THEORY OF THE RADIO-ECHO METEOR HEIGHT DISTRIBUTION IN A NON-ISOTHERMAL ATMOSPHERE

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Summary

The theory of the distribution in height of the echoing points of shower meteors and of sporadic meteors belonging to a homogeneous velocity group, is extended to the case of a model atmosphere in which the scale height is a linear function of height. The mean and the r.m.s. deviation from the mean of the height distribution in this atmosphere are obtained in terms of tabulated functions, and the dependence of these parameters upon the scale height gradient is evaluated. Neither parameter varies strongly with the scale height gradient unless the incident meteors include a large proportion of massive particles. Experimental cut-off and the approximations made in the formulation of the theory limit the accuracy with which atmospheric scale height and density can be determined from observed heights of sporadic meteors.

I. Introduction

In two papers, Kaiser (1954*a*, 1954*b*) has presented a theory of the height distribution of the echoing points of meteors in an isothermal atmosphere, for both shower and sporadic meteors. He has shown that, apart from a small correction depending on the particular aerial system in use, the r.m.s. deviation of the height distribution for meteors of a particular shower, or for sporadic meteors in a narrow velocity range, depends only on the meteor mass distribution and on the scale height of the isothermal atmosphere. The atmospheric pressure and density at the mean height measured for meteors of a homogeneous velocity group are related to the physical properties of meteors (latent heat, density, velocity), the zenith angle of the meteor radiant, and the ionizing probability of an evaporated meteor atom.

The theory has been applied by Evans (1954, 1955) to the determination of atmospheric properties from simultaneous measurement of the height and velocity of sporadic meteors. He deduces an atmospheric scale height which increases by some 15 per cent. over the height range 90–100 km and atmospheric pressures and densities of the same order as those found by other techniques, e.g. rocket measurements. These results, in common with measurements by other methods, suggest that an isothermal atmosphere may not be the most appropriate model for the interpretation of the radio height measurements, and that an improvement in internal consistency and a more efficient use of observational data can be expected from a theory of the radio-echo height distribution based on a non-isothermal model atmosphere. The formulation of such a theory, starting with

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an atmosphere whose scale height is a linear function of height, is the programme of this paper. This model atmosphere includes the isothermal atmosphere as a special case. It offers the distinct advantage that the moments of the theoretical height distribution, which form the link between theory and observation, are obtained in terms of tabulated functions. This feature greatly facilitates examination of the properties of the distributions.

II. Electron Density Profiles in a Linear Atmosphere

We assume a model atmosphere in which the scale height is a linear function of height. In such an atmosphere the scale height at any height h may be defined by

$$H=H_0+\{\delta/(1-\delta)\}(h-h_0), \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where H_0 is the scale height at some datum level h_0. The constant δ specifies uniquely the variation of the scale height with height. The density is

$$\rho=\rho_0(H/H_0)^{-1/\delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

from which it is seen that $H\rho^{\delta}$ is an invariant for a particular atmosphere. This property is useful in changing the datum level.

The distribution of electron line density α along a meteor trail formed in this atmosphere is found by integrating the basic equations describing the evaporation of the meteor particle during its flight (Herlofsen 1947). By inserting the expression for the density gradient in this atmosphere, namely,

$$\frac{\mathrm{d}\rho}{\mathrm{d}h}=-\frac{\rho^{1+\delta}}{(1-\delta)H_0\rho_0^{\delta}},$$

at the appropriate points and accepting the usual approximations (see below), we obtain

$$\frac{\alpha}{\alpha_{\text{max.}}}=\frac{9}{4}\left(\frac{1-2\delta/3}{1-\delta}\right)^2\frac{\rho}{\rho_{\text{max.}}}\left[1-\frac{1}{3-2\delta}\left(\frac{\rho}{\rho_{\text{max.}}}\right)^{1-\delta}\right]^2, \quad \ldots . \quad (3)$$

$$\alpha_{\text{max.}}=\frac{4}{9}(1-\delta)^2(1-2\delta/3)^{-3}\{\beta/(\mu H_{\text{max.}})\}m_{\infty}\cos\chi,$$

