A STUDY OF "SPREAD-F" IONOSPHERIC ECHOES AT NIGHT AT BRISBANE

IV. RANGE SPREADING

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Summary

In the course of investigations of satellite echoes from the F region of the ionosphere, it was noted that the F and E_s traces recorded at night, on $h't$ equipment at frequencies well below vertical, are broader than anticipated and tend to change in a characteristic manner as the gain of the receiver is lowered. In this paper, a quantitative explanation of these phenomena is elaborated, based on the postulate of a " rough " ionosphere.

This theory leads to a method whereby, from the swept-gain $h't$ records, estimates of roughness index can be formed. These estimates compare satisfactorily, on a statistical basis, with estimates by other methods. The theory is extended to the case of multiple-hop reflections, and to the satellite traces ; general agreement with experiment is found. Evidence is presented that the ionosphere appears rougher when transmitter and receiver are adjacent than when they are widely separated, and a tentative explanation is suggested. From the roughness indices, the relative intensities of Z- and O-mode F echoes for Brisbane are computed and the rare appearance of Z-traces on Brisbane records is satisfactorily explained.

I. Introduction

The fixed-frequency $h't$ recorder described by McNicol, Webster, and Bowman (1956) in Part I of this series of papers gives F traces which in almost all cases are substantially broader than would be anticipated from the duration of the pulse, even when allowance is made for the finite bandwidth of the receivers used and for the finite size of spot of the cathode-ray tube. In the swept-gain records discussed by those authors it is found that, as the gain decreases, the trace contracts until, just before extinction, it is no wider than would be anticipated on the basis of pulse duration, receiver response, and C.R.T. spot size (cf. Plate 1, Fig. 1). Occasionally, at a somewhat higher gain, the trace divides, indicating the presence of a " satellite " (McNicol, Webster, and Bowman, loc. cit.) which was not separated from the main trace when the gain was maximal (Plate 1, Fig. 2). If we restrict consideration to those swept-gain records in which there is no such unresolved satellite, it is found that the patches on the swept-gain records tend to take up a shape approximating to a trapezium (cf. Fig. 1). Individual patches depart from this shape, but the average shape of a number of successive patches resembles it reasonably well. The angle BAD (Fig. 1) can change markedly from one hour to the next, but the angle ABC is more nearly constant. The distance BE (where angle CDE is made equal to BCD) usually

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corresponds, roughly, to the trace width calculated from pulse duration and receiver characteristics. Examination of swept-gain records of double-hop and triple-hop echoes shows similar effects and such effects also appear frequently (*a*) on the records of " satellites " and (*b*) on records obtained when there is 100 km separation between transmitter and receiver.

In the present paper a tentative theory of this broadening is worked out, on the basis of a uniformly rough ionosphere, employing assumptions similar to those used by Briggs and Phillips (1950) in their study of the fading of reflected pulses, and quantitative experimental data obtained from swept-gain records are then discussed in relation to this theory. While this paper was in preparation, Moore and Williams (1957) have published an analysis of a somewhat analogous problem (ground scatter of radio emissions from an airborne transmitter). Their results appear to be generally consistent with those given here, but a detailed comparison is scarcely feasible.

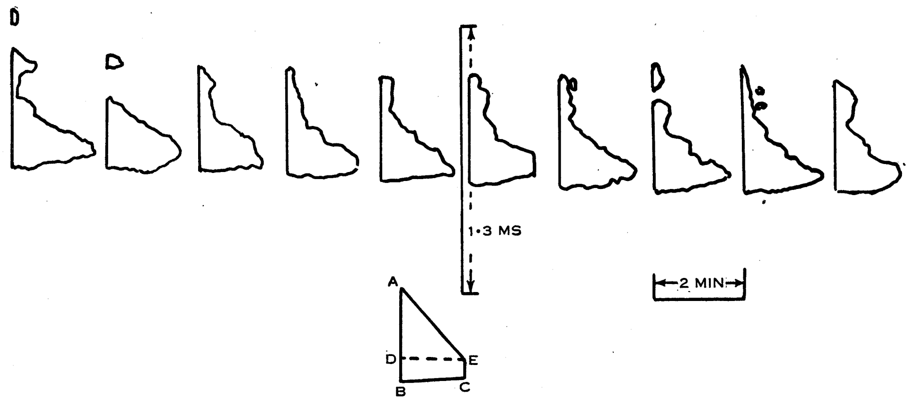

Fig. 1.—Outlines of swept-gain patches and their idealized trapezoidal form.

II. General Theoretical Considerations

Briggs and Phillips (1950) show that the energy which is returned to a transmitting station by a small element of area of the ionosphere subtending a small solid angle $d\Omega$ at the transmitter and located at an angular distance θ from the zenith is given by

$$dW = \varkappa \cos^n \theta \, d\Omega,$$

where $\varkappa$ is a constant depending on the transmitter power, antenna gain, etc. and n is an exponent dependent on the state of roughness of the ionosphere, antenna patterns, etc. This conclusion is based on reasonable assumptions and leads to a consistent explanation of fading data.

They also assume that the total energy received from the whole ionosphere is obtained by adding the power contributions from the different elements. This amounts to assuming that the signals scattered from different areas have random differences of phase. In the case of specular reflection, this cannot be true, but in that case the effective value of n is so large that no discrepancy arises. The smaller the value of n the more nearly the ionosphere approaches complete roughness. Values of n below 3 are impossible and values below 10 very improbable, under the conditions of observation used in the Brisbane experiments.

