THE DETECTION OF TIME-CORRELATED PHOTONS BY A COINCIDENCE COUNTER

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Summary

The existence of a correlation between the arrival times of photons has been confirmed by measurements with a coincidence counter having a resolving time of $3 \cdot 5 \times 10^{-9}$ sec in three different experiments. In the first experiment it was found that the number of coincidence counts recorded from two photomultipliers, the apertures of which were optically superimposed, was significantly greater than when the light beams were incoherent. Furthermore, the number of these correlated counts was in satisfactory agreement with that predicted by theory. In the second experiment the change in the number of excess coincidences was measured as the degree of coherence of the light was altered by increasing the apparent separation of the photocathodes, and in this case also there was reasonable agreement between theory and experiment. In the final experiment it was shown that there was a significant difference between the number of coincidences observed when the light beams were in identical as opposed to orthogonal polarizations, and this last result especially makes it extremely improbable that the correlation could be caused by some spurious effect, such as plasma oscillations in the source, since the light source itself was found to be completely unpolarized.

I. Introduction

Hanbury Brown and Twiss (1956*a*, 1958*a*) have proposed a new type of stellar interferometer which is based upon the existence of a correlation between the times of arrival of photons at two points illuminated by coherent beams of light.

The reality of this correlation has been checked by a preliminary trial on Sirius (Hanbury Brown and Twiss 1956*b*, 1958*b*) and by two series of tests in the laboratory (1956*a*, 1957), in which the fluctuations in the anode currents of two phototubes were cross correlated in a linear multiplier.

On the other hand a negative result was obtained by Adam, Janossy, and Varga (1955) and also by Brannen and Ferguson (1956), who looked for a correlation by the direct method in which a coincidence counter recorded every event when two photoelectrons were simultaneously emitted from the cathodes of the two phototubes.

It was pointed out by Purcell (1956) and also by Hanbury Brown and Twiss (1956*c*) that these different results were not mutually inconsistent since the sensitivity of these counter experiments was too low by several orders of magnitude to yield a positive result. However, to remove any possible doubt as to the reality of the effect and to show that the correlation can be detected

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directly with a coincidence counter, it was decided to repeat the experiment of Brannen and Ferguson with more sensitive equipment.

A preliminary account of such an experiment has been published elsewhere (Twiss, Little, and Hanbury Brown 1957) and in the present paper we shall give a more detailed discussion of this work together with an account of later experiments which were designed to test the existence of a correlation between photons in alternative ways. A similar test by Rebka and Pound (1957) has been reported, which also led to positive results in reasonable agreement with theory.

II. Basic Theory and Design of the Experiments

Consider the simple case in which a plane wave of randomly polarized light with a very narrow spectral bandwidth B is incident upon two identical photocathodes of quantum efficiency α. Let us assume that we are looking for coincidences between the emission times of electrons from these two photocathodes by means of an idealized coincidence counter which registers a count when, and only when, the difference between these emission times is less than τ_c, the resolution time.

If there were indeed zero correlation between the arrival times of photons we should expect, on the average, to observe $N_r(T_0)$ random coincidences in a time interval T_0 where

$$N_r(T_0)=2\alpha^2N_0^2\tau_cT_0, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and where αN_0 is the average number of electrons emitted in unit time from either photocathode. However, if the arrival times of photons are partially correlated according to the theory given by Hanbury Brown and Twiss (1957) in a paper hereafter referred to as PI, and by Purcell (1956), we should expect to find an additional number $N_c(T_0)$ of coincidence counts where

$$N_c(T_0)=\tfrac{1}{2}\alpha^2N_0^2\tau_0T_0, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and where τ_0, the *coherence time* of the light source defined in Appendix II, is approximately equal to $1/B$, the reciprocal light bandwidth. This expression is only valid as long as $\tau_c/\tau_0\gg1$ and this condition applies to all the experiments discussed in this paper.

In a physically realizable case the incident light beam will consist of a pencil of plane waves and N_c will be reduced by the *partial coherence factor* $\Delta(D,\nu_0)$, defined in PI, which is a function of the parameter

$$\pi aD\nu_0/cR_0,$$

where

a, D are the linear dimensions of the light source and of the photocathode respectively,

R_0 is the distance of the light source,

ν_0 is the midband frequency of the light.

If the photocathodes are not optically superimposed, the number of correlated coincidence counts is yet further reduced by the normalized *correlation factor* $\Gamma^2(d,\nu_0)$ which is a function of the parameters

$$\pi aD\nu_0/cR_0,\ \pi ad\nu_0/cR_0,$$

where d/R_0 is the apparent angular separation of the photocathodes as seen from the light source.

Accordingly, the expected number of correlated coincidences in a practical case may be written

$$N_c(T_0)=\tfrac{1}{2}\alpha^2 N_0^2 \tau_0 T_0 \Delta(D,\nu_0)\Gamma^2(d,\nu_0)\gamma_1\gamma_2, \quad \ldots\ldots\ldots\ldots (3)$$

where γ_1, γ_2 are factors introduced to allow for any loss of correlation in the optical and electronic system and for the effects of dark current and background light.