$$\rho_{\text{max.}}=\frac{8}{3}(1-2\delta/3)^{-1}\rho_m l r_{\infty}\cos\chi/(H_{\text{max.}}v_{\infty}^2),$$

$$r_{\text{max.}}=\frac{2}{3}(1-\delta)(1-2\delta/3)^{-1}r_{\infty}.$$

In these approximate expressions, the subscripts max., ∞ refer respectively to the point of maximum electron density in the trail and to the initial state of the meteor. $\beta=\beta_0 v_{\infty}^{n}$ is the ionizing probability, i.e. the probability that an evaporated meteor atom will produce a free electron; the power-law form is assumed. The other quantities are: r, m, v, ρ_m=radius, mass, velocity, and density of the meteor; χ=zenith angle of meteor radiant; μ=mass of individual meteor atom; l=latent heat of evaporation of a meteor atom, corrected for the efficiency of heat transfer.

The equations of which (3) is an approximate solution apply strictly to the evaporation of a spherical body with drag coefficient of unity. The following assumptions have also been made: (*a*) meteor deceleration is very small at all times; (*b*) $v^2 \gg 12l$; (*c*) $v^2 \gg 3l(2+\eta)$. The validity of these assumptions is open to question for slow meteors unless $l \ll 10^{11}$ erg/g (if v is measured in cm/sec) and $\eta \sim 0$ (Weiss 1958). Although these approximations have been retained in the interests of simplification, the departures of the approximate from the true electron density profiles are so large that consideration of their effects on the height distribution will be necessary at a later stage.

With $\delta=0$, the expressions (3) reduce to those for the isothermal atmosphere. Normalized electron densities are plotted in Figure 1 for three values of δ.

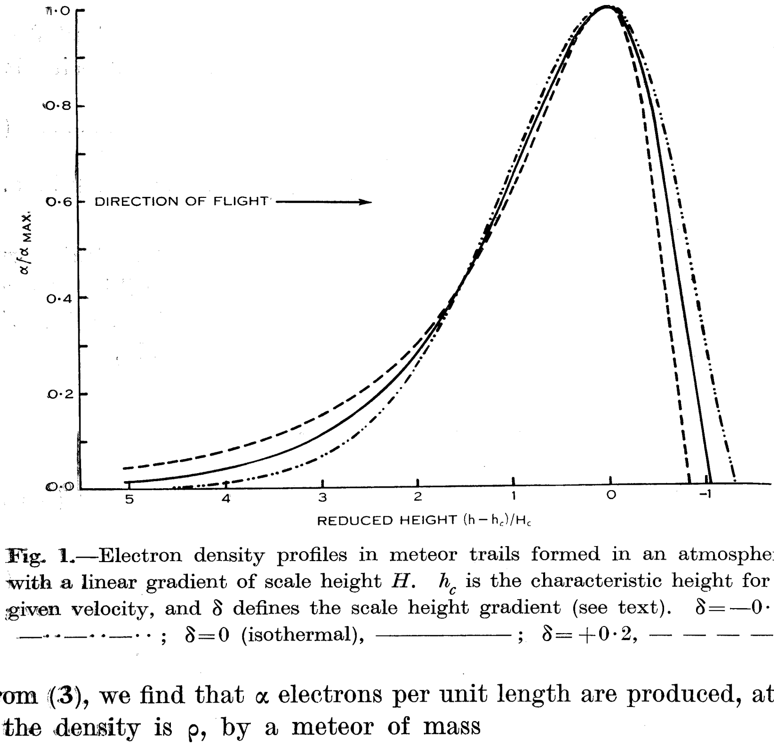

Fig. 1.—Electron density profiles in meteor trails formed in an atmosphere with a linear gradient of scale height H. h_c is the characteristic height for a given velocity, and δ defines the scale height gradient (see text). $\delta=-0\cdot2$, —··—··—·· ; $\delta=0$ (isothermal), ———— ; $\delta=+0\cdot2$, — — — —.