It will be assumed that the dispersion of the ionosphere is so small that it does not affect the pulse width. This is not valid near the penetration frequency.

On the basis of these assumptions and assuming also an ionosphere devoid of major irregularities or curvatures, but having such minor irregularities as

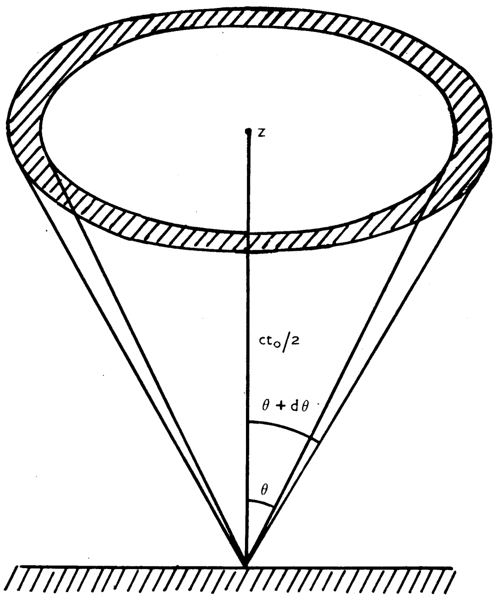

Fig. 2.—Scattering from a rough ionosphere, transmitter and receiver contiguous. Z is point on reflecting surface vertically above station.

to lead to a finite value for n, it follows immediately that the energy received back in a time dt from a strip of the ionosphere bounded by cones of semi-angles θ and $\theta+\mathrm{d}\theta$ (Fig. 2) must be proportional to $F(\theta)$ where

$$F(\theta)\mathrm{d}\theta=2\pi \cos^n \theta \sin \theta \,\mathrm{d}\theta. \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

The time taken for this radiation to travel from the station and back is given by

$$t=t_0 \sec \theta,$$

where t_0 is the time taken for vertically reflected radiation.

If the ionosphere behaved as a specular reflector, the shape of the received pulse would depend only on

(*a*) the shape of the pulse emitted by the transmitter;
(*b*) distortion (due to finite bandwidth) of the receiver.

It is convenient to lump these two effects together, assuming a distortionless receiver with a transmitter pulse adjusted in shape to correspond to the pulse as it appears in the receiver when specular reflection has occurred. This procedure would appear to be valid if the receiver is linear. We shall represent the shape by the arbitrary power function $P(\tau)$, i.e. $P(\tau)$ is the radiated power at time τ after the beginning of the pulse.

At a time t after the start of the pulse, where $t > t_0$, radiation which originated at the start of the pulse will be arriving from a ring on the ionosphere at an angle θ_1, given by

$$t = t_0 \sec \theta_1.$$

At the same time t, radiation will also be arriving after reflection from more nearly zenithal ionospheric rings; this radiation will have been emitted at a later time τ within the pulse. In fact, all parts of the pulse, up to a time τ_1, will be contributing, where

$$\tau_1 = t - t_0,$$

and corresponds therefore to zenithal reflection. Still later parts of the pulse do not contribute to the radiation being received at time t. If $t - t_0 > \tau_0$, the whole pulse contributes.

The energy ΔW returning in an interval Δt at time t from an infinitesimal ring enclosed between cones of angles θ and $\theta + \mathrm{d}\theta$ $(\theta < \theta_1)$ is therefore proportional to the energy emitted during the corresponding interval τ to $\tau + \Delta\tau$, where

$$\tau = t - t_0 \sec \theta,$$
$$\tau + \Delta\tau = t + \Delta t - t_0 \sec \theta.$$

Since it is also proportional to $F(\theta)\mathrm{d}\theta$ we may write

$$\Delta W = kP(\tau)\Delta\tau F(\theta)\mathrm{d}\theta = kP(t - t_0 \sec \theta)\Delta t F(\theta)\mathrm{d}\theta,$$

where k is a constant.

The total rate of return of energy at time t may be obtained by adding the contribution of the several rings and thus becomes

$$I = k\int_0^{\theta_1} P(t - t_0 \sec \theta)F(\theta)\mathrm{d}\theta. \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

The shape of the swept-gain patches is determined by the dependence of $\log I$ on t. In order to evaluate the integral we must make some assumptions regarding the form of $P(\tau)$.

III. Ideal Pulse

It is instructive to consider first the idealized case where the pulse is strictly rectangular in shape. We shall take its duration as τ_0. Thus

$$\begin{aligned} P(t - t_0 \sec \theta) &= P \qquad (t_0 + \tau_0 > t > t_0), \\ &= 0 \qquad (t > t_0 + \tau_0). \end{aligned}$$

Two cases arise, according to whether $t - t_0 \gtreqless \tau_0$, i.e. $\tau_1 \gtreqless \tau_0$.

Case A. $0<\tau_1<\tau_0$. Then the function $P(t-t_0 \sec\theta)$ is the same for all values of θ less than θ_1, and

$$I=2\pi kP\int_0^{\theta_1}\cos^n\theta\sin\theta\,d\theta$$
$$=\frac{2\pi kP}{n+1}\left\{1-\left(\frac{t_0}{t}\right)^{n+1}\right\}. \qquad (3)$$

The quantity within the braces increases from zero (for $t=t_0$) to a value which, if n is large, can be near unity (for $t=t_0+\tau_0$).

