GAIN MEASUREMENTS OF LARGE AERIALS USED IN INTERFEROMETER AND CROSS-TYPE RADIO TELESCOPES

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Summary

A method has been developed for measuring the gain of large interferometer and cross-type radio telescope aerials. Use is made of the strong discrete radio sources, whose intensity need not be known, to allow comparison of the gains of the aerials with that of a standard.

The aerials of the 3·5 m Mills Cross radio telescope at Sydney have been calibrated in this way, using a dipole with a plane reflector as a standard. The five radio sources, Pictor-A (IAU 05S4A), Hydra-A (IAU 09S1A), Hercules-A (IAU 16N0A), Virgo-A (IAU 12N1A), Taurus-A (IAU 05N2A), were used in the calibration and the flux densities of these sources were then derived.

I. Introduction

Although many improvements in radio telescopes have been made over the last few years, the accurate measurement of the flux densities of discrete sources still presents a challenging technical problem. A solution to this can only be achieved by careful attention to the calibration of both receiving and aerial systems. The calibration of the latter is particularly difficult.

If the aerial is large, the resulting narrow beam and high gain combine to give a high signal-to-noise ratio and good discrimination against sources other than the one under observation. For accurate flux density measurements both these factors are important, but for large aerials they are counterbalanced by the difficulty of determining the gain with precision. Computations of the gain of such aerials are often unsatisfactory because of difficulties in accounting for the effects of ground reflections, losses, and mutual impedances, whilst direct gain measurement by conventional methods is a formidable task and is quite often impossible. If, on the other hand, small aerials are used, the situation is reversed. The gain may be calculated and measured quite accurately but the signal-to-noise ratio is poor and the low angular resolution results in confusion between sources, so that flux density measurements are again unreliable.

However, as will be shown, it is possible to calibrate large aerials in terms of a small one whilst still retaining the best features of each system. The method is particularly applicable to the calibration of those interferometer or cross-type aerial systems which have two or more large aerials. These two and the smaller standard aerial can be used to form three interferometer pairs, and, from three separate observations of a discrete source, the gains of the two large aerials

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can then be compared with the known gain of the smaller. Since it is a comparison method, neither the absolute strength of the source nor the absolute calibration of the receiver need be known. Thus, used in conjunction with accurate receiver calibration techniques, this method allows accurate flux density measurements to be made.

The aerials of the 3·5 m Mills Cross (Mills *et al.* 1958, in press) at Sydney have been calibrated in this way using a half-wave dipole above a plane reflector as a standard. Five of the strongest radio sources were used in the calibration, and from the measured values of the gain the flux densities of these sources have been calculated.

II. Method

Consider the output of a phase-switched two-aerial interferometer with a square-law detector. The increment of receiver output power due to a source of flux density S in each aerial beam is given by

$$\Delta P_1 = KS\sqrt{\{G_A(\theta, \Phi)G_B(\theta, \Phi)\}}, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where K is a constant and the square-root factor is the geometric mean of the aerial power gains. θ and Φ are the coordinates of the source relative to the aerials, measured in the horizontal and vertical planes respectively.

If a third aerial of known gain $G_C(\theta, \Phi)$ is now paired with each of the aerials, two more equations are obtained,

$$\Delta P_2 = KS\sqrt{\{G_A(\theta, \Phi)G_C(\theta, \Phi)\}}, \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\Delta P_3 = KS\sqrt{\{G_B(\theta, \Phi)G_C(\theta, \Phi)\}}. \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

The measurable quantities ΔP_1, ΔP_2, ΔP_3 may be expressed in terms of equivalent diode noise generator currents I_1, I_2, I_3, and using (2) and (3) with (1) we have

$$G_A(\theta, \Phi) = G_C(\theta, \Phi)(I_1/I_3)^2, \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$G_B(\theta, \Phi) = G_C(\theta, \Phi)(I_1/I_2)^2. \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Thus the gains of two of the aerials are obtained in terms of the measurable power ratios (I_1/I_3), (I_1/I_2), and the gain of the third aerial.