From (3), we find that α electrons per unit length are produced, at a height where the density is ρ, by a meteor of mass

$$m=\left[\left(\frac{\alpha}{A_1\rho}\right)^{\frac{1}{2}}+\frac{\rho^{1-\delta}}{A_2\cos\chi}\right]^3, \qquad (4)$$

where

$$A_1=\{\pi\beta v^2/(2\mu l)\}\left(\frac{4}{3}\pi\rho_m\right)^{-2/3},$$

$$A_2=\{8\rho_m l/(v^2 H_0\rho_0^{\delta})\}\left(\frac{4}{3}\pi\rho_m\right)^{-1/3}.$$

Since the deceleration is assumed to be small, the velocity v in A_1 and A_2 refers to the initial state of the meteor and the subscript ∞ has been dropped.

III. The Elementary Height Distribution

Following the geometry of Figure I of Kaiser (1954*a*) and his development of the theory, we will restrict the treatment in this section to the echoing points of a homogeneous velocity group of meteors with a well-defined radiant, which are detected in a narrow sector $\theta-(\theta+d\theta)$ of the echo plane. We will further assume that the minimum detectable line density $\alpha_{\text{min.}}$ is a constant within this sector, independent of the range R of the echoing point. For given aerial gain and $\alpha_{\text{min.}}<10^{12}$ electrons/cm, $\alpha_{\text{min.}}$ is proportional to $R^{3/2}$. The heights of most echoing points, at least of sporadic meteors, extend from 70 to 120 km, so that in the given sector $\alpha_{\text{min.}}$ varies by a factor $(1\cdot 7)^{1\cdot 5}$, i.e. about 2. Neglect of this range factor will result in slight distortion in the elementary height distributions derived below, in the sense that the upper tail of the theoretical height distribution will be exaggerated relative to the lower tail.

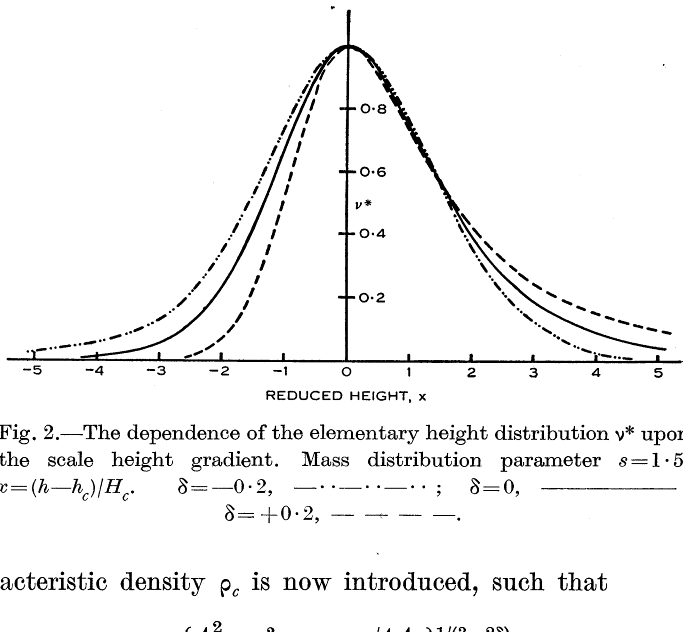

Fig. 2.—The dependence of the elementary height distribution ν^* upon the scale height gradient. Mass distribution parameter $s=1\cdot 5$. $x=(h-h_c)/H_c$. $\delta=-0\cdot 2$, —··—··—··; $\delta=0$, ————————; $\delta=+0\cdot 2$, — — — —.