Case B. $\tau_1>\tau_0$. In this case vertically reflected radiation would have had to originate after the end of the pulse. In fact therefore no radiation is being received from within a cone of semi-angle θ_2, where

$$t_0\sec\theta_2=t-\tau_0.$$

θ_2 is the angle corresponding to the end τ_0 of the emission. Thus in this case

$$I=2\pi kP\int_{\theta_2}^{\theta_1}\cos^n\theta\sin\theta\,d\theta$$
$$=\frac{2\pi kP}{n+1}\left\{\left(\frac{t_0}{t-\tau_0}\right)^{n+1}-\left(\frac{t_0}{t}\right)^{n+1}\right\}. \qquad (4)$$

The function has its maximum value when

$$t=t_0+\tau_0,$$

this value being

$$I_m=\frac{2\pi kP}{n+1}\left\{1-\left(\frac{t_0}{t_0+\tau_0}\right)^{n+1}\right\}.$$

Thus

$$\frac{I}{I_m}=\frac{\{t_0/(t-\tau_0)\}^{n+1}-(t_0/t)^{n+1}}{1-\{t_0/(t_0+\tau_0)\}^{n+1}}. \qquad (5)$$

The shapes of the swept-gain patches indicate that n is usually a large number. Since in the records discussed τ_0/t_0 is of the order $0\cdot05$, it is clear that the first terms in the numerator and denominator far exceed the second and the latter can thus be neglected. Thus, to a good approximation,

$$I/I_m=\{t_0/(t-\tau_0)\}^{n+1}, \qquad (6)$$
$$\log I=\log I_m+(n+1)\log t_0-(n+1)\log(t-\tau_0).$$

The slope of the upper edge of the swept-gain patch (i.e. beyond its maximum) should thus correspond to S_1, where

$$S_1=\frac{1}{\partial(10\log I)/\partial t}=\frac{t-\tau_0}{10\log e}\cdot\frac{1}{n+1} \qquad (7)$$

S_1 would thus vary sufficiently slowly with t to be sensibly constant over the width of a normal trace. Note that it is proportional to $1/(n+1)$ and thus the measurement of the slope allows n to be determined.

IV. Triangular Pulse

The actual shape of the pulse, received under specular reflection conditions, is far from rectangular. In appearance it approximates roughly to an error function, but as this function does not yield conveniently integrable expressions, calculations have been made assuming a triangular pulse, defined by

$$\begin{aligned} P(\tau)&=b\tau/\tau_0 && (0<\tau<\tau_0/2),\\ P(\tau)&=b(\tau_0-\tau)/\tau_0 && (\tau_0/2<\tau<\tau_0),\\ P(\tau)&=0 && (\tau>\tau_0). \end{aligned}$$

In this case three cases arise, corresponding to these three conditions, namely: *Case* (*a*). $0<\tau_1<\tau_0/2$.

$$I=\frac{2\pi kb}{n+1}\left[\frac{t-t_0}{\tau_0}-\frac{1}{n}\cdot\frac{t_0}{\tau_0}\left\{1-\left(\frac{t_0}{t}\right)^n\right\}\right]. \quad\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Case (*b*). $\tau_0/2<\tau_1<\tau_0$.

$$I=\frac{2\pi kb}{n+1}\left[\frac{t_0-t+\tau_0}{\tau_0}+\frac{1}{n}\,\frac{t_0}{\tau_0}\left\{1-2\left(\frac{t_0}{t-\tau_0/2}\right)^n+\left(\frac{t_0}{t}\right)^n\right\}\right]. \quad\ldots\ldots (9)$$

Case (*c*). $\tau_1>\tau_0$.

$$I=\frac{2\pi kb}{n(n+1)}\,\frac{t_0}{\tau_0}\left(\frac{t_0}{t-\tau_0}\right)^n\left\{1-2\left(\frac{t-\tau_0}{t-\tau_0/2}\right)^n+\left(\frac{t-\tau_0}{t}\right)^n\right\}. \quad\ldots (10)$$

To examine the general character of these results we shall again have recourse to approximation. If n is a large quantity (specifically if $n\tau_0\gg t$) then the results reduce, respectively, to

(*a*)
$$\frac{2\pi kb}{n+1}\left[\frac{t-t_0}{\tau_0}-\frac{1}{n}\,\frac{t_0}{\tau_0}\right]. \quad\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

(*b*)
$$\frac{2\pi kb}{n+1}\left[\frac{t_0-t+\tau_0}{\tau_0}+\frac{1}{n}\,\frac{t_0}{\tau_0}\right]. \quad\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

(This approximation is invalid if t is very near $t_0+\tau_0/2$.)

(*c*)
$$\frac{2\pi kb}{n+1}\,\frac{1}{n}\,\frac{t_0}{\tau_0}\left(\frac{t_0}{t-\tau_0}\right)^n. \quad\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It will be noted that for the upper edge of the swept-gain patch we now obtain

$$S_1=\frac{1}{\partial\,(10\log I)/\partial t}=-\frac{t-\tau_0}{10\log e}\cdot\frac{1}{n}, \quad\ldots\ldots\ldots (14)$$

which is not distinguishable (for large n) from the result obtained previously. This suggests that (**14**) may be approximately valid also for the error function form of $P(\tau)$.