The method may be applied to both steerable and transit type instruments, but when applied to the latter we need consider only one of the coordinates, the elevation Φ, as variable ; the coordinate θ is fixed usually at $\theta=0$, where the gain in this direction is a maximum. It is essential to measure the gain as a function of Φ, the elevation, because it is not necessarily constant. Tiltable aerials, for example, are affected by ground reflections for low elevations, whilst fixed arrays of dipoles are subject to gain changes as the phasing of the elements is altered to swing the beam from the vertical. Thus, when this method of aerial calibration is applied to transit instruments, a calibration curve as a function of Φ can be obtained by using several sources each at a different elevation.

As already noted, the equations given above are independent of the strength of the source and the absolute calibration of both the receiver and noise generator.

Nevertheless, it is desirable to use strong sources for good signal-to-noise ratios. At the same time, the sources should be much smaller in angular size than the narrowest aerial beam, otherwise uncertain corrections have to be applied. The detector must be a square-law one for the above equations to hold ; if a linear detector is used, the equations have to be modified to include the effects of the background radiation on the amplitude of the recorded signal. This correction would be different for different combinations of aerials.

In the foregoing, it has been assumed that only one source at a time is present in the aerial beam, that is, there is no confusion. Although this is not true of the smaller standard aerial, enough discrimination can be provided by the other aerial of the interferometer pair if it is sufficiently directive. Consequently, the use of the method is restricted to the calibration of pairs of large aerials such as are found in the large cross or interferometer-type radio telescopes. At short wavelengths, where the Sun is by far the brightest object in the sky, smaller aerials could be calibrated by the method, using the Sun without danger of confusion effects, although for these aerials conventional methods of calibration may be more suitable.

The accuracy of this method is limited by the signal-to-noise ratio. An accuracy of 5 per cent. in the noise generator current ratios is required for the gain to be obtained to 10 per cent., and in the presence of noise such accuracies can be difficult to achieve. High signal-to-noise ratios are therefore required. A further difficulty is introduced by ionospheric scintillations, which effectively add to the noise fluctuations and in serious cases can even render an observation useless.

These difficulties can be overcome by taking a sufficient number of observations.

III. Application to a Cross Aerial

The method has been applied to the calibration of the 3·5 m Mills Cross aerial at Sydney. The experimental arrangement is shown in Figure 1. A half-wave dipole and plane reflector were placed at a distance from the centre of the Cross and pairs of aerials were connected to a phase-switched receiver to be described elsewhere (Mills *et al.* 1958, in press). The output of this was displayed on a Speedomax recording millivoltmeter.

The choice of a standard aerial appears to be limited to either a dipole-reflector combination or a horn. Seeger (1956) has shown what can be achieved with a horn at short wavelengths, but when used at a long wavelength they are clumsy and are susceptible to ground reflection effects, which in the case of the dipole with reflector can be eliminated altogether. Hence this type of aerial has been used here. The dipole was made of $\frac{1}{4}$ in. diameter copper tubing, spaced 0·21 wavelength above a wire mesh reflector which measured approximately 2 by $1\frac{1}{2}$ wavelengths. This whole structure was placed on the ground, which then acts as an infinite extension of the reflecting screen. From the conventional definition for directivity (see for example Schelkunoff and Friis

(1952)) the following expression for the gain of this system in the direction Φ can be derived

$$g_A = 4\frac{R_D}{R_A} g_D \sin^2\left(\frac{2\pi a}{\lambda}\cos\Phi\right), \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

where the g's are the isotropic gains and the R's the aerial input resistances. The subscript D refers to a dipole in free space and A to a dipole at a distance " a " wavelengths above a plane reflector. λ is the wavelength. The angle Φ is measured from the vertical in the north-south plane which is normal to the axis of the dipole. The response of the dipole in this direction is a constant. The calculated maximum value for g_A was $6 \cdot 0$, neglecting ohmic losses, which amount to less than $\frac{1}{2}$ per cent. for the aerial used.

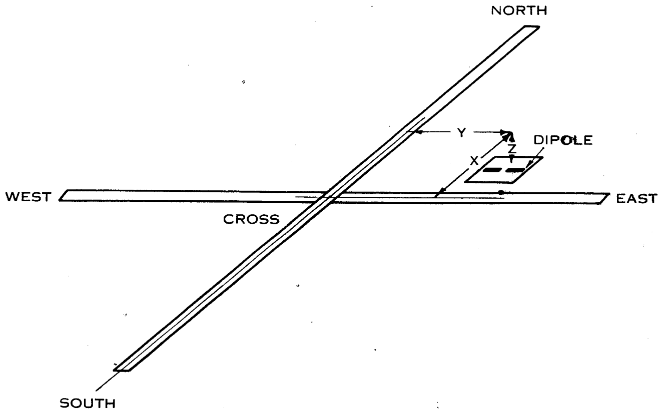

Fig. 1.—Experimental arrangement of aerials.