A characteristic density ρ_c is now introduced, such that

$$\rho_c=\{A_3^2 \cos^2\chi \,.\, \alpha_{\text{min.}}/4A_1\}^{1/(3-2\delta)}, \quad \ldots\ldots\ldots\ldots \quad (5)$$

where

$$A_3=A_2/(1-\delta).$$

ρ_c determines the height of detection of the smallest meteor of velocity v which it is possible to detect with the given equipment in the given sector. At any other height the smallest detectable meteor has a mass

$$m=\left(\frac{\alpha_{\text{min.}}}{A_1}\right)^{3(1-\delta)/(3-2\delta)}\left(\frac{2}{A_3\cos\chi}\right)^{3/(3-2\delta)}\left[z^{-\frac{1}{2}}+\frac{z^{1-\delta}}{2(1-\delta)}\right]^3, \quad \ldots \quad (6)$$

where $z=\rho/\rho_c$. With the usual frequency law for the distribution of meteor masses, namely,

$$N_m dm=m^{-s}dm, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

the normalized elementary height distribution becomes

$$\nu^* = \left[\frac{3-2\delta}{2(1-\delta)z^{-\frac{1}{2}}+z^{1-\delta}}\right]^{3(s-1)}, \qquad \ldots\ldots\ldots\ldots \quad (8)$$

in which z may be written

$$z=\rho/\rho_c=\{1+x\delta/(1-\delta)\}^{-1/\delta}. \qquad \ldots\ldots\ldots\ldots \quad (9)$$

x is the reduced height, measured in units of scale height H_c from the level of the characteristic density ρ_c.

With $\delta=0$, (8) and (9) reduce to Kaiser's elementary height distribution for the isothermal atmosphere. Figure 2 illustrates the dependence of the height distribution upon the gradient of the scale height for $s=1\cdot5$.

IV. Properties of the Elementary Height Distribution

The mean and the r.m.s. deviation from the mean of the distribution (8) are determined through the integrals

$$I_p=\int_{-\infty}^{\infty} \nu^* x^p \mathrm{d}x, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

with $p=0, 1, 2$. Evaluation of these integrals is straightforward. Use of (9) together with the substitution

$$z^{3/2-\delta}=2(1-\delta)(1-w)/w$$

leads to

$$I_p=K(s,\delta)\left(\frac{1-\delta}{\delta}\right)^{p+1}\sum_{r=0}^{p}\binom{p}{r}(-1)^{p-r}\{2(1-\delta)\}^{-2r\delta/(3-2\delta)}\mathrm{B}(a,b), \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

in which

$$K(s,\delta)=2\delta(3-2\delta)^{3s-4}\{2(1-\delta)\}^{-\varepsilon},$$

$\mathrm{B}(a,b)$ is the complete beta function $\int_0^1 w^{a-1}(1-w)^{b-1}\mathrm{d}w$ with

$$a=\{6s(1-\delta)-6+2\delta(r+4)\}/(3-2\delta),$$
$$b=\{3(s-1)-2\delta(r+1)\}/(3-2\delta),$$
$$\varepsilon=a-2r\delta/(3-2\delta).$$

The mean reduced height is

$$\bar{x}=I_1/I_0, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

and the mean square deviation from the mean is

$$(\Delta x)^2=I_2/I_0-\bar{x}^2. \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

For the isothermal atmosphere

$$I_0=\int_{-\infty}^{\infty}\nu^*\mathrm{d}x=\{3^{3s-4}/2^{2s-3}\}\mathrm{B}(2s-2,\ s-1),$$

but for higher orders (12) and (13) reduce to indeterminate forms and the mean and r.m.s. deviation must be found as limits as $\delta\to 0$. Figure 3 is a plot of the parameters of the height distribution for three values of s.

One feature arising in the evaluation of the moments through the integrals (**10**) is worthy of comment. The beta function $B(a,b)$ becomes infinite if either argument is zero. With $r=2$, $b=3(s-2\delta-1)/(3-2\delta)$. Values of s found for shower meteors using isothermal theory range from $s=1\cdot4$ to $s=2\cdot7$. In the actual atmosphere δ is essentially positive, at least in all but the lowest levels of the meteor region. With $s=1\cdot5$ and $r=2$, $b=3(0\cdot5-2\delta)/(3-2\delta)$, and quite moderate values of δ will produce a height distribution whose r.m.s.

