r')v(r',r)}.$$

The rate of charge exchange between r, $r+\mathrm{d}r$, due to protons with zero velocity between r', $r'+\mathrm{d}r'$, is therefore

$$4\pi r'^2 N(r')\mathrm{d}r'\frac{4\sigma(r',r)\mathrm{d}r}{\tau(r')v(r',r)}.$$

The total rate of charge exchange between r, $r+\mathrm{d}r$, due to protons with zero velocity at all points greater than r, is

$$4\pi\int_r^{r_1} 4r'^2\frac{N(r')\sigma(r',r)}{\tau(r')v(r',r)}\mathrm{d}r'\mathrm{d}r.$$

A similar expression is obtained for the rate of charge exchange at all points less than r due to protons with zero velocity in the range r, $r+\mathrm{d}r$.

The minimum value of r (r_0) can be taken as $0 \cdot 45$ m.

If $\lambda(r', r)$ is the mean free path for exchange between r, $r+\mathrm{d}r$, by a proton with zero velocity at r',

$$\frac{\sigma(r',r)}{v(r',r)}=\frac{1}{\lambda(r',r)}. \quad \ldots\ldots (22)$$

Thus, in a steady state,

$$n_1(r)\zeta(r)+\int_r^{r_1} 4\left(\frac{r'}{r}\right)^2\frac{N(r')}{\tau(r')}\left\{\frac{1}{\lambda(r',r)}+\frac{\zeta(r)}{v(r',r)}\right\}\mathrm{d}r'=\frac{4N(r)}{\tau(r)}\int_{r_0}^{r}\frac{1}{\lambda(r,r')}\mathrm{d}r'. \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

n_1, ζ, τ, and v can all be derived from previous equations and figures, and $\lambda=41 \cdot 5$ m (Keene 1949). These quantities were matched approximately by simple functions, giving the approximate equation for the steady state,

$$0 \cdot 646\times 10^{13}(0 \cdot 73-r)+\int_r^{0\cdot 73}N(r')\left(\frac{r'}{r}\right)^2\frac{r'-0 \cdot 5}{r'-0 \cdot 181}[1+0 \cdot 015\{(r'-0 \cdot 5)^2-(r-0 \cdot 5)^2\}^{-1/2}]\mathrm{d}r'$$
$$=N(r)\frac{(r-0 \cdot 45)(r-0 \cdot 5)}{r-0 \cdot 181}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)^*$$

* In equations (**24**) to (**27**) numerical values are in metre units.

Equation (**24**) can be solved iteratively. The solution to two terms of a series is

$$N(r)=1\cdot 23\times 10^{13}\frac{(r-0\cdot 181)(0\cdot 73-r)}{(r-0\cdot 5)r^2}\{1+0\cdot 105(0\cdot 73-r)^{1/2}\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{25})$$

(The maximum value of the second term is 1/18 of the first.) $N(r)$ is plotted in Figure 12.

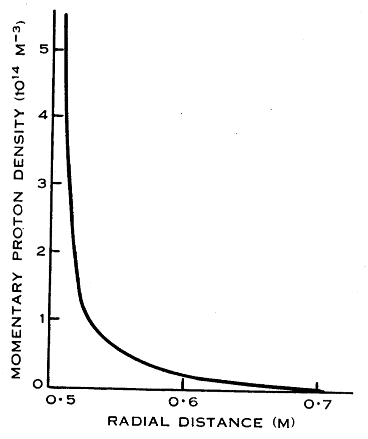
Now, from equations (**20**) and (**25**),

$$n_2(r)=0\cdot 783\times 10^{13}\int_r^{0\cdot 73}\frac{(0\cdot 73-r')\{1+0\cdot 105(0\cdot 73-r')^{1/2}\}}{r^2\{(r'-0\cdot 5)^2-(r-0\cdot 5)^2\}^{1/2}}\mathrm{d}r'. \quad \ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{26})$$

Between $r=0\cdot 51$ and $0\cdot 73$ m equation (**26**) was integrated graphically; $n_2(r)$ is plotted in Figure 13. Between $r=0\cdot 5$ and $0\cdot 51$ m the following approximation is made:

$$n_2(r)=\frac{0\cdot 185\times 10^{13}}{r^2}\cosh^{-1}\left(\frac{0\cdot 23}{r-0\cdot 5}\right). \quad \ldots\ldots\ldots (\mathbf{27})$$

Although the above treatment yields $n_2(r)$ tending to infinity as the core surface is approached, in fact the electron density distribution must merge into that of the core. However, when $4\pi r^2 n_2(r)$, obtained from equation (**27**), is integrated with respect to r between the limits $r=0\cdot 5$ and $0\cdot 51$ m the answer



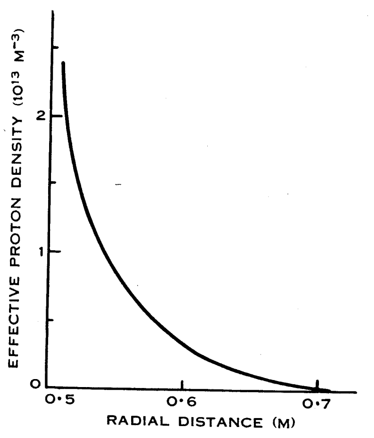


Fig. 12.—$4\pi r^2\mathrm{d}r\times$ordinate=number of protons which momentarily come to rest in the range r to $r+\mathrm{d}r$. Abscissa is distance from the core centre.

Fig. 13.—Effective proton density (n_2) *v.* distance from the core centre.

is finite. The integration gives the total number of electrons in the distribution $n_2(r)$ between these limits, as $1\cdot 12\times 10^{12}$, which is negligible compared with $7\cdot 85\times 10^{15}$, the number in the corresponding region ($r=0\cdot 49$ to $0\cdot 5$ m) just inside the core. In the region between $r=0\cdot 5$ and $0\cdot 51$ m the electrons have energy of the order of the electrons just inside the core and are in much the same circumstances. The tendency (according to the previous treatment) of $n_2(r)$

to go to infinity as $r=0\cdot5$ m is approached is therefore of little physical significance.

The total electron density $n_1(r)+n_2(r)$ reaches the critical value for the beginning of field attenuation, $3\cdot4\times10^{14}$ m^{-3}, at $r<0\cdot51$ m and actually very close to $r=0\cdot5$ m.

The rate of flow of proton-electron pairs into the core is obtained by integrating equation (**21**) between the limits $0\cdot5$ and $0\cdot73$ m.

The integral was evaluated (in part graphically) and yielded the rate as $1\cdot05\times10^{17}$ sec^{-1}, which is negligible compared with $2\cdot45\times10^{20}$ sec^{-1}, the rate of flow inwards of neutral hydrogen atoms.

To sum up, it can be said that the neutral gas present has a negligible effect on the oscillating fields and the flow of matter and energy into the core.