

THE THEORETICAL EVALUATION OF THE TOWNSEND COEFFICIENT α/p FOR HYDROGEN*

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Introduction

The theoretical evaluation of the first Townsend ionization coefficient α/p (ionizations $\text{cm}^{-1} (\text{mm Hg})^{-1}$) requires a knowledge of the precise form of the velocity distribution of electrons in the gas. This distribution was determined theoretically by Morse, Allis, and Lamar (1935) for the particular case when only elastic collisions take place in the gas, but its form is not known for the more complex situation when inelastic collision processes also occur. For this reason theoretical calculations of the coefficient α/p , which were made some years ago (Emeléus, Lunt, and Meek 1936), were based on an assumed distribution of electron velocities and the subsequent comparison with experimental values used as a criterion for assessing the validity of any particular distribution. For the particular case of hydrogen, assuming a Maxwellian distribution of velocities, values of α/p were calculated as a function of the parameter E/p ($\text{V cm}^{-1} (\text{mm Hg})^{-1}$) throughout the range $13 < E/p < 155$. These calculated values (α_c/p) were compared with those measured experimentally (α_e/p) by Ayres (1923) and the ratio α_c/α_e is shown graphically in Figure 1 (a). It can be seen that, whereas agreement between the calculated and experimental values is good for values of $E/p > 30$, there is a considerable discrepancy at the lower values of E/p . It is clear that as E/p is reduced a greater proportion of electron collisions must be elastic and for this low range of E/p Deas and Emeléus (1949) assumed an elastic collision distribution function based on the condition that the mean free path l is independent of the electron velocity u . The agreement, however, was even less satisfactory. In hydrogen, a more valid assumption may

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possibly be that l/u is constant, but even under these conditions this same elastic distribution function reduces to the Maxwellian form already considered by Emeléus, Lunt, and Meek. There appeared to be no satisfactory explanation therefore of the lack of agreement between theory and experiment at low values of E/p .

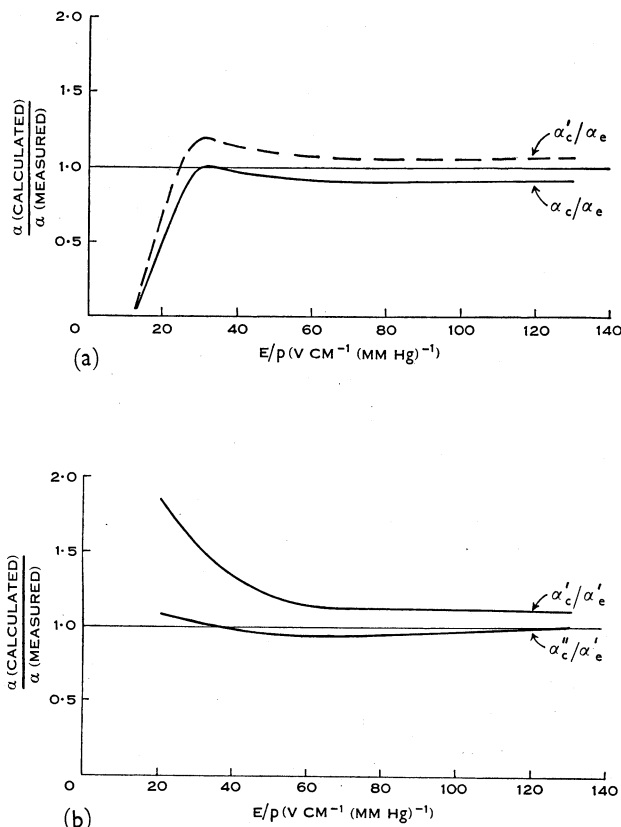


Fig. 1.—Ratio of calculated to measured values of α in hydrogen as a function of E/p . α_e =experimental values of Ayres (1923); α'_e =recent experimental values (Rose, de Bitetto, and Fisher 1956); α_c =original calculated values of Emeléus, Lunt, and Meek (1936); α'_c =values of Emeléus, Lunt, and Meek, using values of drift velocity corrected for a Maxwellian distribution of velocities; α''_c =values of α'_c corresponding to a 5 per cent. reduction in the mean electron energy.