As an example of the effect on the shape of swept-gain patches of the (effective) emitted pulse shape, the dependence of log I on $t-t_0$ has been computed for the following conditions :

$$t_0=10^{-3}\text{ sec},\qquad n=500,$$
$$(a)\ \text{Rectangular}\ \tau_0=5\times10^{-5}\text{ sec},$$
$$(b)\ \text{Triangular}\ \tau_0=10^{-4}\text{ sec},$$

and the results depicted in Figure 3 (with time-zero displaced appropriately).

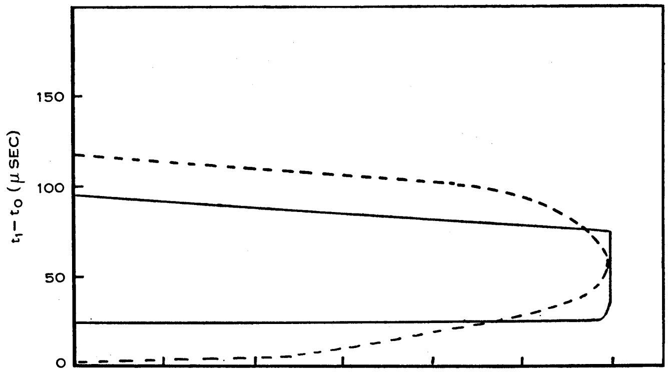

Fig. 3.—Theoretical shape of swept-gain patches, $n=500$, $t_0=10^{-3}$ sec, $\tau_0=5\times10^{-5}$ sec (rectangular pulse), 10^{-4} sec (triangular pulse).

V. Double-hop Reflections

A complete treatment, along the same lines, of double-hop reflections is scarcely feasible because of the wide variety of possible paths. Scattering can occur, not only at each reflection from the ionosphere, but also at the ground (e.g. Dieminger 1951). It seemed, however, worth while to consider briefly

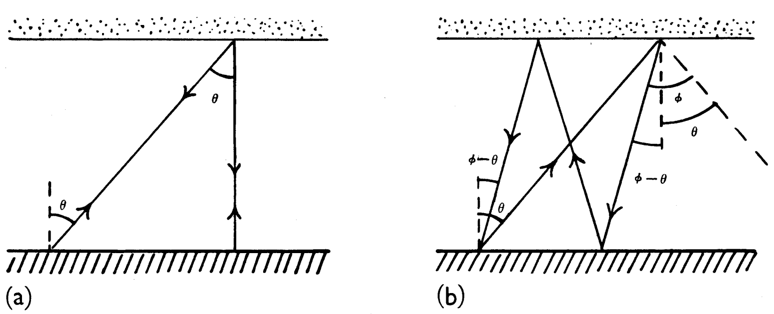

Fig. 4.—Scattering from a rough ionosphere in double-hop reflections (specular reflection at ground). (*a*) Outward and inward paths coincident, two scattering processes ; (*b*) outward and inward paths different, one scattering process.

two limiting cases, in both of which specular reflection from the ground is assumed. In the case represented in Figure 4 (*a*), radiation is scattered (non-specularly) twice from the ionosphere, in the case represented in Figure 4 (*b*) once only.

Comparing the first of these cases with the single-reflection case, the radiation is scattered twice through an angle θ instead of once through 2θ. Thus the scattering factor (cf. Briggs and Phillips, loc. cit.) becomes $(\cos\theta\cos^q\theta)^2$ instead of $\cos\theta\cos^q 2\theta$ and the overall intensity is proportional to

$$\cos^{a+2q+8}\theta \text{ instead of } \cos^{a+4q+6}\theta$$

where $a=t+r$, t, r representing the properties of transmitter and receiver antenna systems in Briggs and Phillips' notation.

In the experiments to be described, a is small (of the order of 2 or 3), while the shape of the patches indicates a large value of n. Thus q is the preponderant term in the index.

To forecast the overall slope of the swept-gain patch we have to take into account also the fact that in this case

$$t=t_0(1+\sec\theta_1)$$

(with t_0 corresponding to a single vertical reflection).

Applying the same arguments as in Section III, and also introducing the approximation

$$t-\tau_0=2t_0,$$

it is possible to show that the slope of the swept-gain patch, in the double-hop case, should correspond to

$$S_2=-\frac{t_0}{10\log e}\cdot\frac{1}{n_2}, \qquad \textbf{(15)}$$

where $n_2=a+2q+9$.

For the single-hop case, using the corresponding approximation ($t-\tau_0=t_0$) we obtain

$$S_1=-\frac{t_0}{10\log e}\cdot\frac{1}{n_1},$$

where $n_1=a+4q+7$. Thus

$$S_2/S_1=n_1/n_2=2 \text{ (approx.)}, \qquad \textbf{(16)}$$

if q is large.

In the case represented in Figure 4 (b) the important factor in the expression for the intensity of the scattered wave is $\cos^q\varphi$, while we have

$$t=\frac{t_0}{2}\sec\theta+\frac{3t_0}{2}\sec(\varphi-\theta).$$

A numerical calculation indicates that, if we set

$$\sec\chi=t/2t_0,$$

then, roughly,

$$\varphi=2\cdot 25\chi, \text{ and thus, if } q \text{ is large,}$$
$$\cos^q\varphi=\cos^{5q}\chi.$$

In view of the smallness of a, it is permissible to use χ in place of θ and $(\varphi-\theta)$ in the remaining factors and write

$$\mathrm{d}W \propto \cos^{a+5q+6}\chi.$$

For the slopes of the swept-gain patches we now have

$$S_2' = \frac{-t_0}{10 \log \mathrm{e}} \frac{1}{n_2'}, \qquad (17)$$

where $n_2' = a+5q+6$,
and thus

$$S_2'/S_1 = n_1/n_2' = 0 \cdot 8 \text{ (approx.)}. \qquad (18)$$

Thus the ratio of slopes is expected to lie between $0 \cdot 8$ and $2 \cdot 0$ dependent on the relative importance of the mechanisms responsible.