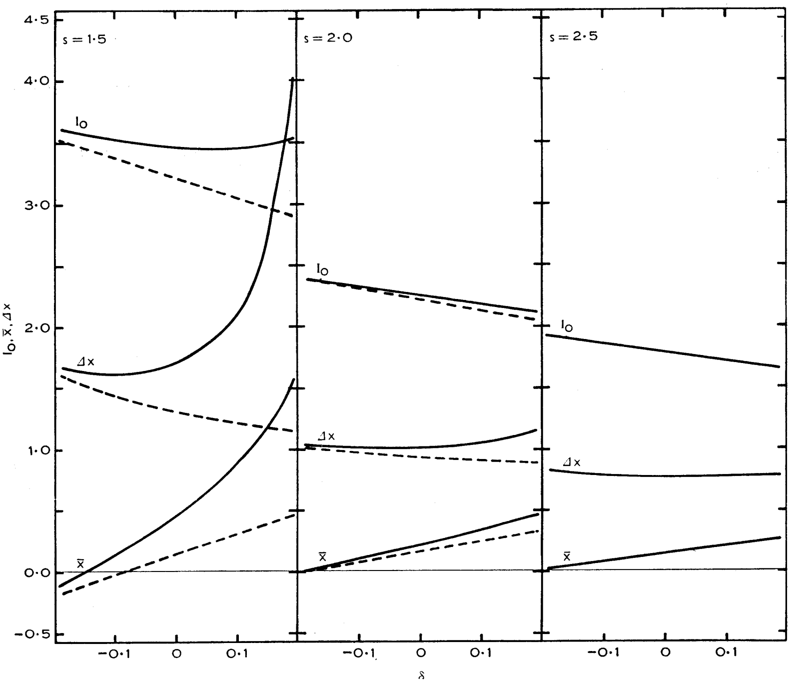

Fig. 3.—Properties of the elementary height distribution as functions of the scale height gradient and the mass distribution parameter s. I_0=zero moment, $\bar{x}$=mean, Δx=r.m.s. deviation from the mean, all in units of reduced height, $x=(h-h_c)/H_c$. ———— Full distribution ; — — — — distribution truncated at $x=3$.

deviation Δx is large (compare Fig. 3). This behaviour is associated with the long upper tail of the height distribution, which in turn is the result of the slow rise of the meteor trail to maximum electron density. Even with $s=2\cdot0$, the value for sporadic meteors, Δx is quite sensitive to δ if $\delta>0$.

The contribution of the upper tail of the height distribution to $\bar{x}$ and Δx will now be examined in more detail, with the aid of numerical examples. If the height distribution is truncated at the height x_0, the contribution of the truncated tail to the integral I_p is

$$I'_p=\int_{x_0}^{\infty} \nu^* x^p \mathrm{d}x. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{14})$$

For $x>3$ and $\delta<0 \cdot 2$, the term $z^{1-\delta}$ in ν^* is less than 2 per cent. of the other term $2(1-\delta)z^{-\frac{1}{2}}$ and may be neglected. Hence

$$I'_p=\left[\frac{3-2\delta}{2(1-\delta)}\right]^{3(s-1)}\int_{x_0}^{\infty} z^{3(s-1)/2}x^p\mathrm{d}x, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{15})$$

For positive δ, with the substitution

$$w=\{1+x\delta/(1-\delta)\}^{-1}$$

and use of (**9**), (**15**) takes the form

$$_+I'_p=\left[\frac{3-2\delta}{2(1-\delta)}\right]^{3(s-1)}\left(\frac{1-\delta}{\delta}\right)^{p+1}\mathrm{B}_{w_0}(c_+,d), \quad \ldots\ldots\ldots \quad (\mathbf{16})$$

where $\mathrm{B}_{w_0}(c,d)$ is the incomplete beta function $\int_0^{w_0} w^{c-1}(1-w)^{d-1}\mathrm{d}w$