The ratio ρ_c of the numbers of correlated to random coincidences is therefore given by

$$\rho_c=\frac{N_c(T_0)}{N_r(T_0)}=\frac{1}{4}\frac{\tau_0}{\tau_c}\Delta(D,\nu_0)\Gamma^2(d,\nu_0)\gamma_1\gamma_2, \quad \ldots\ldots\ldots\ldots (4)$$

The r.m.s. fluctuation in the number of random coincidences is given by

$$n_r(T_0)=\left[\overline{\{N_r(T_0)-\overline{N_r(T_0)}\}^2}\right]^{\frac{1}{2}}=\alpha N_0\sqrt{(2\tau_c T_0)}, \quad \ldots\ldots (5)$$

assuming that these coincidences obey a Poisson distribution, and to achieve a significant result is it necessary that the signal-to-noise ratio S/N defined by

$$\frac{S}{N}=\frac{N_c(T_0)}{n_r(T_0)}=\tfrac{1}{2}\alpha N_0\tau_0\Delta(D,\nu_0)\Gamma^2(d,\nu_0)\gamma_1\gamma_2\left(\frac{T_0}{2\tau_c}\right)^{\frac{1}{2}} \quad \ldots\ldots (6)$$

for the case of an unpolarized light source should be appreciably greater than unity.

The quantity $N_0\tau_0\Delta(D,\nu_0)$ is proportional to the average number of photons emitted into unit angle in unit frequency interval from unit area of the source, so that the sensitivity of the experiment is affected only by the brilliance, not by the bandwidth of the light source. This last fact is essential to the success of the experiment since there are two important reasons why the primary photocurrent must be held to a low value. In the first place, the coincidence counter would start to saturate once the average time interval between successive photoemissions was no longer appreciably greater than the resolution time of the counter ; in the second place, the photomultipliers would lose stability at the high gains ($\approx 10^8$) needed to operate the coincidence circuit from a single primary electron, if the anode current were appreciably to exceed 1 mA. These requirements made it impractical for us to employ the high-pressure mercury arc source used in the correlation experiment with a linear multiplier described in PI, and led us instead to the use of an electrodeless low-pressure ^{198}Hg isotope lamp.

If all the parameters in equations (4) and (6) are known it is possible to obtain a direct experimental check of the theory from the ratio of the number of coincidence counts obtained with coherent, as opposed to incoherent, light beams of the same intensity ; this procedure was adopted in the first experiment described in the present paper. However, it is difficult to make an accurate measurement of the effective resolution time τ_c, especially if τ_c is decreased as far as possible in order to improve the sensitivity ; furthermore, there is always the possibility of a loss of correlation in the electronic system, caused, for example, by a difference in the electron transit times through the photomultipliers. The effect of these uncertainties can be eliminated by taking the ratio of the "correlated" coincidences observed with two different values of $\Gamma^2(d,\nu_0)\Delta(D,\nu_0)$ which depends upon the geometry of the system but not upon τ_c, γ, or τ_0; this procedure was followed in the second experiment described in the present paper.

Finally, an experiment was carried out to find the ratio of the number of coincidences observed when the light beams reaching the photocathodes were linearly polarized first parallel and then orthogonal to each other.

III. Description of the Apparatus

(a) The Optical Equipment

A simplified outline of the optical equipment is shown in Figure 1. The light source consisted of an electrodeless ^{198}Hg isotope lamp, of the type developed by Meggers and Westfall (1950), which was excited by an EC 55 triode oscillator producing 1·5 W at 800 Mc/s; the visible area of the source was limited by a circular pinhole in a brass tube which fitted tightly over the discharge tube. In the first experiment the lamp bulb was cooled by an air blast; the tube was

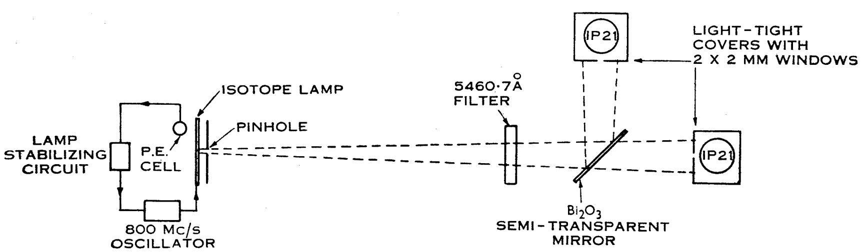

Fig. 1.—Arrangement of the optical system.

later fitted with a water jacket so that a circulating water system could be used. This latter arrangement was temperature-controlled by a thermostat to $\pm\frac{1}{2}$ °C. With a water temperature of 40 °C the light flux was measured to be 0·0013 W/cm^{-2} steradian^{-1}, which is about 30 per cent. of that emitted by the special lamp developed by Forrester, Gudmundsen, and Johnson (1955). To minimize the effects of variations in output light flux, the lamp was provided with a stabilizing circuit as shown in Figure 1.