VI. The Broadening of Resolved Satellite Traces

According to the views elaborated by McNicol and Webster (1956), the presence of a satellite trace indicates the existence of a large-scale irregularity, often of considerable extent, in the F layer. Although this irregularity can be regarded, as a first approximation, as a straight wave-front, there is evidence that minor irregularities are usually present along its length and also that the ionospheric surfaces on either side of it are not specular reflectors.

Since the conditions specified in Section II are no longer valid, we might expect a departure from trapezoidal shape (for the main trace) when satellites are present. Moreover, in the case of the satellites themselves, since the effective scattering surface is now a narrow strip rather than the complete ionospheric plane (strictly, sphere) we might expect the contributions of more remote parts to be less important, i.e. in the swept-gain patch we might expect a smaller slope of the upper edge than for the main trace.

VII. Comparison of Theory with Experiment

In principle, it should be possible to subject the theory to direct check, by deducing from the slope of the swept-gain patches the anticipated correlation coefficient between fading records on spaced receivers and comparing these values with those obtained in fading experiments. An attempt was made to do this using the fading records obtained by Burke and Jenkinson (1957) in measurements of ionospheric drifts. The results were indefinite ; in some cases the calculated correlation coefficient was much higher than the experimental, in other cases much lower. This discrepancy may have been due partly to the effect of the automatic gain control in the fading receivers and partly to inadequacy of the fading samples obtained.

Some confirmation of the accuracy of the conclusions is, however, available from a comparison of the spread of values of n for F reflections determined from the swept-gain patches in a random sample of records, with the spread of values found by Briggs and Phillips (loc. cit.) on almost the same frequency. The results of the present investigation are shown in histogram form in Figure 5 (which should be compared with Figure 8 of Briggs and Phillips' paper). For convenience in comparison, Figure 5 represents, not n, but θ_0, defined by

$$(\cos \theta_0)^n = \tfrac{1}{4},$$

from which, if θ_0 is small, $\theta_0 = 95 \,.\, n^{-\frac{1}{2}}$ degrees.

Figure 5 shows a modal value slightly greater than obtained by Briggs and Phillips. This discrepancy could well be due to the presence of unresolved satellites leading to over-estimates of the slope of the swept-gain patches. Figure 5 includes data obtained from records showing resolved satellites, as well as those showing no resolved satellites. No systematic relationship appears to exist between θ_0-values and the presence or absence of resolved satellites.

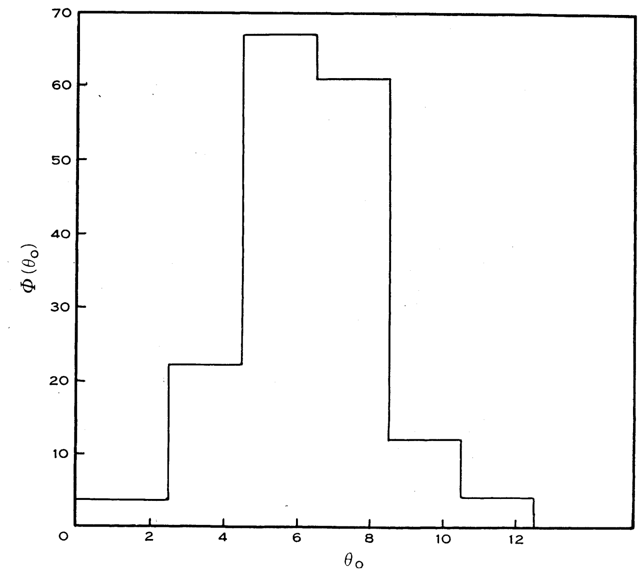

Fig. 5.—Histogram showing distribution of θ_0 values among 160 cases. $\Phi(\theta_0)$ represents number of cases having θ_0 values between limits shown.

The E_s traces have been analysed similarly in a number of cases. As would be expected from the nature of the night-time E_s layer, the mean value of θ_0 is higher than for the F (14 *v.* 5·9). The distribution is roughly consistent with that obtained by Briggs and Phillips for night-time E_s at Cambridge on a similar frequency.

Two-hop F reflections are recorded on some swept-gain films. One film has been examined in detail. In Figure 6, values of θ_0' corresponding to $2n_2$, where n_2 is obtained from the slope of $2F$ patches using equation (**15**), have been plotted against the corresponding values of θ_0 for the single-hop patches. The points are roughly symmetrical about the line corresponding to $n_2 = 0 \cdot 6n$, and two-thirds lie between or close to the lines corresponding to $n_2 = 0 \cdot 5n_1$ and $n_2' = 1 \cdot 25n_1$. The fact that the $2F$ patches conform less closely to the " ideal " shape probably accounts for some discrepancies ; neglect of ground-scatter may also be a factor.