with

$$c_+=\{3(s-1)/2\delta\}-p-1,$$
$$d=p+1.$$

For negative values of δ, the appropriate substitution is

$$w=1+x\delta/(1-\delta),$$

from which

$$_-I'_p=\left[\frac{3-2\delta}{2(1-\delta)}\right]^{3(s-1)}\left(\frac{\delta-1}{\delta}\right)^{p+1}\mathrm{B}_{w_0}(c,d), \quad \ldots\ldots\ldots\ldots \quad (\mathbf{17})$$

where

$$c_-=1-3(s-1)/2\delta.$$

The parameters of the height distribution truncated at $x_0=3$ are also illustrated in Figure 3. The r.m.s. deviations of the full and the truncated distributions differ significantly for large positive values of δ even if s is as large as $2 \cdot 0$. For smaller values of s, the differences rapidly become very large.

V. The Complete Height Distribution

The complete height distribution for shower meteors is obtained by integrating the elementary height distribution (**8**), for the sector $\theta-(\theta+\mathrm{d}\theta)$ of the echo plane, with respect to θ over the whole echo plane, with due allowance for the collection sensitivity of the equipment which is also a function of θ. In the case of sporadic meteors belonging to a homogeneous velocity group a further integration is necessary, over all sporadic meteor radiants on the geocentric celestial sphere. The alterations to the mean and r.m.s. deviation necessitated by this passage from elementary to complete height distribution have been exhaustively discussed by Kaiser for the isothermal atmosphere. In general the corrections are small and only become serious as the mass distribution parameter s decreases well below $2 \cdot 0$; they are also insensitive to the form of the aerial polar diagram.

The parameters of the complete height distribution for the linear atmospheric model could be evaluated in the same way. However, in view of the smallness of the corrections for the isothermal model, there seems little to be gained by carrying through the calculations, which become very tedious. The corrections for the isothermal model, taken over unchanged, should be sufficiently accurate for all practical purposes unless $\delta \gg 0$ and $s \ll 2\cdot 0$.

In the case of sporadic meteors, the extension from elementary to complete height distribution is made by first evaluating the rate of echoes and the height distribution (φ-distribution) observed in an element of solid angle $d\omega$ having coordinates (φ=elevation, θ=azimuth). For an assumed uniform geocentric distribution of radiants, and with $s=2\cdot 0$, the mean $\bar{X}$ and the r.m.s. deviation ΔX of the φ-distribution are (Kaiser 1954*b*):

$$\bar{X}-\bar{x}=-\frac{2}{3}\int_0^{\pi/2} \cos\xi \ln(\cos\xi)d\xi, \quad \ldots\ldots\ldots\ldots \quad (18)$$

$$(\Delta X)^2-(\Delta x)^2=-(\bar{X}-\bar{x})^2+\frac{4}{9}\int_0^{\pi/2} \cos\xi \ln^2(\cos\xi)\, d\xi, \quad \ldots \quad (19)$$

in which ξ is defined by $\cos\chi=\cos\varphi\cos\xi$. It is assumed that the equipment sensitivity is such that the minimum detectable line density is much less than 10^{12} electrons/cm over most of the aerial aperture. The parameters $\bar{X}$, ΔX are illustrated in Figure 4 as functions of δ.

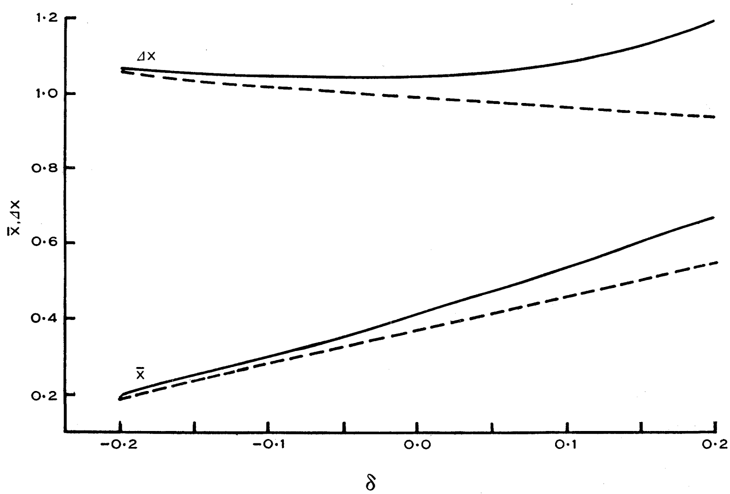

Fig. 4.—Properties of the complete sporadic height distribution observed within a narrow elevation interval. Aerial polar diagram is independent of azimuth. ———— Full distribution; — — — — distribution truncated at $x=3$.