The 5461 Å line of the mercury spectrum was isolated by a Zeiss monochromatic filter having a peak transmission of 75 per cent., and the beam of light was split by a semitransparent dielectric mirror to illuminate the cathodes of the photomultiplier tubes. In the third experiment a sheet of "Polaroid" with a transmission of 64 per cent. in the accepted polarization was placed over each phototube. One of the "Polaroids" could be rotated between two alternative positions 90° apart.

In all cases the photocathode areas were limited by square apertures 2 by 2 mm and the distance from the pinhole to each cathode, which was of the order of 1–2 m, could be adjusted to an accuracy of ± 1 mm.

In the early experiments one of the photomultipliers was mounted on a slide which enabled it to be moved transversely to the line of sight between two positions 5 mm apart. In one position the apertures of the photocathodes as viewed from the source were optically superimposed, in the other position their apparent angular separation was so large that the incident light beams were effectively uncorrelated.

In the second experiment, which was devised to measure the ratio of the correlation with different degrees of optical superimposition, the position of the fixed phototube could be adjusted in a series of steps so that the amount of optical overlap in the " coincidence " position could be varied from 100 to 0 per cent.

In the final experiment, with polarized light, the phototubes were kept fixed throughout the observations.

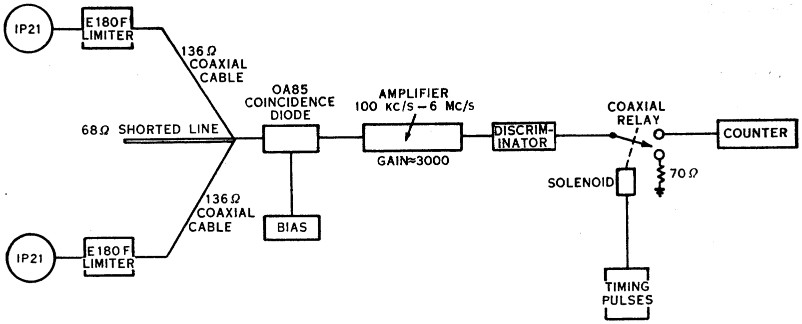

Fig. 2.—Schematic diagram of the coincidence counter.

(*b*) *The Electronic Equipment*

The phototubes were 1P21's working into a high speed coincidence counter essentially the same as that described by Bell, Graham, and Petch (1952), which is shown schematically in Figure 2. One advantage of this circuit is the pulse-limiting action of the pentodes which drive the coincidence diode ; however, to achieve this limiting it is necessary to operate the photomultipliers far in excess of their specified rating. We first tried to maintain an anode voltage of 2300 V, but under these conditions the dark current was excessively high and the stability of the small number of tubes that were tested was very poor. Accordingly, on the advice of Professor Bell (personal communication) we replaced the 6AK5 limiters with the higher gain, sharper cut-off E180F pentodes, which enabled us to obtain adequate limiting when the photomultipliers were operated at 1900 V, and we stabilized the voltages on the last few dynodes of the photomultipliers with voltage regulator tubes as described by Stump and Tallay (1954). With selected 1P21's the dark currents settled down to about 0·1 μA, after the H.T. voltages had been applied for a period of 1 hr, when the photomultipliers were run at room temperature ; the stability of the anode currents was greatly improved also, though occasional small jumps in these output currents were still observed. The stability could no doubt have been improved further by cooling the phototubes, but this complication proved unnecessary.

The output from the coincidence diode was passed through an amplifier having a passband from 0·1 to 6·0 Mc/s and a gain of about 3000 to an amplitude discriminator of the type described by Lewis and Wells (1954), which produced a standard pulse 0·3 μsec in duration and 1·0 V in height to drive a Hewlett-Packard counter-type frequency meter.

The bias on the germanium coincidence diode type 0A85 was maintained at 0·25 V during all the observations. The bias on the amplitude discriminator was set so that it never responded to break-through pulses from the coincidence diode. The screen voltages on the pulse-limiting pentodes were then adjusted so that the amplitude discriminator was never triggered as long as the H.T. voltage was only applied to one of the photomultipliers. Under these conditions a count could only be registered when coincident pulses were received from the pentode limiters, the combined height of which exceeded 0·25 V.

IV. Correlation between Photons Reaching Optically Superimposed Photocathodes

In this first experiment we aimed at getting an experimental check of theory by comparing the number of coincidences observed when the photocathodes were optically superimposed with the number observed when the photocathodes were so widely separated that the light beams incident from the pinhole source were effectively uncorrelated. The effect was expected to be small, of the order of 2 per cent., so to minimize the effects of drift the following experimental procedure was adopted.