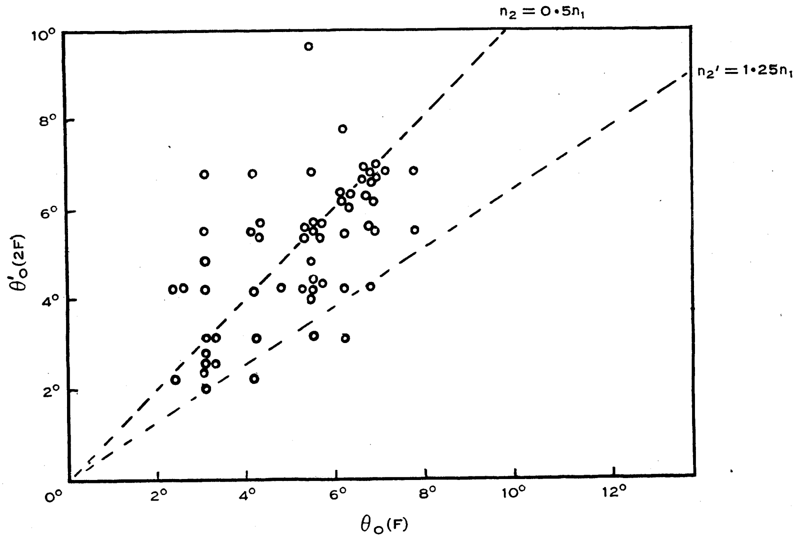

Fig. 6.—Comparison of θ_0' deduced from double-hop F reflections (assuming $n_2 = 0{\cdot}5n$) with θ_0 deduced from the corresponding single-hop reflection. The broken lines indicate theoretical relations for the process of Figures 4 (*a*) (upper line) and 4 (*b*) (lower line).

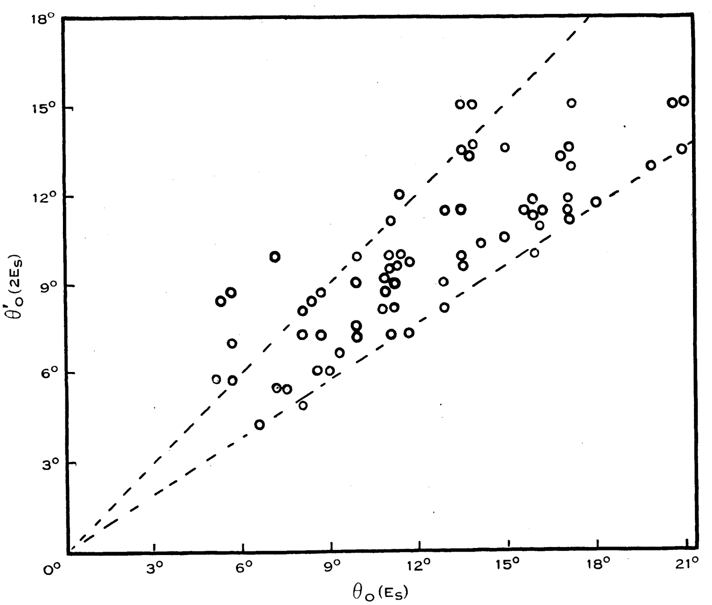

Fig. 7.—Comparison of θ_0' deduced from double-hop E_s reflections (assuming $n_2 = 0{\cdot}5n$), with θ_0 deduced from the corresponding single-hop reflection. The broken lines indicate theoretical relations, the upper line for the process of Figure 4 (*a*), the lower for that of Figure 4 (*b*).

Figure 7 shows, for comparison, the corresponding situation for the E_s and $2E_s$ reflections (on other films). Here rather a higher percentage of points lie between the predicted limits. A few swept-gain patches of triple-hop E_s have been examined. If we write

$$n_3 = \frac{-t_0}{\log e} \frac{\partial \log I}{\partial t}$$

and calculate the values of θ_0 corresponding to $3n_3$, the values obtained are somewhat lower than for the corresponding two-hop reflection.

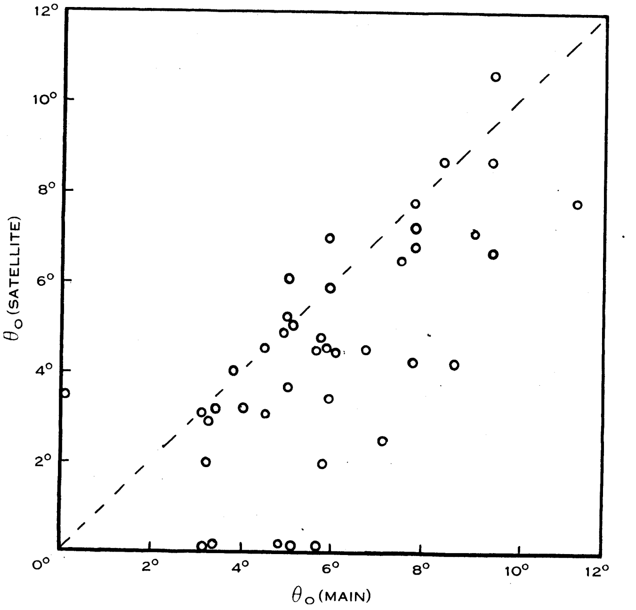

Fig. 8.—Comparison of θ_0 value for F satellite with θ_0 value for corresponding main trace.