The loss in the cable connecting the dipole to the receiver was found to be $2 \cdot 3 \pm 0 \cdot 1$ dB by an insertion loss measurement made between a matched load and a matched diode noise generator.

Two dipoles were constructed. One was half a wavelength long and was matched by a Pi network of known loss at a point half a wavelength from the input terminals. The other was matched by shortening the dipole and by placing balanced series reactances at the dipole terminals. This dipole was thus shorter than a half wavelength by 6 per cent.

The relative gains of these two were checked by observations of the quiet Sun using one dipole at a time with the north-south arm of the Cross. An interference pattern was obtained in the way to be described later and the dipoles were interchanged at transit above the same reflector.

The gains were found to be the same to within 3 per cent., and for mechanical reasons the shortened dipole was used in the following observations.

The three aerial combinations possible with this arrangement produce three distinct types of record as shown in Figure 2. Because of the spacing between the dipole and the centre of the Cross, an interference pattern is produced. This is shown in case (a), where a large number of fringes are observed due to the broad east-west pattern of the north-south aerial. In case (*b*) the narrow pattern of the east-west aerial allows only one fringe to be observed. Case (*c*) is the normal pencil-beam record of the Cross.

For these measurements it was decided to use the strong radio sources, Pictor-A (IAU 05S4A), Hydra-A (IAU 09S1A), Hercules-A (IAU 16N0A), Virgo-A (IAU 12N1A), and Taurus-A (IAU 05N2A), since these culminate at different zenith angles and therefore allow the gain to be obtained as a function of elevation. The use of the Sun is excluded because it is larger in angular size than the aerial beams used in the Cross.

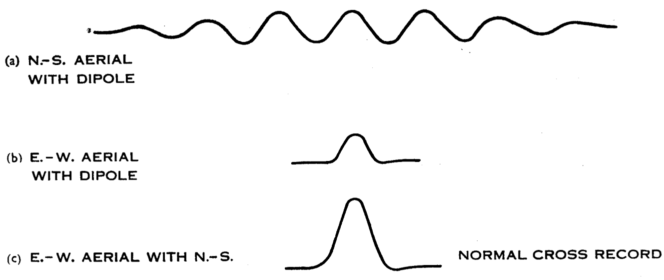

Fig. 2.—East-west patterns produced by different aerial combinations.

Three separate records are then required for each of these sources, but because the cross is a transit instrument it was impossible to get them all on one day. However, it was possible to get two records without losing much information, by first setting up the north-south aerial with the dipole and then near the time of transit changing over to one or other of the remaining aerial combinations for 10 min.

TABLE 1

TOTAL NUMBERS OF RECORDS TAKEN FOR EACH AERIAL COMBINATION AND SOURCE

Source	Aerial Combination		
	Cross	E.-W. Dipole	N.-S. Dipole
Pictor-A, 05S4A	3	3	4
Hydra-A, 09S1A	3	6	4
Hercules-A, 16N0A	6	5	2
Virgo-A, 12N1A	5	4	7
Taurus-A, 05N2A	3	6	3

The numbers of records taken for each aerial combination and source are given in Table 1.

A recording of the source Taurus-A is reproduced in Figure 3. For this case both the north-south with dipole and Cross aerial combinations were used. Noise fluctuations are not troublesome for this source.

A second sample record is shown in Figure 4, and was taken of the source Hydra-A using the north-south with dipole and east-west with dipole aerial combinations. Noise fluctuations are in this case more important. This is particularly so for the single peak observed using the east-west aerial with the dipole. Also, this part of the record had barely enough time to establish a base level from which to measure the height of the peak. As a result of these two factors additional records of the type shown in Figure 5 were taken for all sources.