(a) *Experimental Procedure*

When the photomultiplier currents had settled down to their steady value the photocathodes were optically superimposed for 2 min, the time interval being accurately determined by pulses from a crystal-controlled clock. At the end of this period the counter was isolated for a 30 sec dead period by a fast-acting relay, and the total number n_{1r} of the coincidences observed in the 2-min period were recorded; at the same time one of the photomultipliers was moved to the uncorrelated position. At the end of this dead period the counter was reconnected by the pulsed relay for a further 2 min and then disconnected for a second dead period, during which the total number n_{2r} of the coincidences observed in the " uncorrelated " position was recorded while the photomultiplier cathodes were optically superimposed again. The whole procedure was repeated ten times in a single run, which therefore took a total of 50 min to complete, allowing for the dead periods. The series N_1 and N_2, defined by

$$N_1 = \sum_{r=1}^{10} n_{1r}, \qquad N_2 = \sum_{r=1}^{10} n_{2r} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

were recorded.

If the average light intensity reaching the movable photocathode had been equal in the coincident and displaced positions, the ratio $(N_1 - N_2)/N_2$ would have given a measure of the ratio of the correlated to the random coincidences. In this initial experiment however, there were differences of the order of 0·5 per cent. between these two intensities, and to eliminate the effect of these a comparison run was made after each observation run with an extra length of 136 Ω cable in one channel of the coincidence counter circuit. This cable introduced an extra delay of 15 mμsec, which was about four times the resolution time of the coincidence counter, so that there was no chance of a count being registered by the simultaneous arrival of photons at the two photocathodes.*

The procedure in the comparison run was identical with that in the observation run and from this second set of results we computed the quantity

$$\rho_I = \sum_{r=1}^{10} n'_{2r} / \sum_{r=1}^{10} n'_{1r} = N'_2/N'_1, \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

which was taken to represent the ratio of the light fluxes incident upon the movable photocathode in the "uncorrelated" and "correlated" positions respectively.

The ratio

$$\rho_c = (N_1\rho_I - N_2)/N_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

then represents the experimental value, for the ratio of the correlated to random coincidences in a given run

$$N_c = \rho_c N_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

may be taken as a corrected estimate of the number of correlated coincidences in the same run.

(b) *Experimental Results*

A total of six complete runs was made using the above procedure. Allowing for dead periods and comparison runs the total observation time was 10 hr, which was spread over two successive nights; it was not possible to observe during the day-time because of interference from other equipment in the building. The results are given in Table 1 and can be combined to give an experimental value

$$\rho_{\exp} = 0{\cdot}0193 \pm 0{\cdot}0016 \text{ (p.e.)} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

for the ratio of correlated to random coincidences. The quantity $N_c/2N_2^{\frac{1}{2}}$ in the final column represents the ratio of the number of correlated coincidences to the r.m.s. uncertainty in the number of random coincidences; the factor 2 arises because N_c was calculated from the four quantities N_2, N_1, N'_2, N'_1 which, it was assumed, were statistically independent.

As a final test a dummy run was made with a tungsten filament light source in place of the isotope lamp; no significant correlation was observed in this case and none was expected when using a source of such low surface brightness per unit bandwidth.

(c) *Calibration of the Equipment*

In order to compare the experimental results with theory an estimate is necessary of the parameters in equation (4), some of which could be measured directly, while others had to be calculated theoretically. Of the two in the

* This was confirmed by tests with a millimicrosecond pulse generator in which pulses were simultaneously applied to the grids of the pentode limiter tubes.

latter class, $\Gamma^2(d,\nu_0)$ could be taken as unity by definition in the case in which the photocathodes were optically superimposed. The values of $\Gamma^2(d,\nu_0)$ at the displaced position and of $\Delta(D,\nu_0)$ were then calculated on the assumption that the photocathode sensitivity was uniform over the 2 by 2 mm aperture* from the theory and numerical results given in PI. For the case of the present

TABLE 1

RATIO OF CORRELATED TO RANDOM COINCIDENCES WITH SUPERIMPOSED PHOTOCATHODES

Run No.	$\rho_c \times 100$ Percentage Ratio of Correlated to Random Coincidences	N_2 Number of Random Coincidences	N_c Number of Correlated Coincidences	$N_c/2N_2^{\frac{1}{2}}$ Signal-to noise Ratio*
1	2·20	135,446	2986	4·03
2	1·70	128,975	2190	3·05
3	1·73	136,250	2369	3·21
4	2·19	99,640	2185	3·46
5	1·94	96,326	1864	3·00
6	1·88	93,848	1761	2·87
Dummy run with white light source	0·22	81,576	185	0·32

* Ratio of correlated coincidences to r.m.s. uncertainty in random coincidences.

experiment in which the light source was a pinhole 0·360 mm in diameter at a distance of 1·25 m,

$$\Delta(D,\nu_0)=0{\cdot}475, \qquad (12)$$

and

$$\Gamma^2(d,\nu_0)<0{\cdot}01 \quad \text{for } d=5 \text{ mm}, \qquad (13)$$

so that the decorrelation at the comparison position is effectively complete.