More recently, however, the Townsend coefficient α/p_0 (ionizations $\text{cm}^{-1}(\text{mm Hg})^{-1}$ at 0°C) in hydrogen has been redetermined experimentally in several different laboratories for a wide range of the parameter E/p_0 ($\text{V cm}^{-1}(\text{mm Hg})^{-1}$ at 0°C) (see Blevin, Haydon, and Somerville 1957). As a result of these experiments it has been established that for values of E/p_0 in the range $50 < E/p_0 < 150$ small quantities of the common gaseous impurities have little influence on the measured values of α/p_0 and both recent and earlier

measurements (Ayres 1923) are in very good agreement. At lower values of $E/p_0 < 50$, however, considerable differences occur between the two sets of measurements. It is therefore clearly necessary to reassess the theoretical values of α_c/p in this particular range and, in view of the fact that in hydrogen the particular assumption that l/u is constant is valid over an extended range of E/p values, it seems particularly desirable to establish the validity or otherwise of a Maxwellian form of velocity distribution.

The Evaluation of α_c/p

The values of α_c/p used in determining the ratio α_c/α_e for Figure 1 were calculated assuming a Maxwellian distribution and using the experimental data (Tate and Smith 1932) for the efficiency of ionization $P(V)$. The expression for α_c/p is then

$$\alpha_c/p = 1.23 \times 10^8 W^{-1} \bar{V}^{-1.5} \int_{V_i}^{\infty} P(V) \cdot V \cdot \exp(-1.5V/\bar{V}) dV, \dots (1)$$

where \bar{V} is the mean energy of agitation of the electrons, W is the average electron drift velocity in the direction of the electric field E , and V_i is the ionization potential.

The mean energy \bar{V} and drift velocity W were taken from the measured values of Townsend and his co-workers. In addition a correction term was applied to the mean energy \bar{V} to allow for deviations from an isotropic velocity distribution. At low values of E/p_0 this correction was negligibly small and became appreciable only at the larger values of E/p_0 . Before we can proceed to compare values calculated in this way by Emeléus, Lunt, and Meek (1936) with recent experimental determinations two comments are necessary:

1. The values of \bar{V} and W quoted by Townsend were calculated from formulae which had not been corrected for a distribution of velocities. Consequently, in calculations of α_c/p by the above procedure, correction must be made to the values of the drift velocity given by Townsend (Healey and Reed 1941).
2. When comparing values (α_c/p) thus calculated with the measured values it is necessary to correct the latter to the temperature (15 °C) at which Townsend determined the drift velocity and mean energy.

The effect of correction 1 is illustrated in Figure 1 (a) by the dashed curve.

Comparison of Corrected and Recent Experimental Values of α_c/p

Values of α'_c/p measured recently (Rose, de Bitetto, and Fisher 1956; Blevin, Haydon, and Somerville 1957) have been used for comparison with the corrected values (α_c/p) and are shown as a variation of α'_c/α'_e with E/p in Figure 1 (b).

The most striking feature of this variation is the absence of a maximum value at $E/p \sim 30$, the ratio α'_c/α'_e now decreasing monotonically with increasing E/p . This is particularly significant in that slight modification to the assumed Maxwellian distribution function itself and/or a small systematic change in the mean energy \bar{V} (see Hopwood, Peacock, and Wilkes 1956) can now lead to better agreement with experiment throughout the whole range of E/p . In order to

estimate the magnitude of the change required in the mean energy \bar{V} , an approximate expression

$$P(V) = \beta(V - V_i)$$

may be assumed for the efficiency of ionization $P(V)$ so that

$$\alpha/p = 5.48 \times 10^7 \beta W^{-1} \bar{V}^{0.5} (V_i + 1.33 \bar{V}) \exp(-1.5 V_i / \bar{V}),$$

from which it follows that

$$\frac{\Delta(\alpha/p)}{(\alpha/p)} = \frac{1.5(x^2 + 5x/3 + 1.33)}{x + 1.33} \frac{\Delta \bar{V}}{\bar{V}} = K \frac{\Delta \bar{V}}{\bar{V}},$$

where $x = V_i / \bar{V}$.