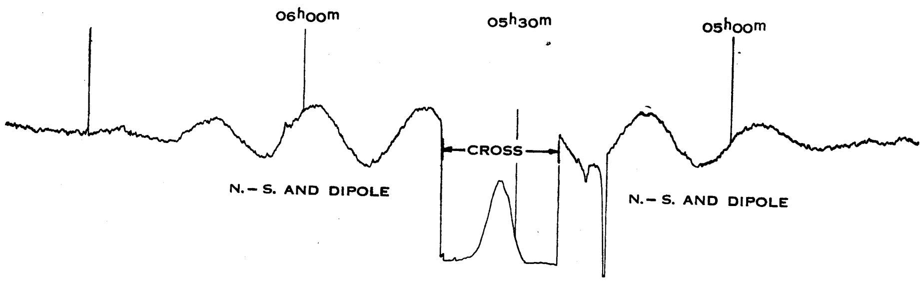

Fig. 3.—Records taken on August 21, 1956 of the discrete radio source Taurus-A using the aerial combinations as shown. The large negative spike at about 0520 is due to interference.

Those illustrated were taken on consecutive days of the Hydra-A source using only the east-west with dipole aerial combination. Some of the variations are due to scintillations and some to noise. Nevertheless, the smoothed amplitudes of these records do not differ by more than 10 per cent.

In the absence of scintillations, the normal Cross records, such as shown in Figure 3, are reproducible from day to day with an accuracy of 3–4 per cent.

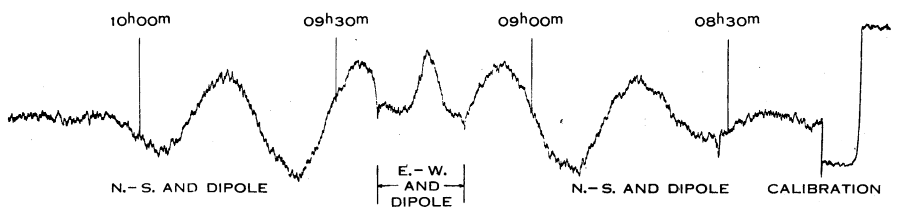

Fig. 4.—Records taken on May 15, 1956 of the discrete source Hydra-A using the aerial combinations as shown.

From all these records it can be seen that the signal-to-noise ratios obtained during these measurements are adequate for calibration purposes, without having to make large numbers of observations. One of the sources, Hercules-A, showed confusion effects. These became apparent when different readings were obtained for different positions of the test dipole with respect to the Cross arms. This source is in a region of strong galactic radiation, and the effect was due to confusion between local concentrations in this radiation and the source. However, such confusion was reduced to negligible proportions when the position of the test dipole was fixed in the position $Y=15\cdot8\lambda$, $X=11\lambda$, $Z=-0\cdot296\lambda$. The coordinate system is shown in Figure 1.

In general, confusion effects due to extended sources may be reduced by using sufficiently large aerial spacings.

All the measurements were taken with the dipole in the position just given, with the exception of those of Pictor-A, which had been completed with the dipole at $Y=13\cdot4\lambda$, $X=1\cdot85\lambda$, $Z=-0\cdot28\lambda$ before the difficulty with the Hercules-A source arose.

A calibration signal similar to that shown in Figure 4 from a diode noise generator was placed on each record. A square-law detector was used and hence the recorded source amplitudes could then be measured off directly as an equi-