Since the action of the pentode limiters was not perfect, there were variations in the amplitudes of the current pulses driving the coincidence circuit, and it was found experimentally that the resolution time of the counter was a function of the sum of the amplitudes of these current pulses. To find the effective value of the resolution time of the counter the resolution time associated with a given pulse amplitude must be averaged over the amplitude probability distribution. The procedure for finding these quantities is given in Appendix I; under the conditions of this experiment it was found that

$$\tau_c=3{\cdot}5\times10^{-9} \text{ sec.} \qquad (14)$$

* In the case of the 1P21, in which a system of grid wires lies between the window and the photocathode, the assumption of uniform cathode sensitivity is not rigorously valid, but from the approximate treatment of an idealized case it seemed that the error introduced would not be significant, at least within the accuracy of the present experiment.

As discussed in Appendix II, the "coherence time" τ_0 of the light was measured with the aid of a Kösters interferometer (Bruce 1956) and it was found that

$$\tau_0 = 0 \cdot 73 \times 10^{-9} \text{ sec}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

so that $\tau_0 \ll \tau_c$.

The factor γ_1 in equations (4) and (5), which represents the loss of correlation in the optical and electronic part of the equipment, receives a significant contribution from polarization effects in the semitransparent mirror. As discussed in Appendix III, this loss of correlation can be determined by measurements with an optical polarizer and, for the Bi_2O_3 mirror used in this experiment, contributed a factor of $0 \cdot 935$ to γ_1. Loss of correlation can also arise because of a difference in the electron transit times through the photomultipliers, and, if this were comparable with the mean resolution time of the counter, the effect would be serious, since one would tend to lose coincidences between small current pulses. However, in view of the fact that the total electron transit time through a 1P21 with an anode voltage of 2000 V is probably less than 10^{-8} sec, it seems reasonable that the differential delay should be appreciably less than $3 \cdot 5 \times 10^{-9}$ sec, the mean resolution time of the counter.

The factor $\gamma_1\gamma_2$ in equations (4) and (5) represents the fraction of random counts due to coincidences between pulses in one phototube and dark current or stray light pulses in the other. A typical value of γ_2, under the conditions of the present experiment, was $0 \cdot 92$, so that we took

$$\gamma_1\gamma_2 = 0 \cdot 86 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

as the factor representing the loss of correlation in the complete equipment.

(*d*) *Comparison with Theory*

Substituting into equation (4) the values of the various parameters given in equations (12)–(16), gives a theoretical value for ρ,

$$\rho_{\text{theor}} = 0 \cdot 0207. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Perhaps the most uncertain quantity in this calculation is τ_c, the mean resolution time of the counter, though there could have been some additional loss of correlation in the coincidence counter for which no allowance has been made; furthermore, the coherence time τ_0 was a function not only of ambient temperature at the light source but also of the output power of the driving oscillator. It is difficult to estimate accurately the combined magnitude of these potential errors, but we estimate that it is unlikely to be appreciably greater than $\pm 0 \cdot 002$, even when we allow for the variations in sensitivity over the photocathode apertures.

A comparison between equations (11) and (17) shows that they are in satisfactory agreement, the difference between the experimental and theoretical values being just less than the probable error.

V. The Variation in Photon Correlation as a Function of Photocathode Displacement

In order to obtain a check on the theory which depended on as few of the parameters of the equipment as possible, an experiment was performed in which the ratio of the number of correlated to random coincidences was compared for several values of the quantity $\Gamma^2(d,\nu_0)\Delta(D,\nu_0)$.

It would have been preferable, in a number of ways, to have kept the photocathodes superimposed, so that $\Gamma^2(d,\nu_0)$ stayed equal to unity, and varied $\Delta(D,\nu_0)$ by varying the apparent angular size of the pinhole source. For one thing, this procedure would have eliminated any possibility that the photocathodes were not illuminated by identical parts of the source. However, in order to vary $\Delta(D,\nu_0)$ appreciably, say by a factor of 4 to 1, it would be necessary to vary the apparent angular size of the source, and, therefore, of the light flux received by the photocathodes by factors of at least 3 to 1 and 9 to 1 respectively. Such a flux change would either have overloaded the photomultipliers at one end of the scale or would have led to an excessively low value of the signal-to-noise ratio at the other.

Accordingly, it was decided to keep $\Delta(D,\nu_0)$ constant and change $\Gamma^2(d,\nu_0)$ by moving the fixed phototube in steps away from the position of 100 per cent. optical overlap.

To improve the signal-to-noise ratio, the temperature of the light, which was controlled by the temperature of the circulating water, was adjusted to maximize $N_0\tau_0$, that is, to maximize the product of the light flux and the "coherence time".* At the same time the resolution time of the counter was decreased by cutting down the length of short-circuited line by a factor of 0·6. As it turned out this move was probably mistaken, since the gain in signal-to-noise ratio was more than offset by a decrease in the stability of the anode photocurrents caused by the high currents, which led to an appreciable increase in the drift in the number of random coincidences. Under these circumstances the probable error in the measurements was almost certainly larger than that set by the fundamental random fluctuations.