In Figure 8 values of θ_0 determined from a " satellite " trace are plotted against θ_0 determined from the corresponding main trace. It will be noted that in the majority of cases the satellite trace gives a smaller value of θ_0, as the theory would suggest. A few satellites without any measurable spreading ($\theta_0 \rightarrow 0$) were recorded. These were usually of rather low (< -40 dB) relative intensity and hence were probably due to particularly narrow wave fronts.

VIII. Trace-broadening using Remote Transmitter

Some swept-gain records were made at Brisbane in which F echoes from transmissions from Toowoomba (100 km away) were recorded simultaneously with F echoes from Brisbane transmissions. In Figure 9, values of θ_R, the angle corresponding to the value of n, estimated from the swept-gain patches for Toowoomba transmissions, are plotted against corresponding values θ_A for Brisbane transmissions. In 25 per cent. of cases values were nearly equal, and in 70 per cent. a lower value was found for the distant than for the local transmitter. Figure 10 gives some indication of the corresponding situation for E_s reflections. The adjustment of the time-base was such that the one-hop E_s reflection from the adjacent transmitter was not recorded. Values of θ' were estimated from the value of n_2 for the two-hop E_s trace, using equation (**16**), and compared with values of θ_R for the one-hop E_s trace from the remote transmitter. For 75 per cent. of the points, θ_R is less than θ'_A. It will be noted that if θ_A had been computed using equation (**18**), the difference would be still more marked.

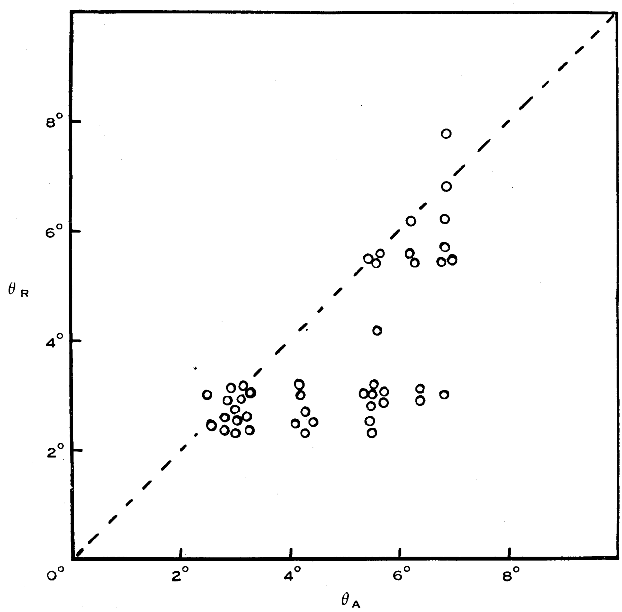

Fig. 9.—Comparison of θ_0 values from F traces arising from a remote transmitter (θ_R), with corresponding θ_0 value from a contiguous transmitter (θ_A).

These results are in line with the observations made during the experiments of McNicol, Webster, and Bowman (loc. cit.), and of Strohfeldt, McNicol, and Gipps (1952), that there is less evidence of (resolved) satellites when the transmitter is remote from the receiver. In searching for an explanation of the effect, it would seem necessary to exclude any mechanism involving refractive effects, since these would be strongly frequency-dependent, whereas range broadening is only slightly, if at all, dependent on frequency. Moreover, except in traces where there are, fairly clearly, imperfectly resolved satellites, the phase-path records usually show straight fringes running right across the F trace. This also, in many instances, applies to the E_s trace. A geometrical model, having approximately the required properties, consists of a flat ionosphere eroded by narrow pits (say, 10 km diameter) of approximately hemispherical shape. A hemispherical pit would reflect back along the path of incidence any ray passing through the centre of the sphere, but rays incident on the hemispherical surface in other directions would only return to Earth after double reflection from the surface. A slight rounding-off of the edges of some of the pits would provide a small amount of " scattering " of the rays from the remote transmitter. It is worthy of note that McNicol and Webster have suggested canyons in the ionosphere in explaining some of the features of satellite traces.