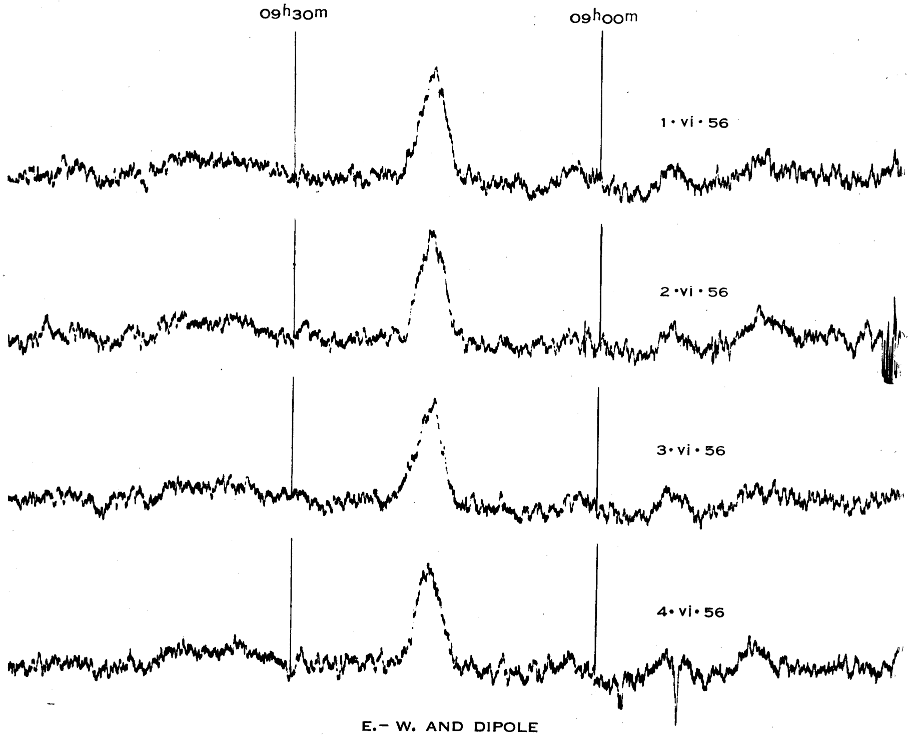

Fig. 5.—Four consecutive records of the source Hydra-A using the east-west aerial with a dipole.

valent noise generator current. A correction was required, however, for the change in amplitude of the interference fringes away from transit. This correction was obtained from a mean of several curves of the observed envelope of the fringe pattern.

From these measurements on the five sources the gains of each arm of the Cross aerial were calculated using equations (4) and (5). Now the gain of the Cross aerial as a whole is given by

$$G_{\text{cross}}=2\sqrt{(G_A G_B)}, \quad \ldots\ldots\ldots\ldots \quad (7)$$

where G_A and G_B refer to the gains of the north-south and east-west arms respectively.

Substituting from equations (4) and (5)

$$G_{\text{cross}}=2G_C\frac{I_1^2}{I_2I_3}. \qquad (8)$$

The values so obtained are given in Table 2. The probable errors of these values are also given and have been obtained from the observed probable errors of the variables in (8).

These results show that the gain of a cross aerial can be determined at suitable points with a probable error due to random effects of about 5 per cent. Systematic errors are believed to be no greater than 10 per cent.

TABLE 2

MEASURED GAIN OF 3·5 M CROSS AERIAL AT DIFFERENT ELEVATIONS AND FLUX DENSITIES OF SOURCES USED FOR THE CALIBRATION

Source	Elevation	Effective Gain of Cross Aerial over Isotropic	Flux Density (10^{-24} W m^{-2} (c/s)$^{-1}$)
Pictor-A, 05S4A	South 78°	820±34	5·7
Hydra-A, 09S1A..	North 68°	640±28	6·7
Hercules-A, 16N0A	North 51°	390±22	8·9
Virgo-A, 12N1A	North 43½°	297±8	24·3
Taurus-A, 05N2A	North 34°	180±10	23

IV. Source Flux Densities

Having thus determined the gain of the aerial we are now in a position to calculate the flux densities of the sources used for calibration. To do this, the calibration of the receiver was checked, and the values of the flux densities are given in Table 2.* The values refer to both planes of polarization and have a probable error of 10 per cent.

It appears that these values are higher than those previously quoted, as reference to Pawsey (1955) and Shakeshaft *et al.* (1955) will show. The reason for this could be that early measurements were affected by confusion effects due to the poor resolving power of the instruments. Furthermore, most other instruments have not been calibrated as a function of elevation and hence ground reflection effects could cause some of the differences which are observed. It is felt therefore that these new values are not subject to the same uncertainties.

V. Acknowledgments

The author wishes to acknowledge the valuable discussions with Mr. B. Y. Mills during the project and the helpful advice and criticism of Dr. J. L. Pawsey in the preparation of the manuscript.

* Baldwin and Smith (1956) have suggested that the Virgo-A source is surrounded by a corona, but our measurements point rather to an extension of the source in the general direction of the jet of the associated nebula M87; this does not have any appreciable effect on the flux density given here.

VI. References

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