The experimental procedure was similar to that adopted in the first experiment, with two major exceptions. Firstly, the degree of optical superimposition in the "correlated" position could be varied by changing the position of that photocathode which remained fixed throughout a given run, and measurements were taken with the cathodes displaced with respect to each other by 0, 1, 2, and 4 mm, that is, by 0, $\frac{1}{2}$, 1, and 2 times the cathode aperture. Secondly, considerable care was taken to equalize the light intensity reaching the movable photocathode in its two alternative positions. By comparing the number of single photon counts recorded by the frequency meter with only a small bias on the coincidence diode and with zero H.T. voltage on the movable photomultiplier, it was concluded that the intensity ratio was equal to unity within $\frac{1}{4}$ of 1 per cent. This result was confirmed by a dummy run with the extra

* The "coherence time" was found approximately in this experiment by measuring the mirror separation in the Kösters interferometer at which the fringes first became invisible.

cable inserted in one arm of the coincidence circuit, in which no significant difference was detected between the counts received in the two positions of the photocathode. Accordingly, it was decided to omit the comparison run which followed every observation run in the first experiment. In each run the photocathode was now alternated a total of 24 times between the " correlated " and " uncorrelated " positions, so that, allowing for the 30-sec dead periods, the total duration of the observations at any given position of the fixed photocathode was 1 hr.

The experimental results are given in Table 2, the symbols in the first three columns having the same meaning as those in Table 1. In the fourth column we have given the theoretical values for $\Gamma^2(d)$, which were calculated from the results given in PI, for the case appropriate to the present experiment for which :

$$\pi aD/\lambda R = 3\cdot 00, \quad D = 2 \text{ mm}.$$

To find the experimental values of $\Gamma^2(d)$ as a function of d, the unknown parameter

$$(\tau_0/4\tau_c)\alpha N_0\gamma_1\gamma_2\Delta(D,\nu_0)$$

TABLE 2

CORRELATION BETWEEN PHOTONS AS A FUNCTION OF CATHODE SEPARATION

d Cathode Separation (mm)	N_R Number of Random Coincidences	N_c Number of Correlated Coincidences	$\Gamma^2_{\text{theor}}(d)$	$\Gamma^2_{\text{exp}}(d)$	p.e.	$\dfrac{\Gamma^2_{\text{theor}}(d)-\Gamma^2_{\text{exp}}(d)}{\text{p.e.}}$
0	350,871	7279	1·0	1·078	±0·085	0·918
1	292,707	3524	0·714	0·621	±0·092	1·01
2	307,412	885	0·251	0·152	±0·091	1·09
4	259,526	—516	0·013	—0·102	±0·097	1·18

was given the value of 0·0193 which gave the best mean squares fit between the theoretical and experimental values of $\rho_c.\Gamma^2{}_{\text{exp}}(d)$, which is plotted in column 5 of Table 2 and shown graphically in Figure 3, was then calculated from the experimental values for ρ_c using equation (4). The probable errors given in column 6 were found on the assumption that the random counts obeyed a Poisson distribution. As may be seen, the residuals are all of the magnitude of the probable error so the agreement with theory is satisfactory, especially in view of the decreased stability in the anode currents of the photomultipliers, which were nearly four times larger than in the first experiment. It may be noted, incidentally, that the lack of a significant difference between the counts registered in the " correlated " and " uncorrelated " position at the maximum cathode spacing is further evidence that the light intensities at the two positions of the movable photocathode were equal.

VI. Experiments with Polarized Light Beams

As long as the light source emits unpolarized light, as was confirmed to be the case for the ^{198}Hg lamp, it is known, from classical radiation theory, that no interference phenomena can arise from the interaction of light beams with mutually orthogonal polarization. In particular, there should be no correlation between the arrival times of photons in orthogonally polarized beams. Hence, if the light source is sufficiently brilliant, there should be a significant difference between the number of coincidences recorded when the light beams reaching the two photocathodes are in identical as opposed to orthogonal states of polarization. To confirm that this is so, a final experiment was carried out with polarized light beams.

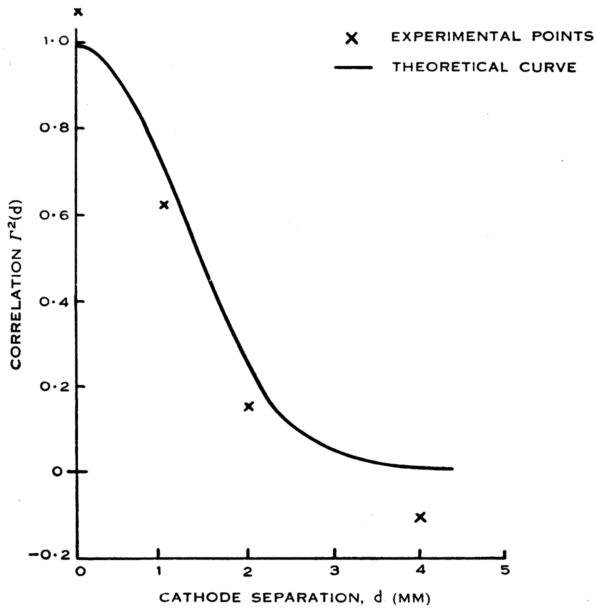

Fig. 3.—The change in correlation with separation of the photocathodes.