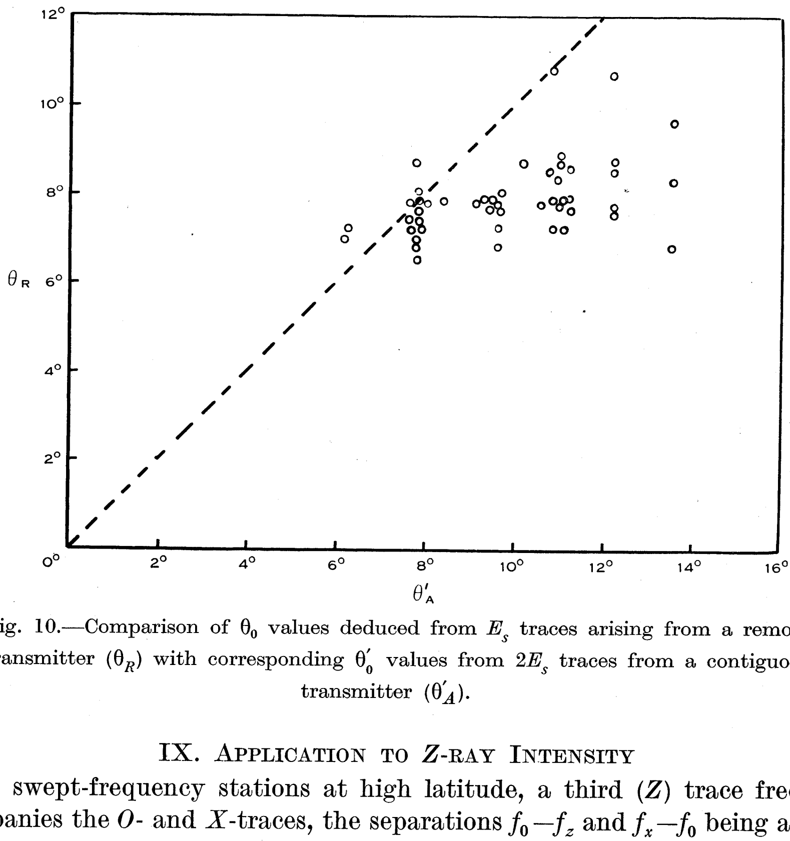

Fig. 10.—Comparison of θ_0 values deduced from E_s traces arising from a remote transmitter (θ_R) with corresponding θ_0' values from $2E_s$ traces from a contiguous transmitter (θ_A').

IX. Application to Z-ray Intensity

At swept-frequency stations at high latitude, a third (Z) trace frequently accompanies the O- and X-traces, the separations f_0-f_z and f_x-f_0 being approximately equal. This phenomenon has never been clearly recorded on swept-frequency records at Brisbane, but it has on a few occasions appeared on fixed-frequency records (cf. Plate 2) especially swept-gain and phase-path (McNicol, Webster, and Bowman, loc. cit.). Ellis (1956) has put forward a theory of the phenomenon which these records make it possible to check. He shows that for rays having directions lying within a small cone surrounding a critical direction θ_c, which at Brisbane would lie at 19° 30′, to the vertical and in the meridional plane, the O-ray would pass beyond the level at which it is usually reflected and would be reflected at a level where (in the absence of collisions) the parameters satisfy the equation

$$x=1+y,$$

where $x=Ne^2/m\omega^2\varepsilon_v$ (using rationalized MKSA units) and $y=\omega_H/\omega$, where $\omega_H/2\pi$ is the gyro frequency.

However, this reflected radiation cannot return to the Earth's surface ; it needs to be scattered back along the direction of incidence to do so. From our previous assumptions the scattered intensity in this direction would be proportional to $\cos^n \theta_c \, \Delta\Omega$ where $\Delta\Omega$ is the effective solid angle of the cone of rays which pass through the O reflection level. Ellis has found, experimentally, that the semi-angle to the half-power points is $0 \cdot 42°$. It is convenient, however, to use an equivalent cone, having full power transmitted for all directions within the cone and zero power for directions outside it. Assuming the actual distribution to be Gaussian, this gives $\Delta\theta = 0 \cdot 52°$ for the semi-angle of the equivalent cone, and $\Delta\Omega = \pi\Delta\theta^2 = 2 \cdot 58 \times 10^{-4}$ steradian.

Since the angle $\Delta\theta$ is much smaller than the separation $\theta_1 - \theta_2$ of the limits of integration, the Z-echo should thus be of the same duration as the pulse itself and show no variation of intensity. The ratio of the intensity of the Z-ray to the maximum intensity of the O-ray should, in the absence of differences in absorption etc., be given by

$$\frac{I_z}{I_m} = \frac{\cos^n \theta_c \, \Delta\Omega}{\{2\pi/(n+1)\}[1 - \{t_0/(t_0+\tau_0)\}^{n+1}]} = \frac{\cos^n \theta_c \, \Delta\Omega}{2\pi\tau_0/t_0} \text{ approx.,} \quad \ldots\ldots\ldots\ldots (19)$$

$$\log \frac{I_z}{I_m} = \log \frac{\Delta\Omega}{2\pi\tau_0/t_0} + n \log \cos \theta_c. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Since, in the Brisbane records, τ_0/t_0 is approximately $0 \cdot 03$, this gives

$$\log I_z/I_m = \bar{3} \cdot 14 - 0 \cdot 0257n = \bar{3} \cdot 14 - 232\theta_c^{-2} \quad (\theta_c \text{ in degrees}).$$

Thus for $\theta_0 = 6°$ the Z-ray intensity should be about 84 dB lower than that of the O-ray.

Experimentally, the Z-ray traces, when appearing on $h't$ records, have the anticipated narrow width and, on swept-gain records, a horizontal upper edge. The relative intensities of Z- and O-rays have been measured in three cases for which θ_0 was near 6°. These were −75, −70, −55 dB. In view of the neglect of absorption, the discrepancy is not serious. It is possible that, in the last case, reflection took place from a tilted part of the ionosphere, thus reducing the angle between the ionization gradient and the magnetic field, and hence the value of θ_c.

At the minimum gain of the swept-gain receiver O-ray echoes are rarely recorded, even when the critical frequency is near $2 \cdot 28$ Mc/s. Since the maximum gain is only 80 dB above this, and since the fixed-gain receivers normally operate with gain intermediate between these two levels, it is not surprising that the Z-ray is rarely recorded.

X. Conclusions

It is concluded that :

(*a*) From an examination of the patches produced by a swept-gain fixed-frequency ionospheric recorder, it is possible to gauge the degree of roughness of ionospheric layers.

(*b*) The effective roughness is a function of the separation of transmitter and receiver, being less the greater the distance between them.

(*c*) The intensity of *Z*-ray echoes recorded in Brisbane is consistent with Ellis's theory.

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