The factor K is larger the lower E/p , and when the percentage change in \bar{V} is 5 per cent. then the percentage change in α'_c/p is ~ 50 per cent. at $E/p = 30$ and ~ 15 per cent. at $E/p = 100$. These are just the sort of percentage changes required to make the agreement between calculated and experimental values satisfactory.

In order to make detailed calculations of the influence of \bar{V} using the exact data given by Tate and Smith for $P(V)$ the integral in equation (1) must be evaluated graphically. In the present analysis the graphical evaluations made by Emel  us, Lunt, and Meek (1936) were assumed, and with that information the effect on the calculated values α'_c/p of a systematic reduction of 5 per cent. in the mean energy \bar{V} throughout the range E/p up to 130 was determined. The results of such calculations (α''_c/p) are also shown in Figure 1 (b), where it can be seen that the agreement is now very good and within the limits of experimental error throughout the whole range of E/p considered.

Possible Sources of Error in the Theoretical Evaluation of α/p

The following comments on the sources of error in the theoretical evaluation may be made at this stage:

1. It is possible that the values of mean energy used by Emel  us, Lunt, and Meek (1936) may in fact be systematically in error by as much as 5 per cent.
2. On the other hand the mean energy values may be correct, in which case a slight modification of the form of the distribution function would be necessary.
3. It may be that the values for the efficiency of ionization $P(V)$, obtained experimentally by Tate and Smith and used in this analysis, are not those appropriate to the particular gas samples used for measuring values of α'_c/p experimentally.* Thus, although the distribution function may be correct, small amounts of impurity gas at low values of E/p may result in an effective efficiency $P(V)$ significantly different from that appropriate to a pure gas. Information about the nature and precise influence of impurities and the respective ionization efficiencies would be required for further analysis along these lines.

* *Note added in Proof.*—Recent work (Craggs, Thorburn, and Tozer 1957) has indicated that Tate and Smith's values for $P(V)$ may be too high for electron energies near the ionization potential. Such an error would have the greatest effect at low values of E/p and could explain the divergence between experimental and calculated values of α/p . In view of this a redetermination of $P(V)$ in pure hydrogen is also necessary.

Conclusions

Several important conclusions may be drawn from the above analysis, namely :

1. If we assume a Maxwellian distribution of velocities for the electrons and the Townsend values of mean energy and drift velocity corrected for this distribution, the calculated values of α/p are from 10 to 20 per cent. higher than the recent experimental determinations for the range of E/p values from 50 to $130 \text{ V cm}^{-1} (\text{mm Hg})^{-1}$.

2. At lower values of E/p with similar assumptions and corrections the calculated values are considerably higher than the recent measured values, being ~ 85 per cent. higher at $E/p = 21 \text{ V cm}^{-1} (\text{mm Hg})^{-1}$.

3. Of the three possible sources of error in the calculated values the assumed values for the mean energy \bar{V} are probably the most important. It has been shown that a systematic error of only 5 per cent. in these measured values is sufficient to reduce the calculated values to within <10 per cent. of the measured values throughout the range $20 < E/p < 130$ for which the calculations were made.

4. In view of this it is clearly necessary in attempting to establish the actual velocity distribution of electrons in gases by comparison of calculated and experimental values of α/p , to determine to a higher order of accuracy values of the mean energy \bar{V} and of α_e/p in the same samples of gas. This is particularly so for conditions corresponding to low values of E/p .

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