A complication now arises because the ratio of the light intensities in the reflected and transmitted beams is very different when the light is vertically from when it is horizontally polarized. However, this ratio should be equal in the case when the orthogonal states of linear polarization are inclined at $\pm 45°$ to the vertical. Accordingly, the optical part of the equipment was set up as follows.

(a) *Setting up the Optical Equipment*

The apertures over the two photocathodes as seen from the source were optically superimposed and a piece of plane " Polaroid " was placed over each photocathode in a rotatable mount. The " Polaroid " in the reflected beam had two adjustable stops between which it could be rotated. The " Polaroid " in the direct beam, which could be fixed at an arbitrary angle to the vertical, was first rotated to maximize the direct photocurrent ; in this position, as can easily be shown, the incident light was polarized in the plane of the mirror. The " Polaroid " was then rotated through 45° and fixed in position. At this stage an auxiliary " Polaroid " was placed over the light source and rotated until the photocurrent in the direct light beam was zero ; the " Polaroid " in the reflected beam was then rotated until this photocurrent was also zero, and one of the adjustable stops was set up at this position in which the light beams incident upon the photocathodes were parallel-polarized. The " Polaroid " was then rotated through approximately 90° : the exact position of the second stop, which determined the " uncorrelated " position of the movable " Polaroid ", was adjusted to ensure the equality of the light intensities reaching the photocathode in the " correlated " and " uncorrelated " positions when the auxiliary " Polaroid " over the light source had been removed.

(*b*) *Operational Procedure and Results*

The operational procedure was very similar to that used in the earlier experiments, the only difference being that the phototubes were kept fixed in position throughout the run while the " Polaroid " in the reflected beam was rotated in successive dead periods between the " correlated " and " uncorrelated " positions. Two runs were made, each of 50 min total duration, and the

Table 3

Difference between Coincidence Counts with Parallel and Orthogonally Polarized Light Beams

Run No.	N_R Number of Random Coincidences	N_c Number of Correlated Coincidences	$\rho\times 100$ Percentage Ratio of Correlated to Random Coincidences	$N_c/\sqrt{2}N_R^{\frac{1}{2}}$ Signal-to-noise Ratio
1	122,848	2512	2·0	5·0
2	104,282	1999	1·9	4·5

results are shown in Table 3. As in the other experiments the fractional difference was of the order of 2 per cent., though a direct comparison with theory was not possible in this case since all the relevant parameters of the optical and electronic equipment were not known.

It may be noted that the number of random coincidences was very much less than in the second experiment, due to the fact that the temperature of the lamp was stabilized at an appreciably lower temperature. Under these conditions the stability of the anode currents of the phototubes was much improved and the drifts were also greatly reduced. Indeed, on the second run the r.m.s. variance in the individual readings, defined by

$$\left\{\frac{\sum_{r=1}^{10}[(n_{1r}-n_{2r})-N_c/10]^2}{10}\right\}^{\frac{1}{2}}$$

was less than the standard deviation, so that in this case, at least, no drifts were detectable.

VII. Discussion and Conclusions

In all the experiments described in the present paper there was positive evidence of a time correlation between photons in coherent beams of light. In the first experiment it was shown that the absolute magnitude of the effect agreed with that predicted by theory within the limits of accuracy of the measurements and in the second experiment it was shown that the correlation decreased in the expected manner as the separation of the photocathodes was increased. These results confirm the more accurate measurements described in PI, which were carried out by a quite different technique.

The objection has been raised (Brannen and Ferguson 1956) that the correlation reported in our original paper (Hanbury Brown and Twiss 1956*a*) might have been caused by spurious intensity fluctuations in the source produced, for example, by plasma oscillations. However, as was pointed out by Hanbury Brown and Twiss (1957) in their discussion of the results obtained with the linear multiplier technique, the quantitative agreement with theory over a whole range of cathode spacings makes this explanation very unlikely, and this conclusion is greatly strengthened by the results reported in the present paper, which were carried out with a radically different technique and with a quite different type of light source. In particular, it is virtually impossible to interpret the results with polarized light, reported above, in terms of spurious effects in the source, in view of the fact that the light beam from the source was found to be completely unpolarized.

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Appendix I

Measurement of the Resolution Time of the Counter

The voltage pulses at the anodes of the photomultipliers have a large spread in amplitude owing to fluctuations in the gain of the electron multipliers. Since the action of the pentode limiters was not perfect there was also a spread in the amplitude of the current pulses at the coincidence diode. It was found experimentally that the resolution time of the counter depended upon the sum of the amplitudes of the current pulses in the pentode limiters. Accordingly, to find the value for τ_c which must be inserted in equations (4) and (6) in the text, one has to measure, firstly, the resolution time as a function of equal pulse amplitudes in the two channels of the coincidence counter and, secondly, the amplitude probability distribution of these current pulses.