For the Adelaide equipment (Robertson, Liddy, and Elford 1953) which operates continuously for extended periods and whose aerial polar diagram is almost symmetrical about the zenithal direction and so is independent of azimuth θ, the assumption of a uniform radiant distribution is justified, and the elementary height distribution will be independent of azimuth. Weighted integration of the φ-distributions is still necessary to build up the complete distribution as observed. Proper weighting would be difficult for the Adelaide equipment, whose echo rate is determined by the line-of-sight velocity of drift of the meteor trails as well as by the aerial polar diagram. By selecting a narrow elevation interval over which the equipment sensitivity may be regarded as substantially constant, the final integration is eliminated (Kaiser 1954*b*) and Figure 4 may be taken as a sufficient approximation to the parameters of the complete sporadic meteor height distribution for a homogeneous velocity group. What constitutes a sufficiently narrow elevation interval can best be ascertained by a preliminary analysis of the records.

VI. Experimental and Theoretical Limitations

Two major limitations to the accuracy with which the derived theoretical height distributions can be expected to represent the measured distributions will now be examined.

Experimentally, the diffusion of the meteor trails will be of some consequence in modifying the upper portion of the height distributions. This acts in two ways. In the first place, diffusion of high meteor trails is so rapid that some are inevitably missed in a photographic radio record. Secondly, Hawkins (1956) has drawn attention to the existence of a diffusion ceiling, a critical height above which Fresnel zones will tend to disappear, thus preventing measurement of velocity even if the height is recorded. The importance of this diffusion cut-off will depend on the operating frequency and the method of recording, and may be ascertained in any particular case from a detailed examination of the records. The numerical examples already given (Figs. 3 and 4) for an elementary distribution truncated at $x=3$ indicate the order of magnitude of the errors which will be introduced by failure to recognize and correct for a severe diffusion cut-off.

The other limitation, a theoretical one, is associated with the assumptions leading to the approximate electron density profiles (**3**). The author (Weiss 1958) has already examined the discrepancy between the true and the approximate profiles for the special case of a spherical meteor evaporating in an isothermal atmosphere. The discrepancy in a linear atmosphere should not be greatly different for reasonable values of δ. An exact evaluation of the errors in the height distributions themselves is precluded by the alterations of the shape of the profiles introduced by the approximations. The approximations tend to understate the length of trail beyond the point of maximum electron density for meteors of all velocities; in addition the trails of slow meteors do not rise to the sharp maximum predicted by the approximations. It seems reasonable to infer that the approximations will not seriously distort the theoretical height distributions for fast meteors but will tend to give too-peaked theoretical distributions for slow meteors.

Some useful conclusions regarding the error in the characteristic height for a given velocity group are also possible. This is so because the mean height for a given velocity will be determined largely by the smaller meteors accessible to the equipment. These small meteors, seen in the largest numbers, will be detected closer to the point of maximum electron density than is the case for the more massive meteors which contribute to the tails of the distribution.

Using the improved approximations suggested by the author (equations (5) of Weiss (1958)) for conditions near the maximum, we obtain the following relation, for $\alpha_{\text{max.}}=$constant:

$$\rho_{\text{max.}}=v_{\text{max.}}^{-(2+\eta/3)}F(\eta)^{-2/3}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

where

$$v_{\text{max.}}=\left[v_{\infty}^{2}-12l\,.\,\ln\left\{\frac{1+2F(\eta)}{2F(\eta)}\right\}\right]^{\frac{1}{2}},$$

$$F(\eta)=1+3l(2+\eta)/v_{\infty}^{2}.$$

From the usual approximation (3)

$$\rho_{\text{max.}}=v_{\infty}^{-(2+\eta/3)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

which is independent of l. These two relations are compared in Figure 5, with 60 km/sec as datum velocity. Evidently the approximate expressions (3) used