To determine the first quantity we used a millimicrosecond pulse generator, which delivered negative-going pulses through a tapped coaxial line to the grids of the pentode limiters. With the bias on the coincidence diode set at the operating level of 0·25 V the amplitude of the pulse generator was adjusted to the level at which the resolution time of the counter was reduced to zero, that is, to the level at which no counts were found for any position of the tap on the coaxial line. One of the pentodes was then cut off and the bias on the coincidence diode was reduced to the level of 0·09 V, at which counts were observed from the current pulses in a single pentode.

The resolution time of the counter was then determined from the distance through which the tap could be moved before the amplitude discriminator ceased to trigger for a range of pulse heights from the generator. The amplitude of these pulses was defined by the bias V on the coincidence diode which just prevented any counts registering from the pulses in a single pentode limiter, where $0\cdot09 \leqslant V \leqslant 0\cdot25$. Finally, the relative number of pulses produced by a single phototube at its operating H.T. voltage, which triggered the counter when the coincidence diode bias was V, was recorded and from this data τ_c was evaluated assuming no significant difference between the resolution time of pulses from the photomultiplier and from the pulse generator respectively. No direct check was made of the truth of this, but, from the fact that the transit time spread in a 1P21 with 5000 V in the anode is only $0\cdot25\times10^{-9}$ sec (Lewis and Wells 1954), it may be concluded that the spread with 2000 V is still only $0\cdot4\times10^{-9}$ sec, which is only a little more than one-tenth the resolution time of the counter. The rise time of the pulse from the generator was also considerably faster than $3\cdot 5\times 10^{-9}$ sec, so the error introduced by any difference in the rate of rise of the two sets of pulses is not likely to be large, especially in view of the fact that the pulse length at the coincidence counter is determined largely by the length of the short-circuited line.

Appendix II

The Measurement of the Coherence Length of the Light

The quantity τ_0, the so-called coherence time of the light, can be defined (Purcell 1956) by the equation

$$\tau_0=\int_{-\infty}^{\infty}|g^2(\tau)|\,d\tau,$$

where $g(\tau)$, the autocorrelation function of the intensity of the incident light, is also the Fourier transform of $f(\nu-\nu_0)$, the normalized spectral response of the light centred on frequency ν_0, so that

$$g(\tau)=\int_{-\infty}^{\infty}f(\nu-\nu_0)\exp 2\pi i\nu\tau d\nu.$$

Instead of calculating $g(\tau)$ by first measuring the spectral response of the light source, a direct measurement was made on a Kösters interferometer (Bruce 1956), with which one can determine $g(2d/c)$ from the visibility of the fringes observed with a mirror spacing d. The fringe pattern was recorded photographically as a function of d at intervals of $0\cdot 25$ in., the visibility curve was obtained from the photographs by means of a microphotometer, and the area under the curve was calculated numerically. As stated in the text, the value found in this way for τ_0 under the conditions of the first experiment was $0\cdot 73\times 10^{-9}$ sec.

Appendix III

The Loss of Correlation due to Polarization Effects in the Semitransparent Mirror

Consider an unpolarized light beam incident on a semitransparent mirror which splits the light into a direct and a reflected beam. The light from the source can be decomposed into two independent orthogonal linearly polarized components of equal intensity such that the electric vector in one polarization state lies in the plane of the mirror. Let us assume that the intensities of the light in the direct beam in these two states of polarization are $\alpha_1^2 I_0$ and $\alpha_2^2 I_0$ respectively so that the associated intensities in the reflected beam are $(1-\alpha_1^2)I_0$ and $(1-\alpha_2^2)I_0$ if we ignore the effects of absorption in the mirror.

The expected number of correlated coincidence counts is then proportional to

$$\alpha_1^2(1-\alpha_1^2)I_0^2+\alpha_2^2(1-\alpha_2^2)I_0^2.$$

However, if there had been no polarization effects in the mirror this number would have been proportional to

$$(\alpha_1^2+\alpha_2^2)(2-\alpha_1^2-\alpha_2^2)I_0^2/2,$$

so that γ_1, the factor representing the loss of correlation in the optical part of the equipment, is given by

$$\gamma_1=\frac{2[\alpha_1^2(1-\alpha_1^2)+\alpha_2^2(1-\alpha_2^2)]}{(\alpha_1^2+\alpha_2^2)(2-\alpha_1^2-\alpha_2^2)},$$

which may be written

$$\gamma_1=\frac{2(1+\sigma_1\sigma_2)}{(1+\sigma_1)(1+\sigma_2)},$$

where $\sigma_1=\alpha_1^2/\alpha_2^2$, $\sigma_2=(1-\alpha_1^2)/(1-\alpha_2^2)$.

The quantities σ_1, σ_2 were determined by placing a sheet of "Polaroid" in front of the light source and measuring the ratios of the maximum to minimum photocurrents both in the direct and in the reflected beams as the polarizer was rotated. For the Bi_2O_3 semitransparent mirror used in the experiments described in the text it was found that $\gamma_1=0\cdot935$.