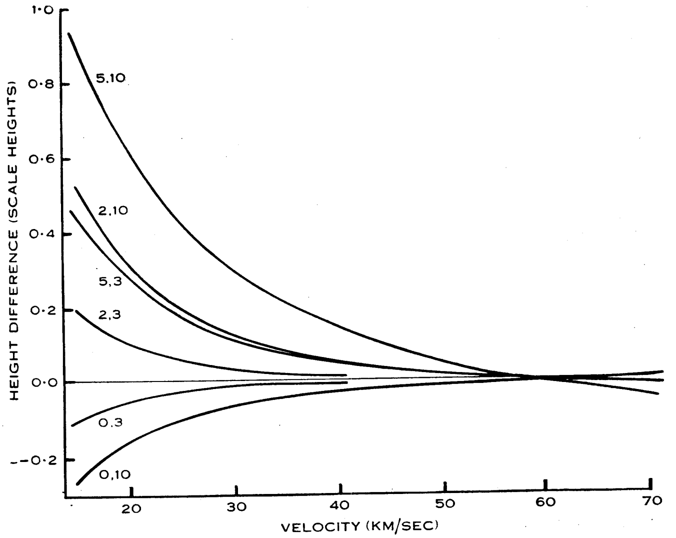

Fig. 5.—Approximate differences between characteristic heights with the usual and the improved approximations for trail shape. The code is (η,l'), where η=exponent in ionizing probability law, $l'=l\times10^{-11}$, l=effective latent heat of evaporation of a meteor atom (erg/g).

in formulating the theory will have little effect on the true relation between meteor velocity and height for fast meteors, and for slow meteors only if $l\gg10^{11}$ erg/g, which is unlikely.

VII. Discussion

Atmospheric densities and scale heights and the gradient of the scale height, evaluated from the radio-echo meteor height distribution through the isothermal model atmosphere, already agree well with the results of measurements by other techniques. Nevertheless, the isothermal model can only make statements about the mean value of the scale height over the whole of the height range over which meteors of a given velocity are detected. Since the gradient of the scale height over the meteor region is not large, it cannot be expected that the linear model atmosphere will lead to any marked improvement in the numerical values of the atmospheric properties, although it will extend the domain over which they may be studied to the whole of the meteor region. The value of the new approach is rather to be sought in the more efficient use of data which it allows, in that the atmospheric properties can be derived from the height measurements treated as a whole, without a preliminary segregation into velocity groups.

The accuracy of numerical deductions from the theory is limited by diffusion and other experimental cut-off and by the approximations accepted in the formulation of the theory. Some allowance can be made for the experimental cut-off, but it seems that little can be done at present about the theoretical limitations, mainly because of ignorance of the absolute values of the meteor constants, particularly the latent heat, and of the velocity-dependence of the ionizing probability of an evaporated meteor atom. Ultimately, the failure of a linear scale height gradient to represent the actual atmosphere will become important, but a preliminary assessment, which indicates $\delta \sim 0 \cdot 15$, suggests that this model is acceptable over that part of the meteor region lying above 90 km.

Finally, it must be stressed that all the preceding considerations are based on the model of a spherical meteor particle and a simple evaporation process. If the phenomenon of fragmentation, advanced by Jacchia (1955) in explanation of anomalies in atmospheric densities derived from photographic meteors, is severe and widespread, this simple model will fail completely as a description of the actual meteor trail. The trails will form higher in the atmosphere and will be shorter in length. There is no real evidence either for or against fragmentation amongst faint radio meteors; photographic meteors are very bright by radio standards and can constitute only a very small fraction of the meteors detected by a normal radio equipment. If the theory of the height distribution of the echoing points of meteor trails formed by evaporation can be made sufficiently precise, it can be used to advantage in assessing the extent to which the evaporation model corresponds with reality.

VIII. Acknowledgments

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